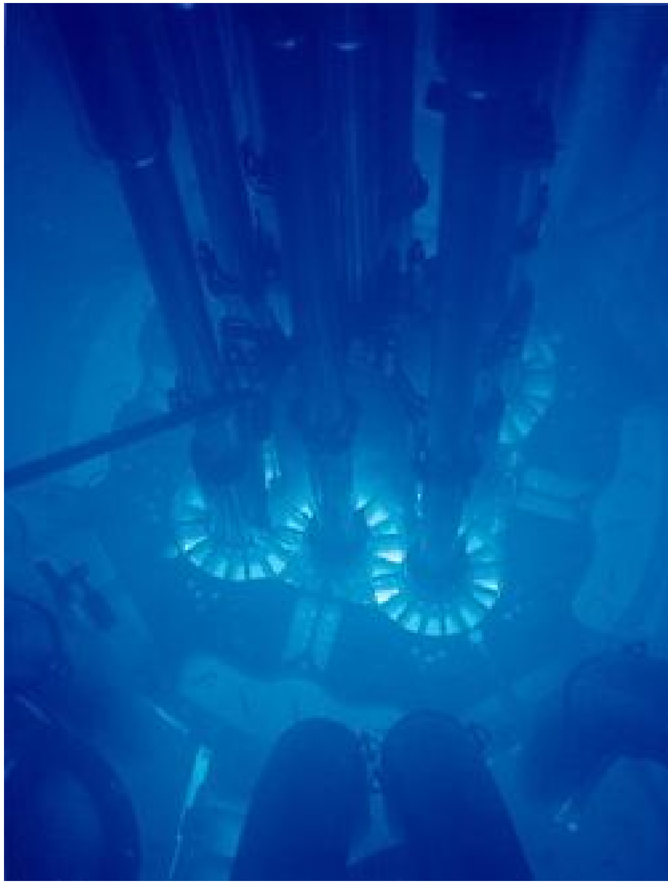
Special relativity

Announcements:

- Homework solutions are on CULearn
- Homework set 3 is on the website and is due Wed at 12:50pm.
- Remember, problem solving sessions today 3-5 and tomorrow 3-4,5-6 in physics help room (G2B90)



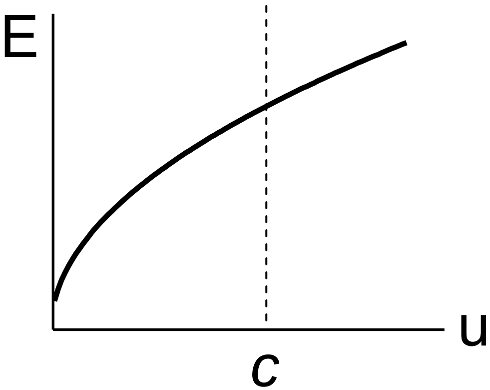
Today we will continue with relativistic energy and the relationship between mass, momentum, and energy.

Clicker question 1 | Set frequency to DA

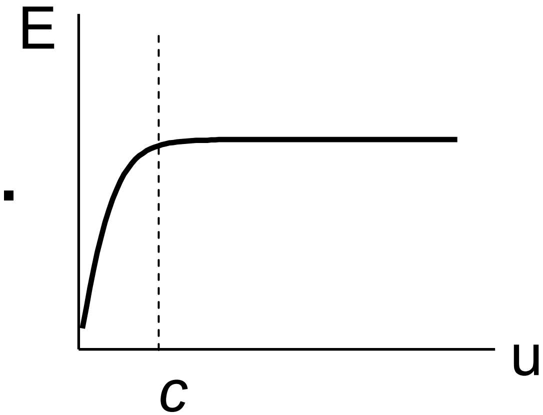
Which of the graphs below is a possible representation of the total energy of a particle versus its speed

