Quantum I (PHYS 3220)

concept questions

Clicker Intro

Do you have an iClicker? (Set your frequency to CB and vote.)

A) Yes
B) No
Have you looked at the web lecture notes for this class, before now?
A) Yes
B) No

(ICALKER frequency is CB)
Have you done the assigned reading for today?
A) Yes – Griffiths only
B) Yes – Web notes only
C) Yes – both text and notes
D) Not really – but I will soon!
E) Nope

Intro to Quantum Mechanics
In Classical Mechanics, can this equation be derived?

\[ \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \]

A) Yes  
B) No

Can this equation be derived?

\[ \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \]

A) Yes  
B) No

(ICLICKER frequency is CB)

Have you done the assigned reading for today?

A) Yes – Griffiths only  
B) Yes – Web notes only  
C) Yes – both text and notes  
D) Not really – but I will soon!  
E) Nope
Postulate #3 says \[ \int |\psi(x)|^2 \, dx = \text{Prob(particle is between } x \text{ and } x+dx) \]
What conclusion can you draw?

A) \( \int |\psi(x)|^2 \, dx \) must be exactly =1
B) \( \int |\psi(x)|^2 \, dx \) is finite, but needn’t =1
C) \( |\psi(x)|^2 \) must be finite at all \( x \).
D) More than one of these
E) One/more are true, but do not follow

Is this wave function normalized?
(This wave function is pure real)

\[ \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx \]

A) Yes  B) No

How would you physically interpret the wave function in the sketch?

A) This doesn’t look very physical…
B) QM doesn’t let you “interpret” wave functions like this
C) It’s a large particle
D) a small particle
E) a particle located at a definite spot \( (x_0) \)
Statistics and Probability

You flip an ordinary coin in the air and get 3 heads in 3 tosses. On the 4th toss, the probability of heads is …

A) greater than 50%
B) less than 50%
C) equal to 50%

Plinko! A marble is released from the same starting point each time. Classical physics says identical systems with exactly the same initial conditions always lead to the same final result, in a deterministic and repeatable way.

Is the distribution of final outcomes for the Plinko game (played 300 times) in this example in conflict with our theories of classical systems?

A) Yes     B) No
The probability density $|\psi|^2$ is plotted for a **normalized** wave function $\psi(x)$. What is the probability that a position measurement will result in a measured value between 2 and 5?

A) $\frac{2}{3}$
B) 0.3
C) 0.4
D) 0.5
E) 0.6

The probability density $|\psi|^2$ is plotted for a **non-normalized** wave function $\psi(x)$. What is the probability that a position measurement will result in a measured value between 3 and 5?

A) $\frac{2}{3}$
B) $\frac{4}{9}$
C) $\frac{1}{2}$
D) 0.6
E) 0.4

Do you plan on attending Tutorial today? (4 PM, basement Tutorial bay)

A) Yes, I’ll be there!
B) Maybe
C) No/can’t come
N independent trials are made of a quantity \(x\). The possible results form a *discrete spectrum* \(x_1, x_2, \ldots, x_i, \ldots x_M\) (\(M\) possible distinct results). Out of \(N\) trials, \(n_i\) of the trials produce result \(x_i\).

If you add up all the results of all \(N\) trials, what is the sum of the results?

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<th>(x_i)</th>
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\[
\sum_i n_i x_i
\]

\[\sum_i x_i \quad \sum_i n_i x_i \quad \sum_i N \]

N = 342 trials, (6 different possible results in each trial)

What is the best estimate of the probability that a token picked from the bag will be an 8?

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\[
A) \text{zero} \quad B) \frac{6}{342} \quad C) \frac{1}{6} \quad D) \frac{80}{342} \quad E) \frac{110}{342}
\]

For a large number \(N\) of independent measurements of a random variable \(x\), which statement is true?

A) \(\langle x^2 \rangle \geq \langle x \rangle^2\) always

B) \(\langle x^2 \rangle \geq \langle x \rangle^2\) or \(\langle x^2 \rangle < \langle x \rangle^2\)

depending on the probability distribution.
A ball is released from rest. You take many pictures as it falls to x=H (pictures are equally spaced in time).

What is \( \langle x \rangle \), the average distance from the origin in randomly selected pictures?

A) H/2
B) larger than H/2 but less than H
C) larger than H
D) smaller than H/2
E) ???

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Waves

A traveling wave is described by

\[ Y(x,t) = 4 \sin(2x - t) \]

All the numbers are in the appropriate SI (mks) units.

To 1 digit accuracy, the wavelength, \( \lambda \), is most nearly...?

A) 1m B) 2m C) 3m D) 4m E) Considerably more than 4m.
Two traveling waves 1 and 2 are described by the equations.

\[ Y_1(x,t) = 8 \sin(4x - 2t) \]
\[ Y_2(x,t) = 2 \sin(x - 2t) \]

All the numbers are in the appropriate SI (mks) units.

