Special relativity

Announcements:

- Homework solutions will be on CULearn by 5pm today.
- Next weeks homework will be on the website by 7pm today.

Hermann Minkowski (1864—1909):

Today we will look at spacetime (Minkowski) diagrams, derive velocity addition and investigate the relativistic Doppler effect.

Recall: transformations

If S′ is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames is related by:

In Galilean relativity

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

In Special relativity

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \frac{vx}{c^2}) \end{aligned}$$

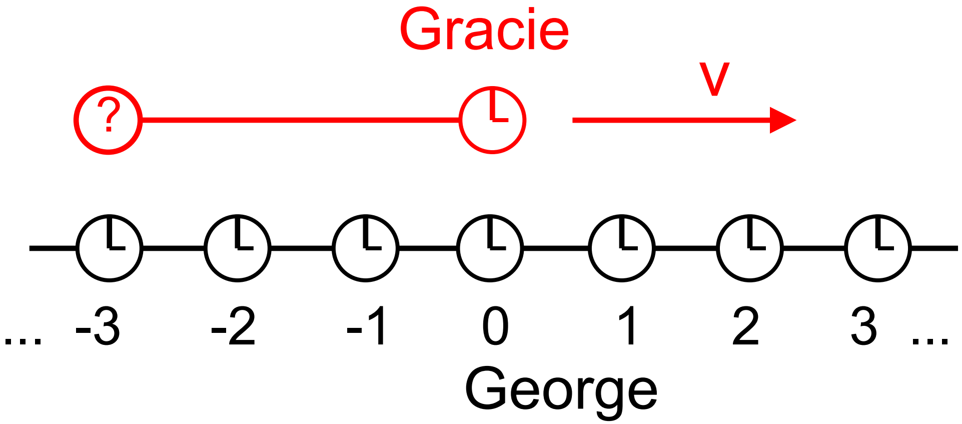
$$\begin{aligned} x &= \gamma(x' + vt) \\ y &= y' \\ z &= z' \\ t &= \gamma(t' + \frac{vx'}{c^2}) \end{aligned}$$

This assumes (0,0) is the same event in both frames.

Note that *v* is the velocity of the S′ frame relative to the S frame and can be positive or negative.

Clicker question 1

Set frequency to DA



$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

George has a set of synchronized clocks in reference frame S, as shown. Gracie is moving to the right past George, and has her own set of synchronized clocks. Gracie passes George at the event (x,t) = (x′,t′) = (0,0) in both frames. The event is the clock at x=0 striking 3 o'clock. An observer in George’s frame checks the clock marked ? at t=0. Compared to George’s clocks, this one reads

A. slightly before 3:00

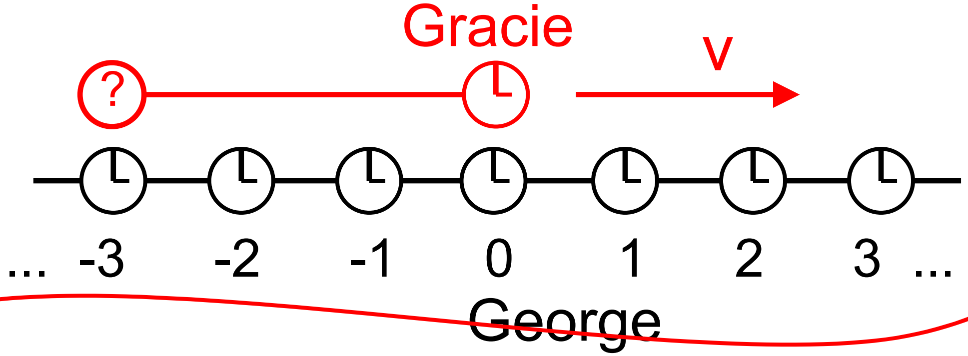
B. slightly after 3:00

C. 3:00

D. Impossible to tell

Clicker question 1

Set frequency to DA



$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Let George be the unprimed frame and Gracie be the primed frame which is moving to the left with velocity *v*.