$$E = \gamma_u mc^2$$

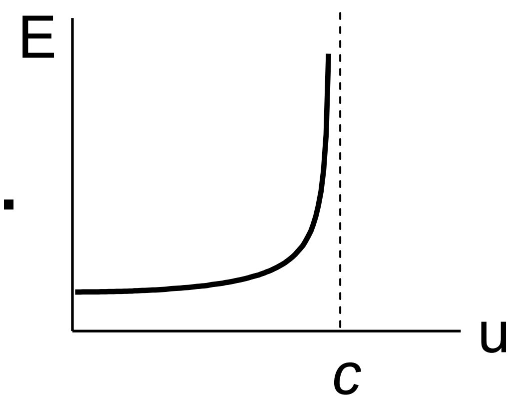
A.



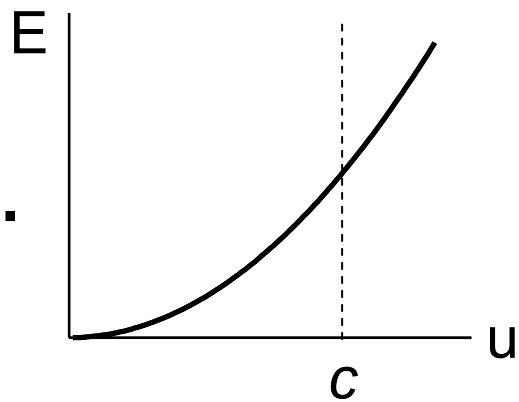
B.



C.



D.



E. None of them

$$\gamma_u = \frac{1}{\sqrt{1 - u^2 / c^2}}$$

As the velocity **u** increases towards *c*, the energy increases. But for $u \to c$, $\gamma \to \infty$ so it is impossible for a particle to reach *c* as it would need infinite energy.

Relativistic momentum and energy

Relativistic momentum: $\vec{p} = \gamma_u m\vec{u}$

Total energy: $E = \gamma_u mc^2$

With these definitions for momentum and energy, conservation of momentum and conservation of (total) energy continue to work in isolated relativistic systems

At rest, γ_u=1 so the rest energy is $E_{\text{rest}} = mc^2$

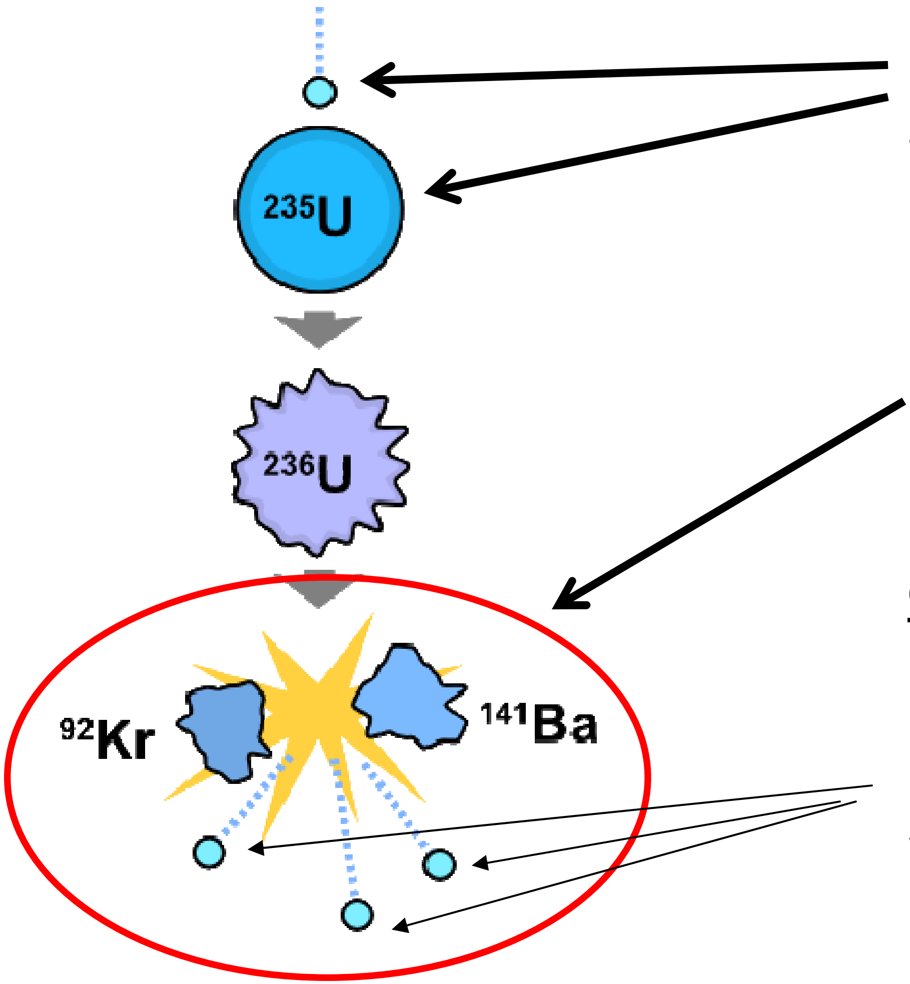
This tells us that mass and energy are equivalent. Mass is a type of energy like motion is a type of energy (kinetic energy).

