$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \qquad \text{div}\vec{H} = \text{div}\vec{M}, \quad \text{curl}\,\vec{H} = \vec{J}_{free}$$

Boundary conditions: $H_{\parallel(in)} - H_{\parallel(out)} = K_{free}$

$$H_{\perp(in)} - H_{\perp(out)} = M_{\perp(in)} - M_{\perp(out)}$$

Paramagnetism: Material contains dipoles (usually unpaired electrons) that partially align with the magnetic field, yielding to the $\vec{m}\times\vec{B}$ torque. So $\vec{M} \propto \vec{B}$, and in same direction.

Diamagnetism: Electron orbit is affected by an external $\vec{B}$. A bogus ~~quasi~~classical argument: Assume a circular orbit of an electron (charge $-e$), radius R, speed v. Time-averaged current is

$$I = \frac{-ev}{2\pi R}$$

So dipole moment is $\vec{m} = I\vec{a} = \frac{-ev}{2\pi R}\pi R^2\hat{z} = \underbrace{-\frac{1}{2}evR\hat{z}}_{}$

where $\hat{z}$ is the rotation axis.

Note — for a given radius, $|\vec{m}| \propto |\vec{v}|$.

opposite to rotation axis (since $q = -e$)

What direction is additional force from $\vec{B}$ on this "current?"

$$\vec{F} = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}$$

if orbital axis is $\hat{z}$, then $\vec{v} = v\hat{\phi}$. For $\vec{B} = B\hat{z}$,

$-e\vec{v}\times\vec{B} = -evB\hat{r}$ inward. So an external $\vec{B}$ will cause orbit to speed up, increasing $\vec{m}$ in a direction against $\vec{B}$.

Diamagnetism is weaker than paramagnetism, so <u>net</u> effect is paramagnetic unless paramagnetism vanishes (such as when all electrons are paired).

For dia/paramagnetic materials, $\vec{M} \propto \vec{B}$: <u>linear</u> materials.
Can write as:

$$\vec{M} = \varkappa \vec{B}, \quad \text{where } \varkappa > 0 \text{ (paramagnetic) or } \varkappa < 0 \text{ (diamagnetic).}$$

Now, $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

$$= \frac{\vec{B}}{\mu_0} - \varkappa \vec{B}$$

$$= \vec{B}\left(\frac{1}{\mu_0} - \varkappa\right)$$

Can also write as $\vec{M}$ in terms of $\vec{H}$:

$$\vec{M} = \varkappa \vec{B} = \varkappa \frac{\vec{H}}{\left(\frac{1}{\mu_0} - \varkappa\right)}$$

$$\chi_m \equiv \frac{\varkappa}{\left(\frac{1}{\mu_0} - \varkappa\right)} \qquad \underline{\text{magnetic susceptibility}}$$

and $\vec{M} = \chi_m \vec{H}$ ⟵ most common formulation!

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right) = \mu_0 \left(1 + \chi_m\right) \vec{H}$$

call $\mu = \mu_0 (1 + \chi_m)$ magnetic <u>permeability</u>.

$\mu > \mu_0$ for paramagnetic materials

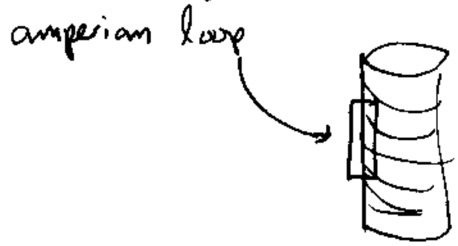
$< \mu_0$ diamagnetic

$\vec{B} = \mu \vec{H}$ can use much the same way as $\vec{D} = \epsilon \vec{E}$ in electrostatics.

χ_m is usually $\sim 10^{-6} - 10^{-4}$, so these effects are generally small. Weird exception is gadolinium: $\chi_m = 0.5$!

Example: Infinite solenoid, where inside filled with a substance of suscep. χ_m.

Use symmetry to find $\vec{H}$:



$(H_{in} - H_{out}) z = nI$; $H_\phi = H_r = 0$ by symmetry

So $\vec{H}_{in} = nI\hat{z}$

and $\vec{B}_{in} = \mu nI\hat{z} = \mu_0(1+\chi_m)\hat{z}$

$\vec{K}_b = \vec{M}\times\hat{n} = \chi_m(\vec{H}\times\hat{n}) = \chi_m nI\hat{\phi}$