Special relativity

Announcements:

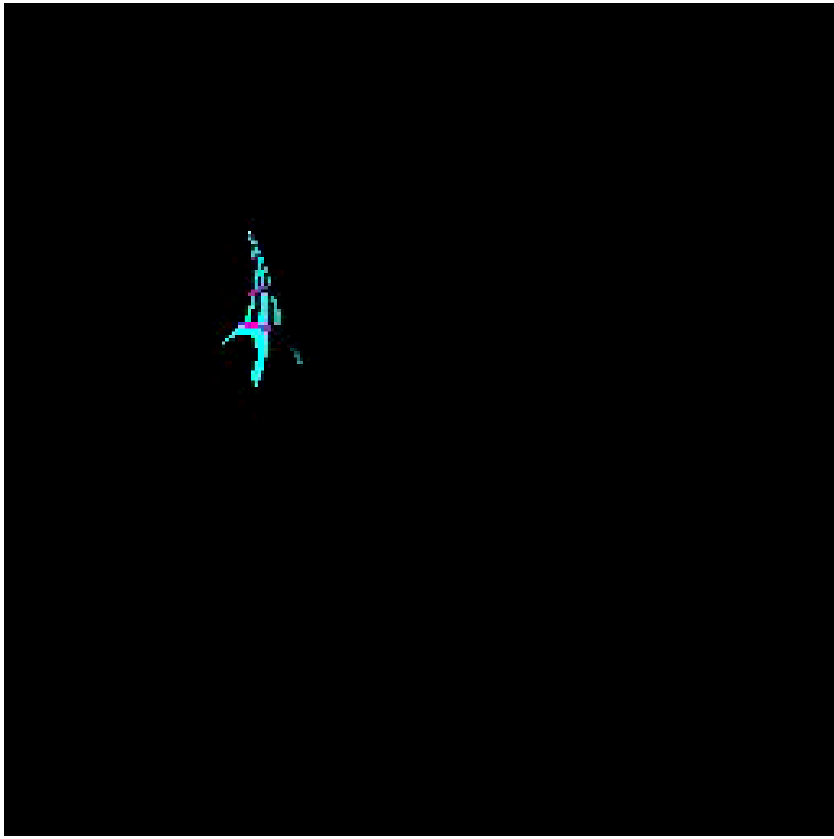
- Homework solutions are on CULearn
- Homework set 3 is on the website and is due Wed at 12:50pm.
- Remember, problem solving sessions Monday 3-5 and Tuesday 3-4,5-6.

Hermann Minkowski (1864—1909):

Today we will investigate the relativistic Doppler effect and look at momentum and energy.

Flyby movies at 0.99c

from http://www.vis.uni-stuttgart.de/~weiskopf/gallery/index.html



Relativistic Doppler shift

The speed of light is the same for all inertial observers

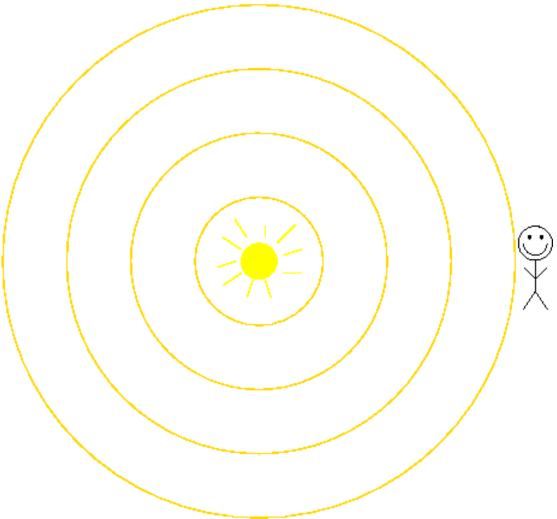
However, the wavelength and frequency change based on *relative* velocity

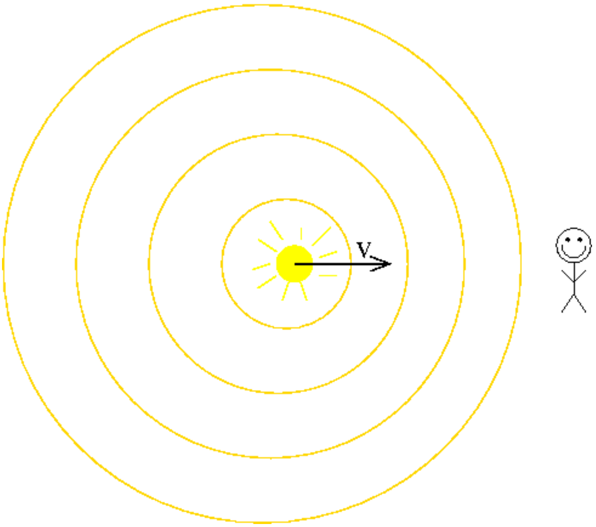
For a source moving toward an observer:

$$f_{obs} = f_{source}\sqrt{\frac{1+\beta}{1-\beta}}$$

For a source moving away switch + and -

It does not matter if it is the source or the observer that is moving; only the relative velocity matters.