Which wave has the higher speed?
A) Wave 1
B) Wave 2
C) Both waves have the same speed

Have you ever studied the (classical) Wave Equation?
A) Yes
B) No
C) Not sure

Let \( y_1(x,t) \) and \( y_2(x,t) \) both be solutions of the same wave equation; that is,

\[ \frac{\partial^2 y_i}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y_i}{\partial t^2} \]

where \( i \) can be 1 or 2, and \( v \) is a constant.

Is the function \( y_{\text{sum}}(x,t) = ay_1(x,t) + by_2(x,t) \) still a solution of the wave equation?
(with \( a, b \) constants)
A) Yes, always
B) No, never
C) Sometimes, depending on \( y_1 \) and \( y_2 \).
Two impulse waves are approaching each other, as shown.

Which picture correctly shows the total wave when the two waves are passing through each other?

A) Two pulses
B) One pulse
C) Two separate pulses
D) None of these

A two-slit interference pattern is viewed on a screen. The position of a particular minimum is marked. This spot on the screen is further from the lower slit than from the top slit. How much further?

A) 2λ  B) 1.5λ  C) 3λ  D) 0.5λ  E) None of these

Two radio antennae are emitting isotropic radio signals at the same frequency \(f\) in phase. The two antennae are located a distance 10.5λ apart (\(\lambda = \frac{c}{f}\)). A technician with a radio tuned to that frequency \(f\) walks away from the antennae along a line through the antennae positions, as shown:

As the technician walks, she notes the tone from the radio is...

A) very loud, all the time.
B) alternates loud and quiet as she walks.
C) very quiet, all the time.
D) quiet at first, and then loud all the time.
Do you plan to attend today’s Tutorial (on interpretation of wave functions, the Schrodinger Eqn, and time dependence)

A) Yes, at the 3 pm “sitting…”
B) Yes, at the 4 pm sitting…
C) Perhaps, more likely at 3
D) Perhaps, more likely at 4
E) No, can’t come/not planning on it.

A linear operator \(L[f(x)]\) has the property 
\[ L(a*f_1+b*f_2) = a*L(f_1)+b*L(f_2), \] 
a and b any constants. How many of these operators are linear operators? (A and B are constants).

I. \(L[f(x)] = (f(x))^2\)
II. \(L[f(x)] = A*d^2f(x)/dx^2\)
III. \(L[f(x)] = \sin(f(x))\)
IV. \(L[f(x)] = A*f(x) + B\)
V. \(L[f(x)] = \exp(f(x)) = e^{f(x)}\)

A) None of these  B) 1 of these  C) 2  D) 3  E) 4 or more of these

Take deBroglie seriously, electrons are waves! 
Assume an integer # of wavelengths of the orbiting electron must “fit” on the circumference of the orbit (why?)

First: derive a formula relating \(\lambda, r\) (radius), and “n” (the number of wavelengths around the circle)

Then: solve for \(L = r*p\) (angular momentum) using deBroglie’s relation for momentum.
Starting with the assumed solution 
\[\Psi(x,t) = A \exp[i(kx - \omega t)] \]
how can one obtain a factor of \(\omega\) (perhaps with other factors)?
Use the operator…

A) \(\frac{\partial}{\partial x}\)  
B) \(\frac{\partial}{\partial t}\)  
C) \(\frac{\partial^2}{\partial x^2}\)  
D) \(\frac{\partial^2}{\partial t^2}\)  
E) \(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\)

Starting with the assumed solution 
\[\Psi(x,t) = A \exp[i(kx - \omega t)] \]
how can one obtain a factor of \(k^2\) (perhaps with other factors)? Use the operator…

A) \(\frac{\partial}{\partial x}\)  
B) \(\frac{\partial}{\partial t}\)  
C) \(\frac{\partial^2}{\partial x^2}\)  
D) \(\frac{\partial^2}{\partial x^2}\)  
E) \(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\)

Do you know the de Broglie relations?

A) Yes  
B) No
Two particles, 1 and 2, are described by plane wave of the form \( \exp[i(kx - \omega t)] \).
Particle 1 has a smaller wavelength than particle 2: \( \lambda_1 < \lambda_2 \).
Which particle has larger momentum?

A) particle 1  
B) particle 2  
C) They have the same momentum  
D) It is impossible to answer based on the info given.

Consider the eigenvalue equation
\[
\frac{d^2}{dx^2} [f(x)] = C \cdot f(x)
\]
How many of the following give an eigenfunction and corresponding eigenvalue?