For George, the event is at (x,t)=(−b,0) where b is positive.

In Gracie's frame, the event in question is at (x′,t′) given by:

$x' = \gamma(-b - v0) = -\gamma b$ (not what we are looking for)

$t' = \gamma\left(0 - \frac{v(-b)}{c^2}\right) = \frac{\gamma vb}{c^2}$ where b>0 so t′>0

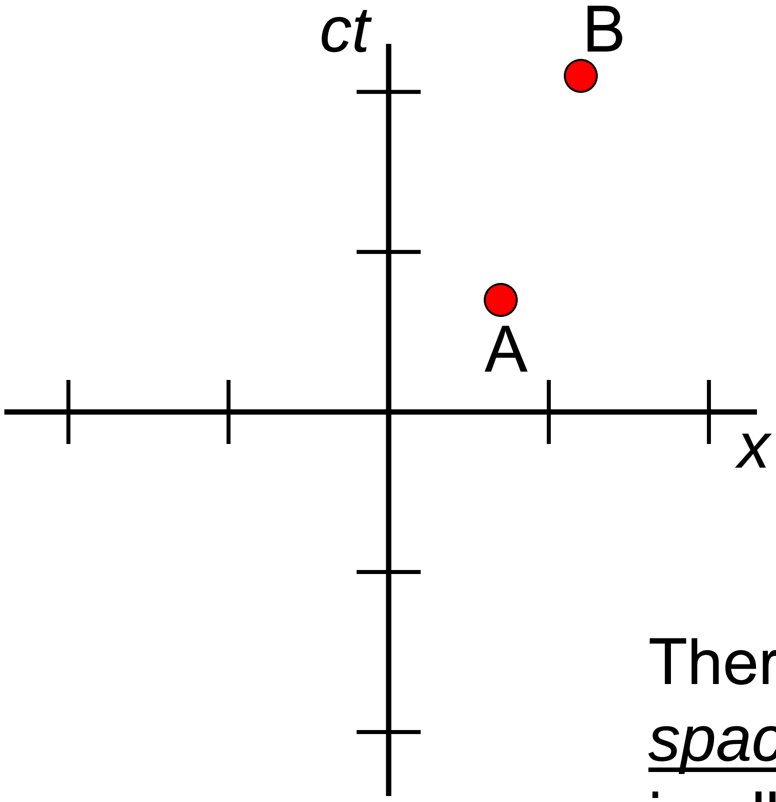
A. slightly before 3:00

B. slightly after 3:00

C. 3:00

D. Impossible to tell

Spacetime (Minkowski) diagrams



Consider two events, **A** and **B**, in spacetime as shown.