Clicker question 1

Set frequency to DA

An alien on his spaceship sends a laser beam toward Earth using a souped up green laser pointer. The people on Earth observe a yellow light from the alien spaceship. Is the spaceship moving toward or away from Earth?

A. Spaceship is headed to Earth
B. Spaceship is headed away from Earth
C. Impossible to tell

Relativistic Doppler shift

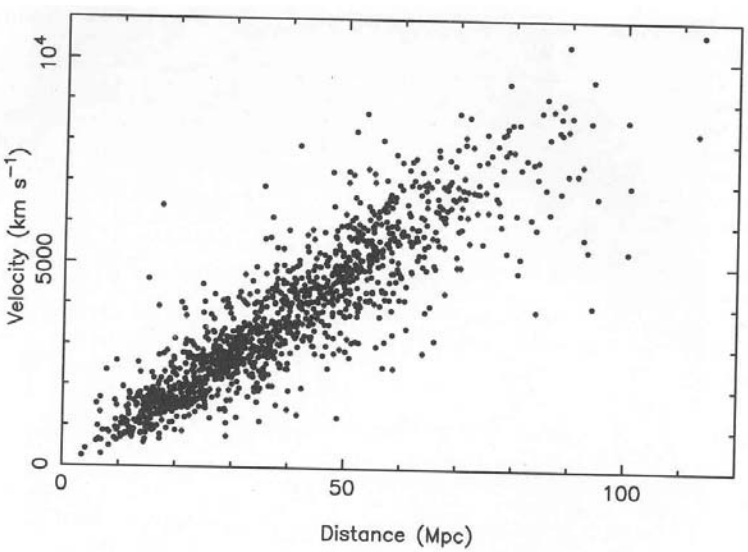
Since c is constant and $c=\lambda f$ then

$$\lambda_{obs} = \lambda_{source}\sqrt{\frac{1-\beta}{1+\beta}}$$

For approaching source, λ is shorter – blueshift

For receding source, λ is longer – redshift

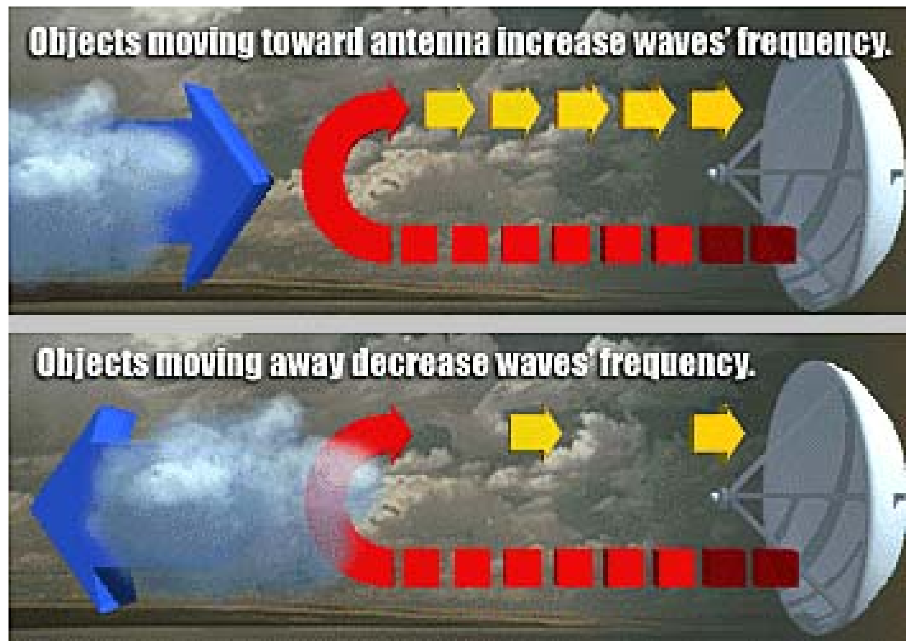
In 1929 Hubble showed the velocity of galaxies (measured using redshift) was proportional to distance. First evidence for the Big Bang theory.



Relativistic Doppler shift

Used to measure velocity in police and baseball radar guns.

Used in Doppler radar to measure the speed of the air/rain.



Moving from kinematics to dynamics

Back in Physics 1110 we started by discussing velocities and accelerations and called this kinematics.

Then we moved to Newton's laws of motion which tells us that it is force that causes acceleration. This is called dynamics.

Finally, we used conservation of momentum and conservation of energy to avoid the complication of calculating accelerations (as long as we had an isolated system).

Let's start thinking about momentum:

Classically, momentum is **p**=m**u** where we continue using **u** to represent the velocity of an object while **v** represents the velocity of a frame.

