LINEAR materials:

$$\vec{M} = \chi_m \vec{H}, \quad \mu = \mu_0(1+\chi_m), \quad \text{and} \Rightarrow \vec{B} = \mu \vec{H}.$$

Usual warning: still can't assume $\text{div}\vec{H} = 0$, since μ changes at boundaries and $\text{div}\vec{M}$ is large (∞) there.

Ferromagnetism: Dipole moments align with the field, but unlike paramagnetism, the adjacent dipoles have very strong mutual interactions: in a local region (generally very large on atomic scales, but still microscopic), all dipoles are pointing the same direction ⇒ local field in one of these domains is very large (teslas).

Domain boundaries have opposite field on either side, ← see figure in Griffiths

a macroscopic scale the different domains average out to a macroscopic magnetization that is smaller than the local $\vec{M}$ where all spins are aligned. Say the macroscopic $\vec{M} = 0$. What happens when an external field turns on? Domain boundaries start to move: aligned domains grow, antialigned domains shrink. Net magnetization $\vec{M} = ?$

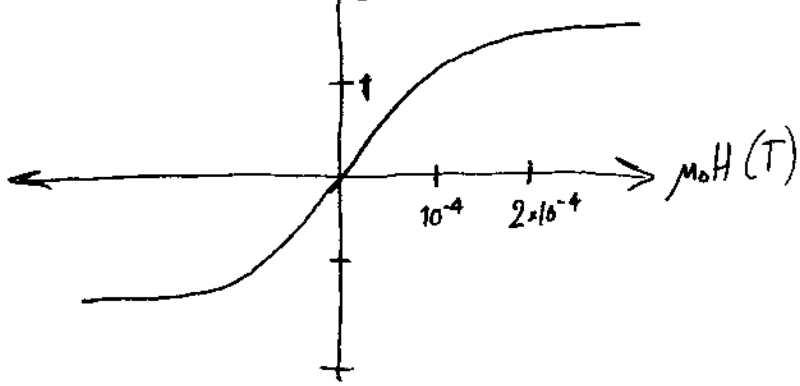
⟶ Much less straightforward than paramagnetism.

For small magnetizations and fields, can treat as linear:

$$\frac{\mu}{\mu_0} \sim 10^4.$$

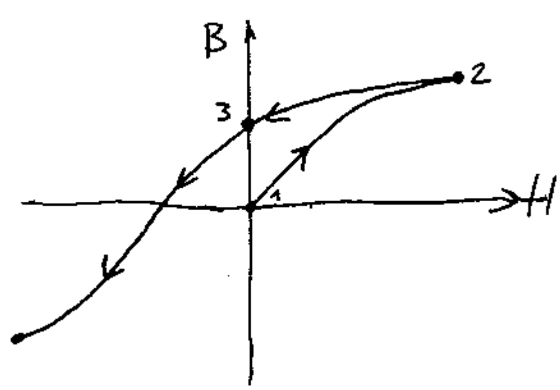
At larger values of H, the material saturates (misaligned domains are gone, so $\vec{M}$ is maximized)
Usually expressed as B vs. H curve: (more physical is $\vec{M}$ vs. $\vec{H}$, but $\vec{H}$ small.)

$\frac{\mu}{\mu_0}$ is slope



So μ is actually a shrinking function of $|H|$, not constant.

This B-H curve is characteristic (approximately) of a material called "soft iron" — an unusual material.
Most ferromagnets exhibit hysteresis: recovery from a near-saturation magnetization is incomplete: B-H curve is thus multivalued. $\vec{B}$ depends on $\vec{H}$ and history:



Start with $\vec{H}=\vec{M}=0$ (1)
- Go to saturation (2)
- then return $\vec{H}$ to 0 (3)
- $\vec{B}$ (really $\vec{M}$) does not return to zero!

Hysteresis is responsible for permanent magnets

Electromagnets: for large field, best to wrap coil around a well-shaped soft iron. What happens to flux away from coil?