Physics 3210, Spring 2010 HW#9 - due Wednesday, Mar. 17

For full credit, you must show all your work and explain your reasoning in complete sentences.

1) A billiard ball of mass $M$ and radius $R$ sits on a flat horizontal billiard table. You want to strike it horizontally with a cue so that it rolls without slipping on the table. At what height $h$ above the table should you strike it? The moment of inertia of a sphere is $\frac{2}{5}MR^2$. (Remark: this is also the principle that sets the height of the bumpers around the table.)

2) The key mathematical step in relating angular momentum to angular velocity (hence in defining the moment of inertia tensor) is to evaluate the quantity $\hat{r} \times (\hat{\omega} \times \hat{r})$. To do this, first prove the "BAC CAB" rule,

$$\hat{A} \times (\hat{B} \times \hat{C}) = \hat{B}(\hat{A} \cdot \hat{C}) - \hat{C}(\hat{A} \cdot \hat{B})$$

Use this to show that Taylor’s expression (10.19) is correct.

3) Consider a solid ellipsoidal object of mass $M$ and uniform density, whose surface is described by the equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

a) Find the components of the moment of inertia tensor for this ellipsoid.

b) What are its principal axes of inertia, and what are the moments about these axes?

(Remark: Any rigid object can be described in terms of its principal moments of inertia. Sometimes it’s useful to replace the real object with a simpler ellipsoid with the same moments, as this is easier to visualize.)

4) Taylor, 10.16. You do not have to work out the moment of inertia tensor as Taylor asks – we did that in class.

5) A flat piece of sheet metal rotates around a fixed point O which is inside the metal. Define coordinates where the sheet lies in the x-y plane, with the z-axis perpendicular to the sheet.

a) Show that the z-axis is a principal axis of inertia of the sheet.

b) Verify that the components of the moment of inertia tensor are not independent, but satisfy $I_{zz} = I_{xx} + I_{yy}$. 