P3310 LECTURE 35 NOTE: PROBLEM SET 10 DUE FRIDAY

Summary of fields, currents, potentials and their relations:

$\uparrow$ derivatives $\downarrow$ integrals

$$\vec J$$

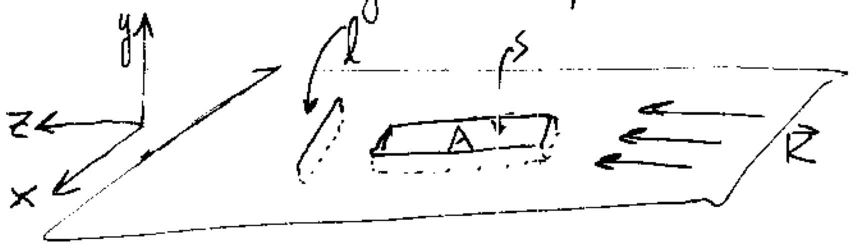
$$\mu_0 \vec J = \text{curl}\,\vec B \qquad \vec B = \frac{\mu_0}{4\pi}\int d^3r' \, \frac{\vec J(\vec r\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\nabla^2 \vec A = \mu_0 \vec J \qquad \vec B \qquad \vec A = \frac{\mu_0}{4\pi}\int d^3r \, \frac{\vec J(\vec r)}{\mathscr{r}}$$

$$\vec B = \text{curl}\,\vec A \qquad \text{(not common)}$$

$$\vec A$$

More on boundary value problems: consider surface current.

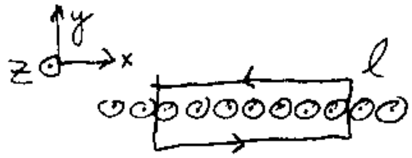


$\vec E_\perp$ discontinuous at surface charge. Check $B_\perp$ at current:
Gaussian surface above:

$$\oint_S \vec B \cdot d\vec A = \left(B_y\,\text{up} - B_y\,\text{dn}\right) A = 0$$

$\Rightarrow B_z$ is <u>continuous</u> at the surface current.

What about $B_\parallel$? Amperian loop l:



$\vec K \perp$ surface $\qquad B_{up\,x}\, l - B_{dn\,x}\, l = \mu_0 I = K l \mu_0$

If orient loop so surface $\parallel \vec K$, then $I = 0$, so
$B_{up\,z}\, l - B_{dn\,z}\, l = I = 0$.

Only component with discontinuity is therefore B_x, which is in the $\vec{K}\times\hat{n}$ direction. In general, at a surface current:

$$\vec{B}_{up} - \vec{B}_{dn} = \mu_0 \vec{K}\times\hat{n}. \qquad (\text{where "up"} \equiv \hat{n})$$

Now, look at boundary conditions on $\vec{A}$:

$\nabla^2\vec{A} = \mu_0 \vec{J}$ so, in analogy with $\nabla^2 V = \frac{\rho}{\varepsilon_0}$ we can say that $\vec{A}$ is continuous across a surface current:

$$\vec{A}_{up} = \vec{A}_{dn}$$

Also, there is an analogous derivative boundary condition:

$$\frac{\partial \vec{A}_{up}}{\partial n} - \frac{\partial \vec{A}_{dn}}{\partial n} = -\mu_0 \vec{K}$$

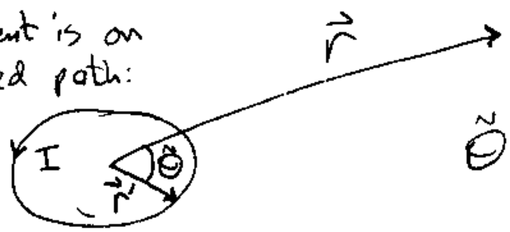
Note that boundary value problems in $\vec{A}$ are not as useful as in electrostatic V. Why? Poisson/Laplace equations still apply, but the physical problems that can be attacked are fewer: there's no convenient equivalent to a conductor (an equipotential surface), so rarely do we set up a problem where, say, "$\vec{A} = \vec{A}_0$" on a surface. Generally only have the derivative boundary conditions to work with.

Multipoles in magnetostatics: Since $\vec{A}$ obeys Laplace equation in regions with no current, then we can use same types of expansions/sep of variables techniques to describe $\vec{A}$ as V at large distances.

Key difference: monopole moment is always zero!

Use same expansion for $\frac{1}{\mathscr{r}}$: $\frac{1}{\mathscr{r}} = \frac{1}{\sqrt{r^2 + r'^2 - 2rr'\cos\tilde{\theta}}}$

where current is on a single closed path:



$\tilde{\theta}$ is angle btw $\vec{r}$ and $\vec{r}'$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(\cos\tilde{\theta})$$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\oint \frac{1}{\mathscr{r}}\, d\vec{l}' = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty} r^{-(n+1)}\oint d\vec{l}'\, r'^n P_n(\cos\tilde{\theta})$$

Lowest term is dipole:

$$\vec{A}_{dip} = \frac{\mu_0 I}{4\pi r^2}\oint d\vec{l}'\, r'\cos\tilde{\theta}$$

$$= \frac{\mu_0 I}{4\pi r^2}\oint (\hat{r}\cdot\vec{r}')\, d\vec{l}' =$$

$$= \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$ where $\vec{m} = I\int d\vec{A} = I\vec{A}$ "vector area" of current loop

$\vec{m}$ = "magnetic dipole moment"