P3310 LECTURE 31

Recap from week before spring break:

$\text{div}\,\vec{B} = 0$ ← Empirical   Force $\vec{F} = Q\vec{v}\times\vec{B}$ where $\vec{v}$ velocity

$\text{curl}\,\vec{B} = \mu_0 \vec{J}$ ← Ampere's Law

Consequence:

→ $$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int d^3r' \, \frac{\vec{J}(\vec{r}')\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

→ $\oint d\vec{l}\cdot\vec{B} = \mu_0 I_{enc.}$ (just Stokes's theorem)

Using Ampere's Law:

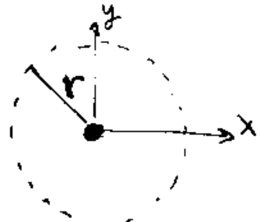
Applications are similar to those of Gauss's Law: calculating a magnetic field from a known current distribution when there is some symmetry to help.

Like Gauss's Law, Ampere's Law is always <u>true</u> but <u>not</u> always <u>useful</u>.

Examples:

Field from a long wire, current I:

$\hat{z}\odot$



Know that by symmetry, $\vec{B}$ can't have any $\hat{z}$ component. By $\text{div}\,\vec{B}=0$, can't have any $\hat{r}$ component. So $\vec{B} = B(r)\,\hat{\phi}$

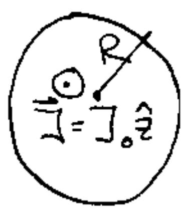
To find $\vec{B}(\vec{r})$, take an "Amperian loop" of radius r around the wire: $d\vec{l} = r\,d\phi\,\hat{\phi}$

So $\displaystyle\int d\vec{l}\cdot\vec{B} = \int_0^{2\pi} d\phi\; r B(r) = 2\pi r B(r) = \mu_0 I$

so $\displaystyle\vec{B}(\vec{r}) = \hat{\phi}\,\frac{\mu_0 I}{2\pi r}$

$\Rightarrow$ same answer as from Biot-Savart integral.

Now, consider uniform current density in a thick wire:



What is $\vec{B}$ everywhere?

Symmetry arguments apply identically: $\vec{B} = B(r)\,\hat{\phi}$.

So $\displaystyle\vec{B}(\vec{r}) = \hat{\phi}\,\frac{\mu_0 I_{encl.}}{2\pi r}$

What is I_{encl}? If outside wire, $I_{encl} = I_{wire} = \pi R^2 J_0$

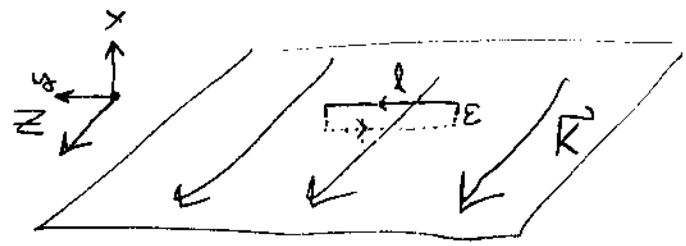
So $\displaystyle\vec{B}_{outside} = \hat{\phi}\,\frac{\mu_0 R^2 J_0}{2r}$

If inside wire, $I_{encl.} = \pi r^2 J_0$, so

$$\vec{B}_{inside} = \hat{\phi}\,\frac{\mu_0 r J_0}{2}$$

Looks very similar to electrostatic problems with similar symmetries: current outside in radius doesn't contribute; current inside contributes as if concentrated at the center. (No analogue to spherical charge distribution, due to $\operatorname{div}\vec{B} = 0$.

Surface current density K: Amperian loop crosses surface:



Be careful of the integration direction!

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc.}$

If $\vec{K}$ is perpendicular to surface, then $\oint \vec{B}\cdot d\vec{l} = B_{y\,up}\,l - B_{y\,dn}\,l$

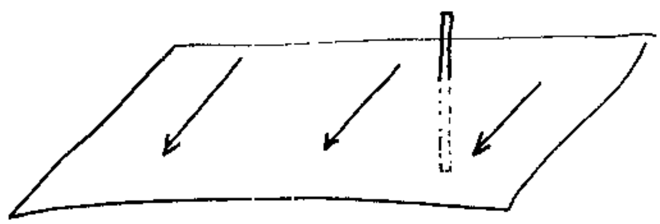
$I_{enc.} = Kl$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 K l \Rightarrow \boxed{B_{y\,up} - B_{y\,dn} = \mu_0 K}$

so if no external field, know by symmetry that $|B_{up}| = |B_{dn}|$

and therefore $B_{y\,up} = \frac{\mu_0 K}{2}$ and $B_{y\,dn} = -\frac{\mu_0 K}{2}$

What is B_x? Turn the surface 90° : take $\varepsilon \to 0$



$I_{encl} = \int d\vec{A}\cdot\vec{J} = 0$ since $d\vec{A} \perp \vec{J}$ everywhere.

So $B_x = 0$.