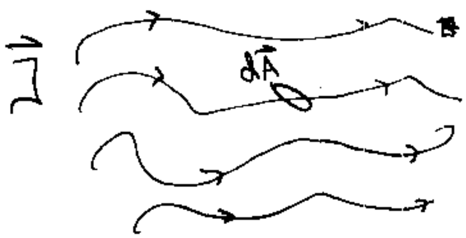
P3310 LECTURE 28

Description of current: $\frac{\text{charge}}{\text{time}}$ dimensions $\left(= A = \frac{C}{s}\right)$

Normally think of current in a wire as a scalar — but it gets more complicated looking closer. The quantity we're interested in generally is the current density $\vec{J}$, which is a <u>vector</u>:

In a substance where charge densities are moving about, we can place an infintesimal area element $d\vec{A}$:



The <u>net</u> charge/time dI through $d\vec{A}$ is $|dI| = \vec{J} \cdot d\vec{A}$.

Note that current is <u>net</u> charge: if equal + and − charge densities are moving with same speed in opposite direction, current $= 0$.
So J has units of $\frac{A}{m^2} = \frac{C}{m^2 s}$.

($\vec{I}$ is, of course, also a vector.)

Since $\vec{J}(\vec{r})$ is a vector field, it obeys the divergence theorem:
current through a closed surface:

$$\oint_S d\vec{A} \cdot \vec{J} = \int_V d^3r \, \mathrm{div} \vec{J}$$

But $\oint_S d\vec{A} \cdot \vec{J}$ is current (out of) a <u>closed volume</u>. Conservation of charge says that must be equal to the decrease/time of the total charge in the volume: $\oint_S d\vec{A} \cdot \vec{J} = -\frac{dQ_{enc}}{dt}$

$$Q_{enc} = \int_V d^3r' \, \rho(\vec{r}')$$

Putting the time derivative inside the volume integral:

$$\oint_S d\vec{A} \cdot \vec{J}(\vec{r}') = -\frac{d}{dt} \int d^3r' \rho(\vec{r}') = \int_V d^3r' \left(\frac{-d\rho}{dt} \right)$$

$$\text{so} \quad \int_V d^3r' \, \mathrm{div} \vec{J}(\vec{r}') = \int_V d^3r' \left(-\frac{d\rho}{dt} \right)$$

$$\Longrightarrow \mathrm{div} \vec{J} = -\frac{d\rho}{dt}$$

Continuity equation: enforces charge conservation.

Magnetostatics involves <u>time-independent</u> currents — generally this means that $\mathrm{div} \vec{J} = 0$.

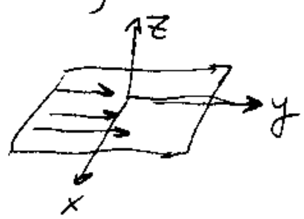
Magnetostatic currents can be thought of sort-of like charge distributions, but...

— $\vec{J}$ is a vector, ρ is a scalar

— No real concept of a "point current" since a moving point charge ($\vec{I} = Q\vec{v}$) is <u>not</u> a static scenario

Surface and path (line) currents:

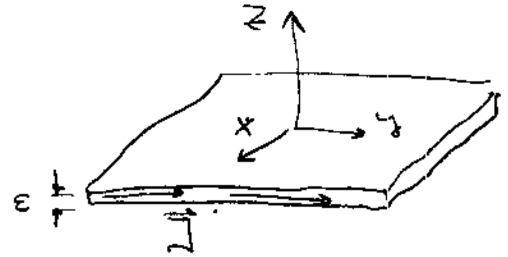
Current may be distributed on a surface (think surface charge σ moving at velocity $\vec{v}$) — in this case $\vec{J}(\vec{r})$ has a delta-function dependence (just as $\rho(\vec{r})$ does for a surface charge):



Surface current $\vec{K} = K_0 \hat{y}$ on xy plane:

$$\underbrace{\vec{J}(\vec{r})}_{\frac{\text{current}}{\text{length}^2}} = \underbrace{K_0}_{\frac{\text{current}}{\text{length}}} \hat{y} \underbrace{\delta(z)}_{\frac{1}{\text{length}}}$$

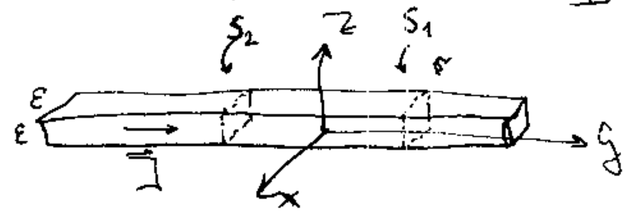
Surface current as an approximation of volume current:



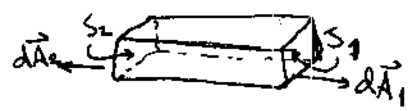
$$\vec{K}(x,y) = \int_0^{\varepsilon} dz'\, \vec{J}(x', y, z)$$

Line/path current:

$$\vec{I}(y) = \int_0^{\varepsilon} dz \int_0^{\varepsilon} dx\, \vec{J}(x,y,z)$$



But, know from charge conservation that if current is static, then current through surface S_1 = current through S_2:
Consider a closed surface with S_1 and S_2 as ends:



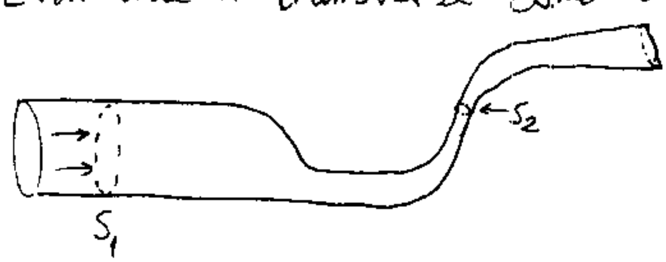
No current through sides, since charge can't leave the wire.

$$\oint \vec{J}\cdot d\vec{A} = \frac{d}{dt} Q_{enc} = 0$$

So $$\int_{S_2} \vec{J} \cdot d\vec{A}_2 + \int_{S_1} \vec{J} \cdot d\vec{A}_1 = 0$$

If the wire curves, this is still true! So consider the static current in a wire to be a position-independent scalar: $\int d\vec{A} \cdot \vec{I} = I$.

Even true if transverse dimensions change:



$$\int_{S_1} d\vec{A}\cdot\vec{J} = \int_{S_2} d\vec{A}\cdot\vec{J} = I,$$