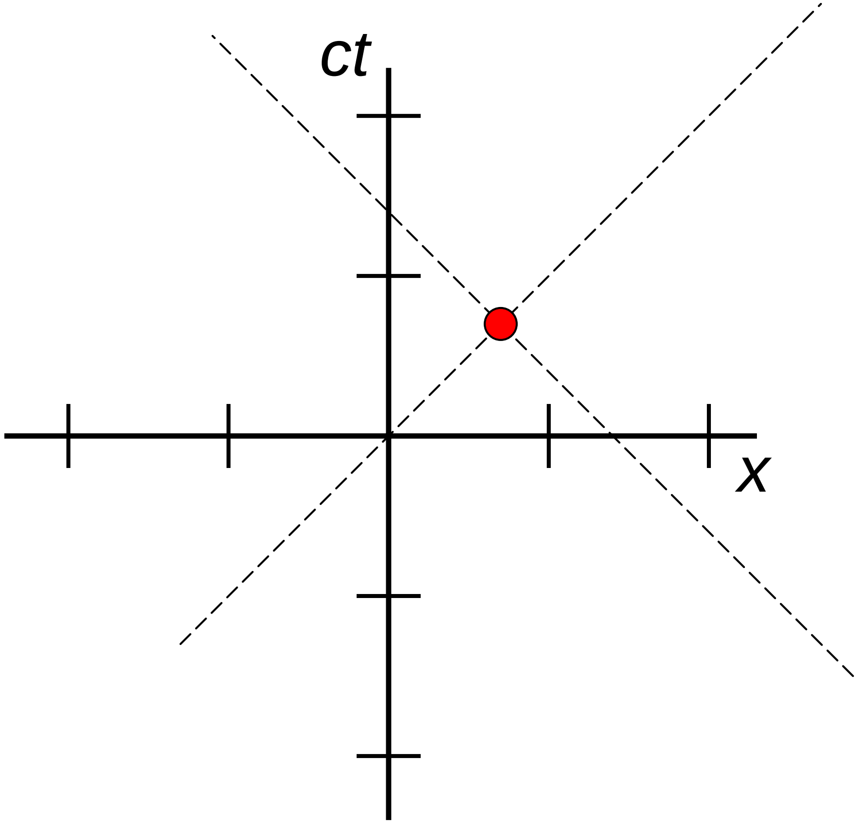
The 3-D distance between them may be different in different inertial frames due to length contraction.

There is a 4-D distance, called the *spacetime interval* which **is** an invariant in all inertial reference frames.

$$\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

If you apply the Lorentz transformations to this quantity you get back the same result.

Spacetime (Minkowski) diagrams



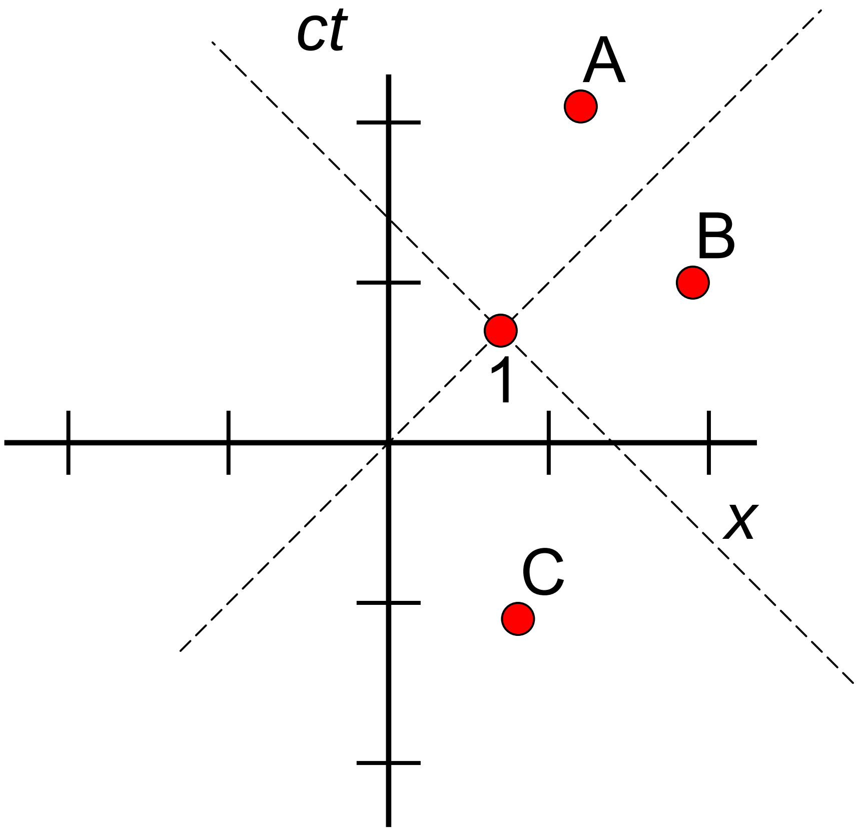
Here is an event in spacetime.

Any light signal that passes through this event has the dashed world lines. These identify the *light cone* of this event.