Kinetic energy is *always* the total energy minus the rest energy:

$$KE = E - E_{\text{rest}} = \gamma_u mc^2 - mc^2 = (\gamma_u - 1)mc^2$$

At *low* speeds, this becomes the familiar $KE \approx \frac{1}{2}mu^2$

Rest Energy is real: Nuclear fission



Initial mass of a neutron and a ^{235}U nucleus.

Final products have less mass, but much more kinetic energy. Conversion of mass to kinetic energy.

Oh yes, and more neutrons, so the reaction can run wild (chain reaction!).

Clicker question 2 | Set frequency to DA

Q. If we could convert mass entirely to energy, how much mass would be required to run a 30W light bulb for a year (a year is approximately $3 \cdot 10^7$ s)?

A. 10 kg
B. 3 g
C. 3 mg
D. 10 μg
E. 10 ng

One year is $3 \cdot 10^7$ s so running a 30 W light bulb for a year requires $E = P \cdot \Delta t = 30\ \text{W} \cdot 3 \cdot 10^7\ \text{s} = 9 \cdot 10^8\ \text{J}$.

To get that energy from rest mass we use $E=mc^2$ and solve for mass.

$$m = \frac{E}{c^2} = \frac{9\times10^8\ \text{J}}{(3\times10^8\ \text{m/s})^2} = 1\times10^{-8}\ \text{kg} = 1\times10^{-5}\ \text{g} = 10\ \mu\text{g}$$

A relativistic collision problem

Object **A** has mass $9m_0$ and speed v_A=0.8c (γ_A=5/3).
Object **B** has mass $12m_0$ and speed v_B=−0.6c (γ_B=5/4).

The objects collide and stick together (completely inelastic collision)

From Physics 1110 we know collisions conserve momentum and that inelastic collisions do **not** conserve kinetic energy

But this is an isolated system, so **total** energy *must* be conserved.

Let's start with momentum conservation.

Classically, what is the total initial momentum?

$$P_{\text{initial}}^{\text{classical}} = m_A v_A + m_B v_B = 9m_0 \cdot 0.8c - 12m_0 \cdot 0.6c = 7.2m_0c - 7.2m_0c = 0$$

What is the total relativistic momentum?

$$P_i = \gamma_A m_A v_A + \gamma_B m_B v_B = \tfrac{5}{3}7.2m_0c - \tfrac{5}{4}7.2m_0c = 12m_0c - 9m_0c = 3m_0c$$

So it does not end at rest as predicted classically!

A relativistic collision problem

Object **A** has mass $9m_0$ and speed v_A=0.8c (γ_A=5/3).
Object **B** has mass $12m_0$ and speed v_B=−0.6c (γ_B=5/4).

