Physics 3210, Spring 2010 HW#2 - due Wednesday, Jan. 27

For full credit, you must show all your work and explain your reasoning in complete sentences.

1) Use the Euler-Lagrange equations to show that for a mass moving in a potential that is independent of the polar coordinate $\phi$, the angular momentum is conserved. Interpret this in terms of the language of angular momentum (torques, etc).

2) Work out the equation of motion for the simple pendulum, using
   a) good old $F=ma$
   b) the Lagrangian formalism.

While both methods are pretty straightforward for this problem, nevertheless you should see that using the Lagrangian is even more straightforward.

3) Taylor’s example 7.3 (p. 255) talks about the “Atwood machine,” which is basically two masses tied to either end of a string, where the string wraps around a pulley. As Taylor does the example, the pulley itself does not rotate, but the string slides (frictionlessly, of course!) over the pulley. Here, you should do the same problem, but assume that the pulley is free to rotate and has moment of inertia $I$. What is the motion of the two masses?

4) A pendulum with a mass $m$ hangs from a string of length $l$.
   a) What is the period of this pendulum under normal gravitational acceleration $g$?
   b) Now the pendulum is in an elevator which is accelerating upward with acceleration $a$. What is the period of the pendulum in this circumstance? Hint: you should start by writing the Lagrangian in an inertial frame, i.e., one that is fixed on the ground and not moving with the elevator. (Historical note: When Einstein recognized the equivalence between uniform acceleration and gravity, it was the starting point of general relativity.)

5) A mass $m$ attached to a spring of spring constant $k$ executes simple harmonic motion in coordinate $x$ with frequency $\omega = \sqrt{k/m}$. But wait! What if the spring also has a mass $M$? To figure out what happens, do the following:
   a) Show that the kinetic energy due to the stretching spring is $\frac{1}{6}M(x')^2$, assuming that the string stretches uniformly.
   b) Show that the motion of the mass-plus-spring system is still harmonic, and the frequency is now $\omega = \sqrt{k/(m+M/3)}$.

6) Taylor, 7.29, the “crazy pendulum attached to a rotating wheel” problem.