P3310 LECTURE 38

Magnetic fields in matter: Magnetization $\vec{M}$:

Recap: $\vec{M}$ = magnetic dipole moment/volume

Can interpret net effect of these dipoles as a bound current:

Volume current $\vec{J}_b = \text{curl}\,\vec{M}$

surface current $\vec{K}_b = \vec{M} \times \hat{n}$

So, $\vec{B}$ (and $\vec{A}$) is a result of the bound currents plus any free current $\vec{J}_{free}$ that might be present —

So, rewriting Ampere's Law with this knowledge:

$$\text{curl}\,\vec{B} = \mu_0 \vec{J}$$
$$= \mu_0 (\vec{J}_f + \vec{J}_b)$$
$$\text{curl}\,\vec{B} = \mu_0 \vec{J}_f + \mu_0\, \text{curl}\,\vec{M}$$

Convenient to define a magnetic equivalent to $\vec{D}$ in electrostatics by grouping the curls together:

$$\text{curl}\,(\vec{B} - \mu_0 \vec{M}) = \mu_0 \vec{J}_f$$

Let $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M} \implies \boxed{\text{curl}\,\vec{H} = \vec{J}_f}$ Ampere's Law in magnetized materials

Stokes's theorem gives integral version: $\oint \vec{H} \cdot d\vec{l} = I_{f\,encl}$

$\implies$ integral of $\vec{H}$ around a loop = free current through the loop.

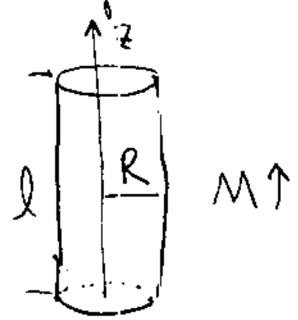
Weird things about $\vec{H}$:

1) Engineers call $\vec{H}$ the "magnetic field." It is not the magnetic field! Often, if $\vec{H}$ is called the field, then $\vec{B}$ is called "flux density" or (worse) "induction" (which has unrelated meanings). We use same terminology as Griffiths: $\vec{B}$ is the magnetic field; $\vec{H}$ is just "$\vec{H}$."

2) No free current does not mean $\vec{H}=0$, just $\text{curl}\,\vec{H}=0$. As in electrostatics, where $\rho_{free}=0$ means $\text{div}\,\vec{D}=0$, but $\text{curl}\,\vec{D}$ generally $\neq 0$: $\text{div}\,\vec{B}=0$ always, but $\text{div}\,\vec{H} = -\text{div}\,\vec{M} \neq 0$ in general.

Free current comes from charges that are free to move: current in a conductor (i.e. a wire hooked up to a battery, or a beam in a cathode ray tube). Some materials (conducting ferromagnets) may have large free and bound currents.

Example: bar magnet. Object with magnetization constant:



$\vec{M} = M_0\hat{z}$

Where are bound currents? Surface currents:

$\vec{K}_b = \vec{M}\times\hat{n}$ $\qquad \hat{n} = +\hat{z}$ top, $-\hat{z}$ bottom, $\hat{r}$ on walls.

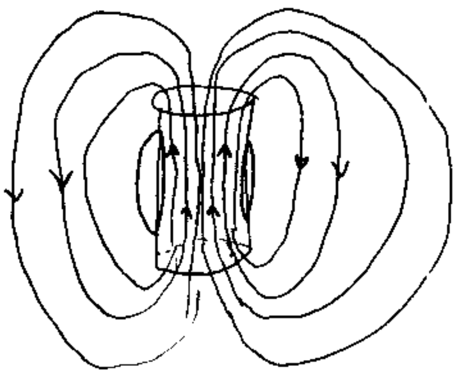
$\vec{M}\times\hat{z}=0$, so no current on top & bottom

$\vec{M} \times \vec{r} = M_0 \hat{z} \times \hat{r} = M_0 \hat{\phi}$

so $\vec{K}_b = M_0 \hat{\phi}$ on wall

$\Rightarrow$ Same field as a finite solenoid.

What do $\vec{B}$ and $\vec{H}$ lines look like?



$\vec{B}$ = vaguely dipole

$\vec{H}$: Consider boundary conditions, with analogy to $\vec{D}$:

$$\underbrace{\text{curl } \vec{H} = \text{curl}(\vec{J}_f) = 0}_{\substack{\text{Parallel component is} \\ \text{continuous at a boundary} \\ \text{where no } \vec{J}_{free}}} \; ; \quad \underbrace{\text{div } \vec{H} = -\text{div}(\vec{M})}_{\substack{\text{Perpendicular component changes at} \\ \text{a boundary if } M_\perp \text{ changes}}}$$

So $\vec{H}$ field lines terminate where $M_\perp$ terminates, but $\vec{B}$ field lines never terminate.

The roles of perpendicular and parallel components are somewhat reversed when comparing $\vec{D}$ to $\vec{H}$.

At boundaries:

$\rightarrow$ top & bottom: No surface current, so:

$\vec{B}$ is continuous

$\vec{H}_{up} = \frac{\vec{B}}{\mu_0}$, $\vec{H}_{dn} = \frac{\vec{B}}{\mu_0} - M_0 \hat{z}$: $H_{up\,z} - H_{dn\,z} = +M_0$

→ wall: $\vec{B}$ discontinuous: ampere's law tells us
$B_{in\,z} - B_{out\,z} = \mu_0 K_b = \mu_0 M_0$; $B_{in\,r} - B_{out\,r} = 0$.

$\vec{H}$ constraint:

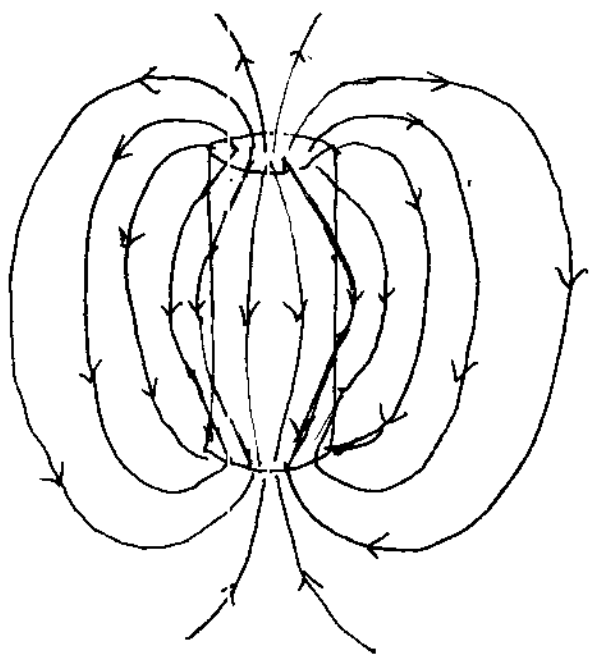
$$H_{in\,z} - H_{out\,z} = \frac{1}{\mu_0}\left(B_{in\,z} - B_{out\,z}\right) - \left(M_{in\,z} - \underbrace{M_{out\,z}}_{0}\right)$$

$$= M_0 - M_0 = 0.$$

$H_{in\,r} - H_{out\,r} = 0$ So $\vec{H}$ continuous at boundary.

Inside, $B_z < \mu_0 M_0$ (since $\mu_0 M_0$ would be field if the bar were infinite), so $H_{z\,in} = \frac{B_{z\,in}}{\mu_0} - M_0 < 0$.

$\vec{H}$ field lines resemble: (Note $\vec{H} = \vec{B}$ outside)



→ Mathematically, looks like $\vec{E}$ field lines from two uniformly charged discs.