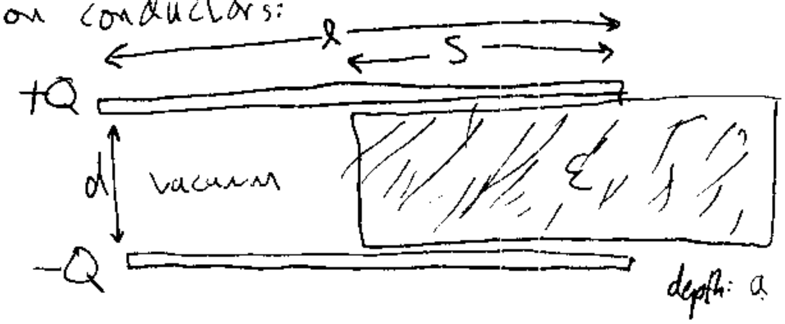
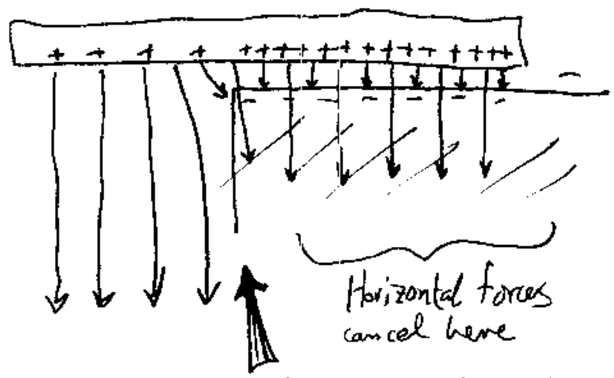
Force on dielectrics: similar (but more complicated) to force on conductors:



Classic problem: dielectric slider in a capacitor.

Look up close at the edge:



Small finge field with noncanceling component. Very difficult to calculate exactly!

Get around this nasty calculation by using an energy argument:

$$(-)dW = F_s\,ds$$

(where the − sign applies or not depending on whether $\vec{F}$ is on dielectric or the capacitor plates) → Force is trying to put system in a lower potential energy state.

The energy is $W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$.

What is C?

Vacuum part: $C = \frac{\varepsilon_0}{d}a(l-s)$

Dielectric part: $C = \frac{\varepsilon_0}{d}asK$

Total $C = \frac{\varepsilon_0 al}{d} + \frac{\varepsilon_0 as}{d}(K-1)$

$= \frac{\varepsilon_0 al}{d} + \frac{\varepsilon_0 as\chi_e}{d}$

→ so $C = C(s)$.

If $F_s = \frac{-dW}{ds}$, then $F_s = \frac{-d}{ds}\left(\frac{1}{2}\frac{Q^2}{C}\right)$

Take $C(s)$:

$$= +\frac{1}{2}\frac{Q^2}{C^2}\frac{dC}{ds}$$

$$= +\frac{1}{2}V^2\left(\frac{\varepsilon_0 \chi_e a}{d}\right)$$

Note that this is $\frac{dW}{ds}$, where Q is fixed (not V).

This is because keeping V fixed while changing C requires external work (like a battery).

→ in general, it's possible to calculate forces using energy/work even when the actual fringe fields that cause the force are very difficult to calculate explicitly.