I. \( f(x) = \sin(kx) \), \( C = +k^2 \)  
II. \( F(x) = \exp(-x) \), \( C = +1 \)  
III. \( F(x) = \exp(i k x) \), \( C = -k^2 \)  
IV. \( F(x) = x^3 \), \( C = 6 \)

A) 1  B) 2  C) 3  D) all 4  E) None

\[
\int_{-\infty}^{\infty} d^2 \Psi^*(x,t) \Psi(x,t) dx = \int_{-\infty}^{\infty} \frac{d}{dt} \left( \Psi^*(x,t) \Psi(x,t) \right) dx
\]

A) Yes, no problemo!  
B) There’s something not right about this…
Which expression below would be the QM equation for $<KE>$?

A) $\int_{-\infty}^{\infty} \frac{-\hbar^2}{2m} \left( \psi^*(x,t) \psi(x,t) \right) dx$

B) $\int_{-\infty}^{\infty} \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \left( \psi^*(x,t) \psi(x,t) \right) dx$

C) $\int_{-\infty}^{\infty} \frac{-\hbar^2}{2m} \psi^*(x,t) \frac{\partial^2}{\partial x^2} \psi(x,t) dx$

D) None of these!  

E) More than one!

After assuming a product form solution $\Psi(x,t) = \psi(x) \phi(t)$, the TDSE becomes

$$i\hbar \frac{1}{\psi} \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{1}{\psi} \frac{\partial^2 \psi}{\partial x^2} + V = E$$

If the potential energy function $V$ in the Schrödinger Equation is a function of time, as well as $x$ [$V = V(x,t)$], would separation of variables still work; that is, would there still be solutions to the SE of the form $\Psi(x,t) = \psi(x) \phi(t)$?

A) Yes, always  
B) No, never  
C) Depends on the functional dependence of $V$ on $x$ and $t$

$\Psi_1(x,t)$ and $\Psi_2(x,t)$ are two solutions of the time-dependent SE.

Is $\Psi_{\text{sum}}(x,t) = a \Psi_1(x,t) + b \Psi_1(x,t)$ also a solution of the TDSE?

A) Yes  
B) No  
C) Depends on $\Psi_1(x,t)$ and $\Psi_2(x,t)$
Do you know what the momentum operator is?

A) Yes
B) No

Do you plan to attend today’s Tutorial (on relating classical to Quantum, and qualitative “sketching” of wave functions)

A) Yes, at 3 pm
B) Yes, at 4 pm
C) Perhaps, more likely at 3
D) Perhaps, more likely at 4
E) No, can’t come/not planning on it.

Given $\Psi_n(x, t)$ as one of the eigenstates of $\hat{H}\Psi_n = E_n\Psi_n$,
what is the expectation value of the Hamiltonian-squared?

$$\langle H^2 \rangle = \int \Psi_n^* \hat{H}(\hat{H}\Psi_n) dx = ?$$

A) $E_n$
B) $E_n^2$
C) zero
D) $E_n^2 - E_n$
E) Something else/it really depends!!
Ψ₁ and Ψ₂ are two energy eigenstates of the Hamiltonian operator. They are non-degenerate, meaning they have different eigenvalues E₁ and E₂. 

\( \hat{H}\Psi_1 = E_1\Psi_1 \) and \( \hat{H}\Psi_2 = E_2\Psi_2 \) and \( E_1 \neq E_2 \).

Is \( \Psi_s = \Psi_1 + \Psi_2 \) also an energy eigenstate?

A) Yes, always
B) No, never
C) Possibly yes, depends!

Given \( u_n(x) = A \sin(kx) + B \cos(kx) \)
the boundary condition, \( u(0) = 0 \), implies what?

A) \( A = 0 \)
B) \( B = 0 \)
C) \( k = 0 \)
D) \( k = n\pi, n = 1, 2, 3 \ldots \)
E) None of these

Given \( u_n(x) = A \sin(kx) \)
the boundary condition, \( u(a) = 0 \), implies what?

A) \( A = 0 \)
B) \( k = 0 \)
C) \( k = n\pi, n = 1, 2, 3 \ldots \)
D) None of these,...
An electron and a neutron have the same speed. Which particle has the shorter wavelength?

A) The electron  
B) The neutron  
C) They have the same wavelength

How does the energy $E_1$ of the ground state $(n=1)$ of an infinite square well of width $a$ compare with the energy of the ground state of a well with a larger width? The larger well has …

A) lower energy  
B) higher energy  
C) the same energy  
D) Need more information

How does the energy $E$ of the $n=3$ state of an infinite square well of width $a$ compare with the energy of the $n=3$ state of a well with a larger width? The larger well has …

A) lower energy  
B) higher energy  
C) the same energy
In an infinite square well, the lowest two stationary states are \( u_1(x) \) and \( u_2(x) \). At time \( t=0 \), the state of a particle in this square well is \( \Psi(x,t=0) = \frac{1}{\sqrt{2}} (u_1(x) + u_2(x)) \).