Light cones are 45° lines on spacetime diagrams.

Clicker question 2

Set frequency to DA



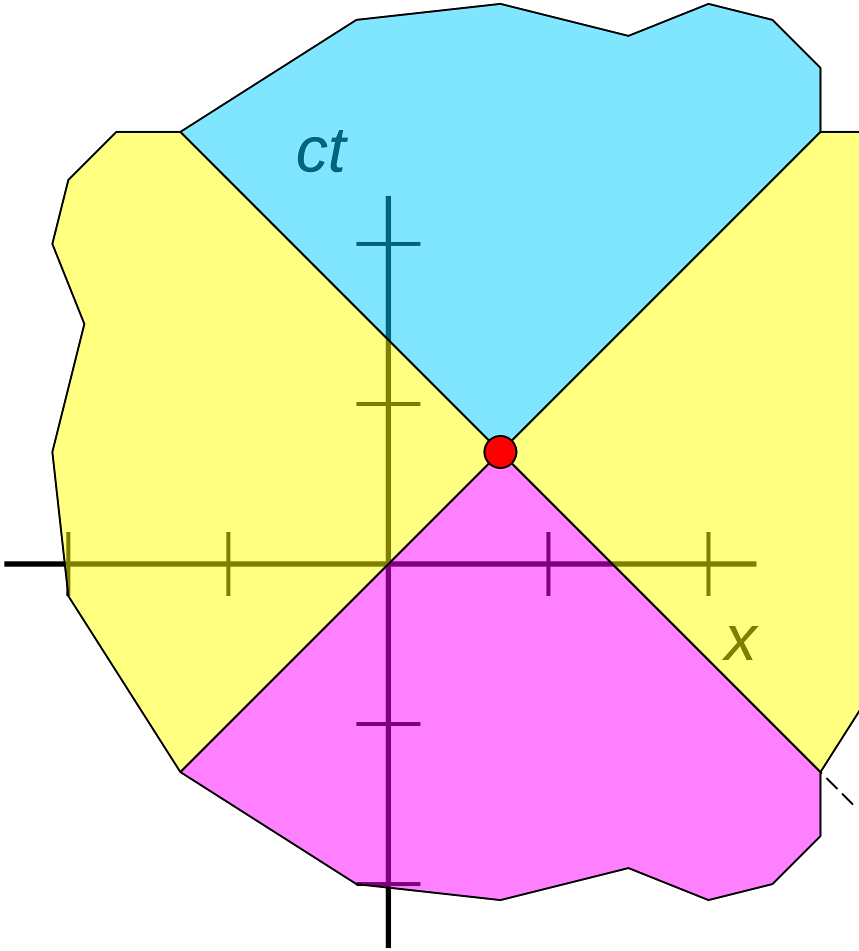
Starting from Event **1**, which other events could be reached? That is, what other events is it possible to travel to?

[✓ A. A]
B. B
C. C
D. More than one of the above
E. None of the above

Reaching point **B** requires going faster than the speed of light

Reaching point **C** requires going into the past.