Momentum conservation $P_i = P_f$ gives us: $3m_0c = \gamma_f m_f u_f$

Remember that m_f may not be m_A+m_B as it would be classically.

Now let's look at the total energy. The initial energy is

$$E_i = \gamma_A m_A c^2 + \gamma_B m_B c^2 = \tfrac{5}{3}9m_0c^2 + \tfrac{5}{4}12m_0c^2 = 15m_0c^2 + 15m_0c^2 = 30m_0c^2$$

So conservation of energy $E_i = E_f$ gives us: $30m_0c^2 = \gamma_f m_f c^2$

Dividing these two equations: $\dfrac{3m_0c}{30m_0c^2} = \dfrac{\gamma_f m_f u_f}{\gamma_f m_f c^2}$ or $\dfrac{1}{10} = \dfrac{u_f}{c}$ so $u_f = 0.1c$

Furthermore: $\gamma_f = (1-0.1^2)^{-1/2} = 1.005$ so we can solve the conservation of energy equation for m_f: $m_f = 30m_0/\gamma_f = 29.85m_0$

A relativistic collision problem

Object **A** has mass $9m_0$ and speed $v_A=0.8c$ ($\gamma_A=5/3$).
Object **B** has mass $12m_0$ and speed $v_B=-0.6c$ ($\gamma_B=5/4$).

Classically, total momentum is 0 but in reality it is $3m_0c$

Classically, $m_f = m_A + m_B = 21m_0$ but in reality, $m_f = 29.85m_0$
so $8.85m_0$ of mass is gained.

Classically, final kinetic energy is 0 and the initial kinetic energy is

$$KE_{\text{initial}}^{\text{classical}} \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = \tfrac{1}{2} 9m_0 \cdot 0.64c^2 + \tfrac{1}{2} 12m_0 \cdot 0.36c^2 = 5.04m_0c^2$$

But in reality, the initial and final kinetic energies are:

$$KE_i = (\gamma_A - 1)m_A c^2 + (\gamma_B - 1)m_B c^2 = \tfrac{2}{3} 9m_0c^2 + \tfrac{1}{4} 12m_0c^2 = 9m_0c^2$$

$$KE_f = (\gamma_f - 1)m_f c^2 = (1.005 - 1)29.85m_0c^2 = 0.15m_0c^2$$

So the change in KE is $KE_f - KE_i = 0.15m_0c^2 - 9m_0c^2 = -8.85m_0c^2$

The "lost" kinetic energy appears as gained mass in the total energy

http://www.colorado.edu/physics/phys2170/

Really, mass gets created!

<u>CERN in Geneva, Switzerland</u>

Before the LHC (Large Hadron Collider) CERN operated LEP, the Large Electron-Positron collider in the same underground tunnel.

Electron and positrons have a mass of 9×10^{-31} kg. They were accelerated to very high energies so when they annihilate, they create a Z^0 particle with a mass of 1.6×10^{-25} kg.

Relativistic momentum and energy

Relativistic momentum: $\vec{p} = \gamma_u m \vec{u}$

Total energy: $E = \gamma_u mc^2$

With some algebra we can eliminate the velocity variable from these two relations and obtain the triangle relation:

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2$$

A new unit of energy is the electron-volt (eV). It's the energy obtained by an electron moving through 1 V. It is not an SI unit but is very common.

$$\Delta E = q\Delta V = 1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ C} \cdot 1 \text{ V} = 1.6 \cdot 10^{-19} \text{ J}$$

Since mc^2 is a unit of energy, dividing energy by c^2 gives a unit of mass. Also, dividing energy by c gives a unit of momentum.

A proton has a mass of 938 MeV/c^2. What is this in kg?

$$938 \text{ MeV}/c^2 = \frac{938\times10^6 \text{ eV} \cdot 1.60\times10^{-19} \text{ J/eV}}{\left(3.00\times10^8 \text{ m/s}\right)^2} = 1.67\times10^{-27} \text{ kg}$$

Also use eV/c or MeV/c units for momentum