Is this particle in a stationary state?
A) Yes, \( \Psi \) is a stationary state.
B) No, \( \Psi \) is not a stationary state at any time.
C) No, \( \Psi \) is a stationary state only when \( t=0 \).
D) Not enough information.

In an infinite square well, the lowest two stationary states are \( u_1(x) \) and \( u_2(x) \). At time \( t=0 \), the state of a particle in this square well is \( \Psi(x,t=0) = \frac{1}{\sqrt{2}} (u_1(x) + u_2(x)) \). What is the wave function at time \( t \)? \( \Psi(x,t) = \\

At \( t=0 \), could the wavefunction for an electron in an infinite square well of width \( a \) \((0<x<a)\) be \( \Psi(x,0) = A \sin^2(\pi x/a) \), where \( A \) is a suitable normalization constant? (Assume it is zero outside the region \( 0<x<a \)).
A) Sure
B) No, it’s not physical
The energy eigenstates, \( u_n \), form an orthonormal set, meaning
\[
\int u_m^*(x)u_n(x)dx = \delta_{mn}
\]
What is \( \int u_m^*(x)\sum c_n u_n(x)dx = ? \)
A) \( \sum c_n \)
B) \( c_m c_n \)
C) \( c_n \)
D) \( c_m \)
E) None of these

Given a particle in a box (size \( a \)), with
\[
\Psi(x,t=0) = \sqrt{\frac{2}{a}} \left[ \frac{\pi}{3} \sin(\pi x/a) + \frac{7\pi}{3} \sin(3\pi x/a) \right]
\]
What is the probability of measuring \( E_1 \)? How about measuring \( E_2 \)?
A) \( \text{Prob}(E_1) = 1/3, \quad \text{Prob}(E_2) = 2/3 \)
B) \( P(E_1) = \text{sqrt}[1/3], \quad P(E_2) = \text{sqrt}[2/3] \)
C) \( P(E_1) = 1/3, \quad P(E_2) = 0 \)
D) \( P(E_1) = \text{sqrt}[1/3], \quad P(E_2) = 0 \)
E) None of these is correct

Assuming your system is isolated, does the probability of measuring \( E1 \) depend on the time \( t \) of the measurement?
A) Yes
B) No
Mathematica has numerically solved the TISE, with \( V(x) = \frac{1}{2} m \omega^2 x^2 \), starting from \( u(0) = 1 \), with an assumed energy \( E \).

What should be our next try?
A) Make \( E \) a little bigger  
B) Make \( E \) a little smaller  
C) Make \( u(0) \) a little bigger  
D) Make \( u(0) \) a little smaller  
E) None of these/more than one/something else...

What is the behaviour of \( u(x) = Ae^{-ax^2} \) as \( x \) goes to infinity?
A) \( u(\infty) \) blows up.  
B) \( u(\infty) \) goes to a nonzero constant  
C) \( u(\infty) \) goes to 0, but \( u(x) \) is not normalizable  
D) \( u(\infty) \) goes to 0, and \( u(x) \) is normalizable  
E) Can’t decide without knowing \( A \) and \( a \).
In QM, what is $\hat{p} f(x)$
(where $\hat{p}$ is the momentum operator)
A) $0$, for all functions $f(x)$
B) $h k \frac{d}{dx} f(x)$
C) $\frac{df}{dx}$
D) $-i\hbar \frac{df}{dx}$
E) None of these/something else

$[x, p] = i\hbar$.
Is $[p, x] = i\hbar$? Is $ip + cx = cx + ip$?
A) Yes and yes
B) Yes and no
C) No and yes
D) No and no

$\hat{a}_x \hat{a}_x = \frac{1}{\hbar \omega} \hat{H} - \frac{1}{2}$
What is $\hat{a}_x \hat{a}_x$
Given $\hat{a}_+ \hat{a}_- = \frac{1}{\hbar \omega} \hat{H} - \frac{1}{2}$
and $\hat{a}_+ \hat{a}_+ = \frac{1}{\hbar \omega} \hat{H} + \frac{1}{2}$

What is $[\hat{a}_+, \hat{a}_-]$?

A) Give me a break, it's a bloody mess
B) 0
C) 1
D) -1
E) It's simple, just not given above.

If $\hat{H}(\hat{a}_+ u_n) = (E_n + \hbar \omega)(\hat{a}_+ u_n)$

What can you say about $\hat{a}_+ u_n$?

A) Nothing much
B) It is a stationary state (an “energy eigenfunction”) and must be equal (or proportional) to the state $u_n$
C) It is a stationary state, but is NOT proportional to the state $u_n$