What we really need momentum for is to use conservation of momentum on problems like collisions and explosions.

Conservation of momentum

Conservation of momentum states that for an isolated system (no net force):

$$\vec{P}_{\text{total}} = \sum_i \vec{p}_i = \text{constant}$$

What if we observe this isolated system in a different inertial reference frame?

Using Galilean transformations we get (in 1D)

$$P'_{\text{total}} = \sum_i m_i u'_i = \sum_i m_i (u_i - v) = \sum_i m_i u_i - \sum_i m_i v$$

so that

$$P'_{\text{total}} = P_{\text{total}} - v \sum_i m_i$$

This just says that the momentum changes by the mass of the system times the relative velocity *v*.

The velocity between these two inertial reference frames (*v*) is constant and mass is constant so if momentum is conserved in one inertial reference frame (P_{total}) then it is conserved in all inertial reference frames (P'_{total}).

Conservation of momentum

But we know that the Galilean transformations are *not* correct at high velocity. If we apply the correct transformations we find that if momentum is conserved in one reference frame it is not necessarily conserved in other inertial reference frames.

So we need a new definition of momentum.

We defined momentum as $p = mu = m\dfrac{\Delta x}{\Delta t}$

We know that Δt depends on which inertial frame you are in but there is one time that stays the same: the proper time. This is the time measured in the rest frame and we will know call it tau ($\Delta\tau$).

We try $p = m\dfrac{\Delta x}{\Delta \tau}$ and remember time dilation: $\Delta t = \gamma \cdot \Delta\tau$

This gives us: $p = m\dfrac{\Delta x}{\Delta \tau} = \gamma m\dfrac{\Delta x}{\Delta t} = \gamma mu$

Conservation of momentum

So the relativistic momentum is: $p = \gamma_u mu$

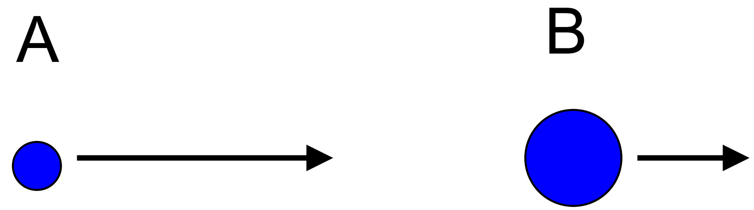
Note the addition of a subscript on γ.

Our previous use of γ was to relate between two different ***frames*** with a relative velocity of *v*. In contrast, γ_u is associated with a particle. If we measure $p=\gamma_u mu$ in one inertial frame we can convert the momentum to another inertial reference frame moving with speed *v* which will introduce another γ which we should probably call γ_v.

It should be clear by context which one we are talking about so I will probably drop the subscript after a while.

Set frequency to DA

$$p = \gamma_u m u$$



Particle A has half the mass but twice the speed of particle B. If the particles' momenta are p_A and p_B, then

A. $p_A > p_B$
B. $p_A = p_B$
C. $p_A < p_B$

Classically, both particles have the same momentum.

γ_u is bigger for the faster particle.

Momentum transformation and energy

Using the old momentum and Galilean transformation to get from S to S′ frame: $P'_{\text{total}} = P_{\text{total}} - v\sum_i m_i$

Using the relativistic momentum and the correct velocity we find:

$$P'_{\text{total}} = \sum_i \gamma_{u'_i} m_i u'_i = \gamma_v \sum_i \gamma_{u_i} m_i u_i - v\gamma_v \sum \gamma_{u_i} m_i = \gamma_v \left(P_{\text{total}} - v \sum_i \gamma_{u_i} m_i \right)$$

Since *v* and γ_v are constants, in order to have conservation of momentum in each frame, the quantity $\gamma_u m$ must also be constant.

What other quantity is conserved when no external forces act?

Energy!

$\gamma_u m$ has units of mass (kg); to give it units of energy, can multiply by c^2 (which we know is constant).

So let us postulate that energy is $E = \gamma_u mc^2$

Energy

Total energy of an object moving at speed *u* is $E = \gamma_u mc^2$

What do we get for the total energy when an object is at rest?

At rest, γ_u=1 so the rest energy is $E_{\text{rest}} = mc^2$

Maybe you have heard of this one before? ☺

Furthermore, we can define kinetic energy as the total energy minus the rest energy:

$$KE = E - E_{\text{rest}} = \gamma_u mc^2 - mc^2 = (\gamma_u - 1)mc^2$$

Remember the binomial approximation for γ is $\gamma = (1-\beta^2)^{-1/2} \approx 1 + \frac{1}{2}\beta^2$ (for small β)

Using this on the kinetic energy gives: $KE = (\gamma_u - 1)mc^2 = (1 + \frac{1}{2}\frac{u^2}{c^2} - 1)mc^2 = \frac{1}{2}mu^2$

So we get the correct kinetic energy at low speed.