Spacetime (Minkowski) diagrams

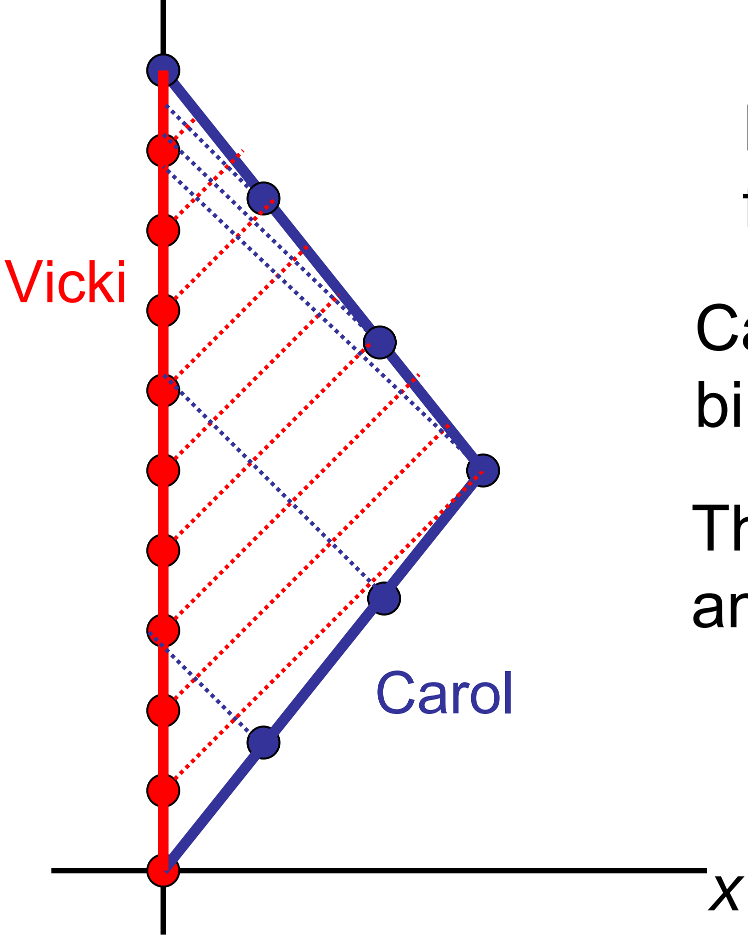


The blue area is the *future* for this event.

The purple area is the *past* for this event.

The yellow area is *elsewhere* and cannot be causally connected to this event because a signal would need to travel faster than light to connect the events.

Twin paradox revisited



From the Earth frame, the worldlines for Vicki and Carol are shown.

Can imagine that each twin sends out birthday greetings every other year.

This gives an idea about what Vicki and Carol actually experience.

Reading quiz 1

Set frequency to DA

Please answer this question on your own.
No discussion until after.

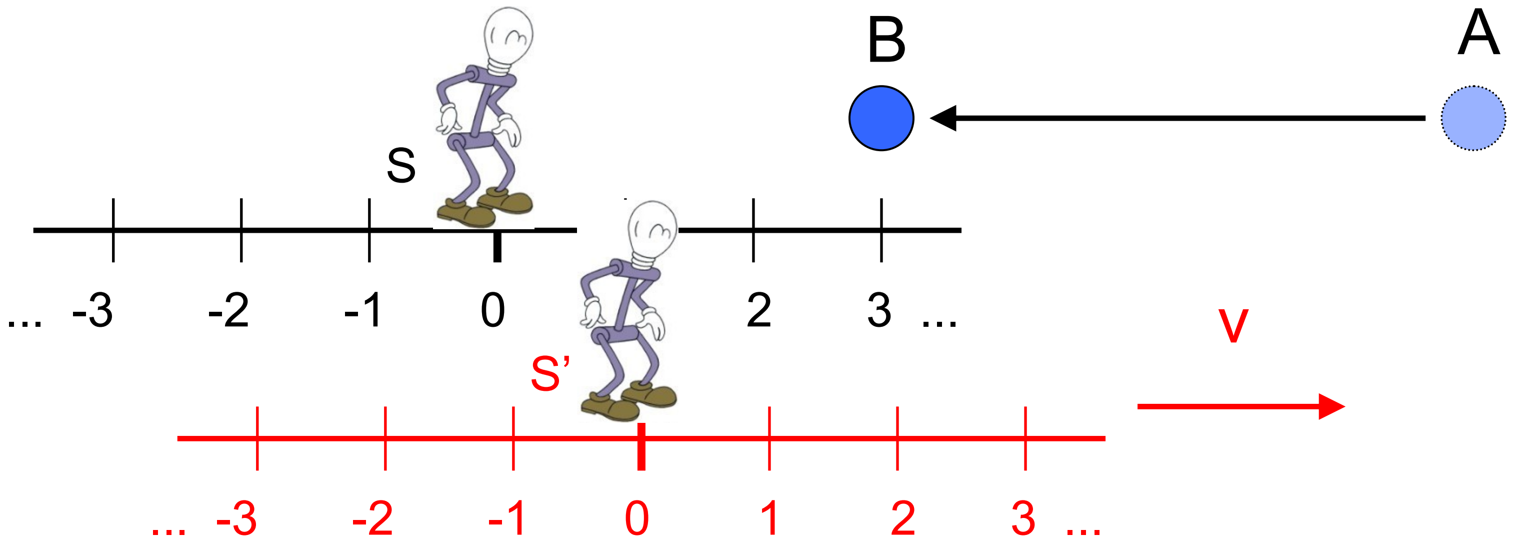
The classical velocity addition formula is $\vec{u}' = \vec{u} - \vec{v}$.
This does not work in special relativity because…

