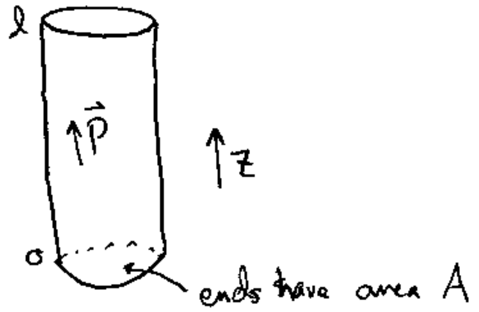
3310 LECTURE 22

Another polarization example: cylinder with uniform polarization:



Remember $\sigma_b = \vec{P}\cdot\hat{n}$; $\rho_b = -\vec{\nabla}\cdot\vec{P}$

So σ on ends is $\pm PA$ if $\vec{P}$ uniform $P\hat{z}$, $\rho = 0$

Long cylinder: Looks like two point charges

Short cylinder: Looks like to plane charges

Nonuniform polarization: $\vec{P} = P(z)\hat{z}$

Long cylinder, small area limit: $\rho = \frac{-dP}{dz}$

$$\longrightarrow \lambda = \rho A = -A \frac{dP}{dz}$$

Looks like a wire (line) charge, with point charges at its end.

Typical real situations don't involve a polarized object just sitting there: usually there's some electric field produced by <u>free</u> charges (e.g. capacitor plates), and the polarization is a complicated response to this externally imposed field. Want a mathematical construct that handles this cleanly: Displacement field.

Consider charge density: total charge $\rho = \rho_{free} + \rho_{bound} = \rho_{free} - \operatorname{div}\vec{P}$

<u>Total</u> electric field, by Gauss's Law: $\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\rho_f - \operatorname{div}\vec{P}\right)$

So $\mathrm{div}\vec{E} + \frac{1}{\varepsilon_0}\mathrm{div}\vec{P} = \frac{1}{\varepsilon_0}\rho_f$

$$\mathrm{div}\left(\vec{E} + \frac{1}{\varepsilon_0}\vec{P}\right) = \frac{1}{\varepsilon_0}\rho_f \quad \Longrightarrow \frac{1}{\varepsilon_0}\vec{D} \equiv \vec{E} + \frac{1}{\varepsilon_0}\vec{P}$$

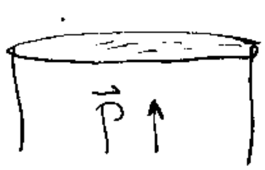
so $\mathrm{div}\vec{D} = \rho_f$ and $\oint_S \vec{D}\cdot d\vec{A} = Q_{f,encl.}$

$\mathrm{div}\,\vec{D}$ depends only on free charge, and can be calculated with no detailed knowledge of the polarization. Of course you need to know $\vec{P}$ to then find $\vec{E}$.

Consider what happens at surface (assume no free charge) of our polarized cylinders

$$\mathrm{div}\vec{D} = 0 \qquad \text{so} \qquad \sigma_b = \vec{P}\cdot\hat{n} = P$$

$\sigma_b \rightarrow$



$$\vec{E}_{up} - \vec{E}_{down} = \frac{\sigma_b}{\varepsilon_0}\hat{z}$$

$$\vec{D}_{up} - \vec{D}_{down} = \hat{z}\left[\varepsilon_0\left(E_{up} - E_{down}\right) + \left(P_{up} - P_{down}\right)\right]$$

$$= \hat{z}\left[\sigma_b + \left(0 - P\right)\right] = \hat{z}\left(\sigma_b - \sigma_b\right) = 0$$

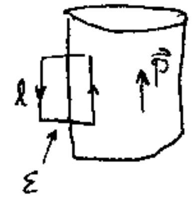
$\Longrightarrow \vec{D}$ is continuous across a surface with only bound charges.

Caveat: $\vec{D}$ can do things $\vec{E}$ can't:

$$\mathrm{div}\vec{D} = \rho_f \quad \text{but} \quad \mathrm{curl}\,\vec{D} = \mathrm{curl}\left(\varepsilon_0\vec{E} + \vec{P}\right) = \mathrm{curl}\,\vec{P}$$

... not necessarily zero.

Look at cylinder walls:



$$\int \vec{P}\cdot d\vec{l} = \int_{\text{surf.}} \text{curl}\,\vec{P}$$

$$= P\ell + 0 + 0 + 0 = P\cdot\ell \neq 0.$$

In general $\vec{P}$ depends on $\vec{E}$ and history of sample. Need to know $\vec{P}(\vec{E})$, ρ_f to specify problems completely.

Dielectric materials: Most insulators have polarization $\vec{P} \propto \vec{E}$:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad \chi_e = \text{susceptibility of material.}$$

$\vec{E}$ of course can depend on polarization too, not just free charges. If calculable, begin with $\vec{D}$:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$= \underbrace{\varepsilon_0 (1+\chi_e)}_{=\,\varepsilon \ \text{permittivity}} \vec{E}$$

$$\varepsilon_r \equiv 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0} \quad \text{"relative permittivity"}$$

usually ~1.01 or less for gases, ~few–tens for solid ins.