A. It is not symmetric between the primed and unprimed frames
B. Velocity is not a well defined concept in special relativity
C. It would allow a measured speed of light which is not the same in all inertial reference frames
D. More than one of the above
E. None of the above

If light has a speed of *c* in **S**, it would have a speed of *c-v* in **S′**.

Velocity

An object moves from event A to event B.



As seen from frame S the velocity is: $u = \dfrac{\Delta x}{\Delta t}$

As seen from frame S′ the velocity is: $u' = \dfrac{\Delta x'}{\Delta t'}$

Velocity transformation

Using the Lorentz transformation for *x*′ and *t*′ we get:

$$u' = \frac{\Delta x'}{\Delta t'} = \frac{\gamma(\Delta x - v\Delta t)}{\gamma(\Delta t - (v/c^2)\Delta x)}$$

Cancel the γ and divide top and bottom by Δ*t* to get

$$= \frac{\frac{\Delta x}{\Delta t} - v}{1 - (v/c^2)\frac{\Delta x}{\Delta t}} = \frac{u - v}{1 - uv/c^2}$$

The velocity addition formula in special relativity is:

$$u' = \frac{u - v}{1 - uv/c^2}$$

Galilean result

New in special relativity

Note that *u* and *u*′ are the velocities of an object in a frame while *v* is the velocity of one frame to another (S′ velocity relative to S).

Clicker question 3

Set frequency to DA

A spacecraft travels at speed v=0.5c relative to the Earth. It launches a missile in the forward direction at a speed of 0.5c. How fast is the missile moving relative to Earth?

$$u' = \frac{u - v}{1 - uv/c^2}$$

- A. 0
- B. 0.25c
- C. 0.5c
- D. 0.8c
- E. c

This actually uses the inverse transformation:

$$u = \frac{u' + v}{1 + u'v/c^2}$$

Have to keep signs straight. Depends on which way you are transforming. Also, the velocities can be positive or negative!

Best way to solve these is to figure out if the speeds add or subtract and then use the appropriate formula.

Since the missile if fired forward in the spacecraft frame, the spacecraft and missile velocities **add** in the Earth frame.

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.5c + 0.5c}{1 + 0.5c \cdot 0.5c/c^2} = \frac{c}{1.25} = 0.8c$$

Velocity addition works with light too!

A Spacecraft moving at 0.5c relative to Earth sends out a beam of light in the forward direction. What is the light velocity in the Earth frame?

$$u' = \frac{u - v}{1 - uv/c^2}$$

$$u = \frac{u' + v}{1 + u'v/c^2}$$

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{c + 0.5c}{1 + c \cdot 0.5c/c^2} = \frac{1.5c}{1.5} = c$$

What about if it sends the light out in the backward direction?

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{-c + 0.5c}{1 - c \cdot 0.5c/c^2} = \frac{-0.5c}{0.5} = -c$$

It works. We get the same speed of light no matter what!