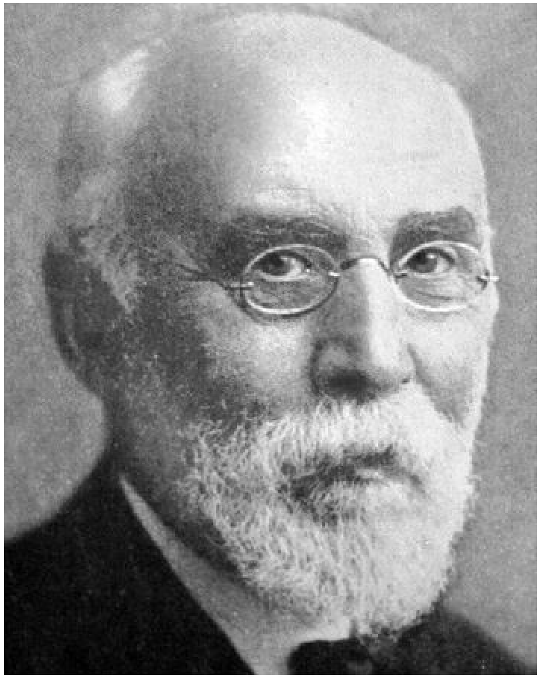
Special relativity

Announcements:

- Homework solutions are on CULearn
- Remember problem solving sessions today from 3-5 and tomorrow from 3-4 and 5-6.
- Homework is due Wednesday at 12:50pm in wood cabinet in G2B90



Hendrik Lorentz (1853—1928):

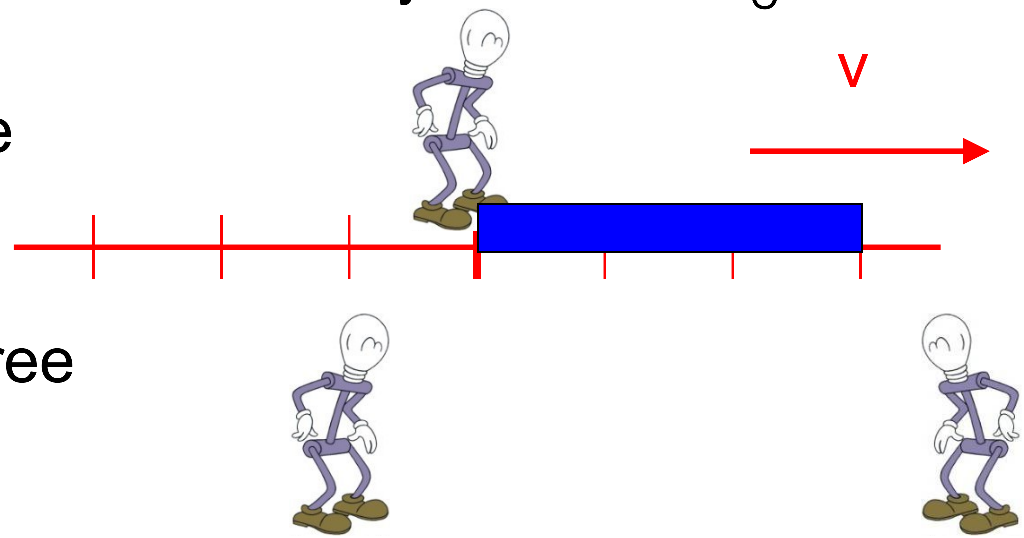
Today we will derive the Lorentz transformation, investigate the twin paradox and look at spacetime diagrams.

Clicker question 1

Set frequency to DA

Curly runs by real fast with a stick he knows to be of length L_C. Larry and Moe are at rest and each measures the stick as it goes by. How are the three measurements related?

Curly measures L_C



Larry measures L_L

Moe measures L_M

A. $L_C < L_L < L_M$
B. $L_C > L_L > L_M$
C. $L_C = L_L = L_M$
D. $L_C < L_L = L_M$
E. $L_C > L_L = L_M$

Curly measures the proper length L_0.

Larry and Moe measure a shorter length due to length contraction: $L = L_0/\gamma$.

Summary of time dilation and length contraction

Factor of gamma always shows up: $$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

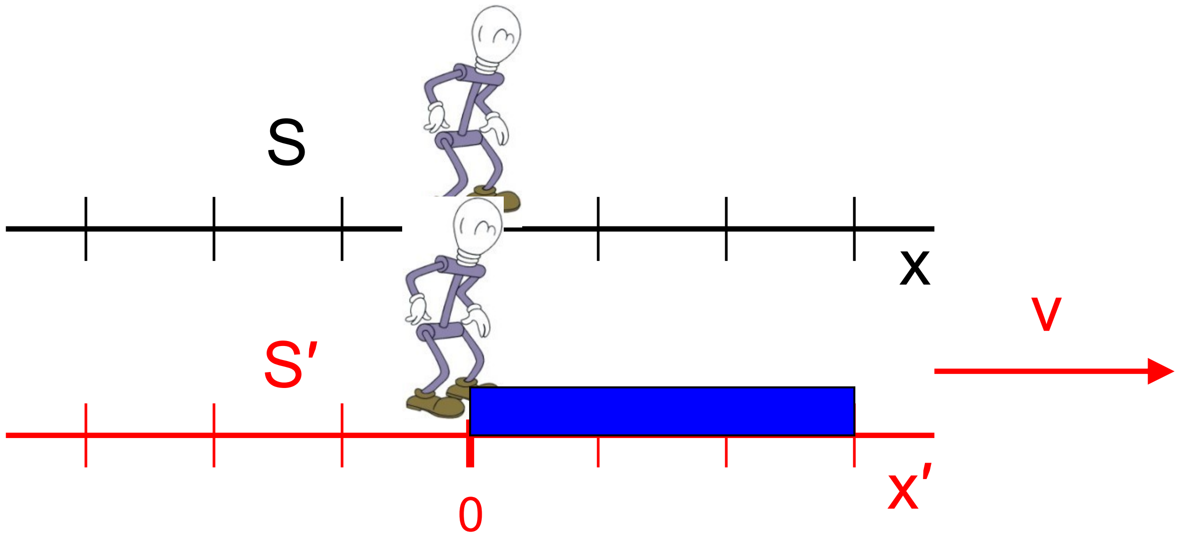
Time dilation – moving clocks run slower: $$\Delta t = \gamma \Delta t_0$$

The rest frame time (proper time) is Δt_0 and is the time in the moving system rest frame. It is shorter than the time measured in the frame where the system is moving.

Length contraction – moving objects are shorter (in the direction of motion): $$L = L_0 / \gamma$$

The length of an object in motion (L) is less than the rest frame length (proper length) (L_0).

The Lorentz transformation

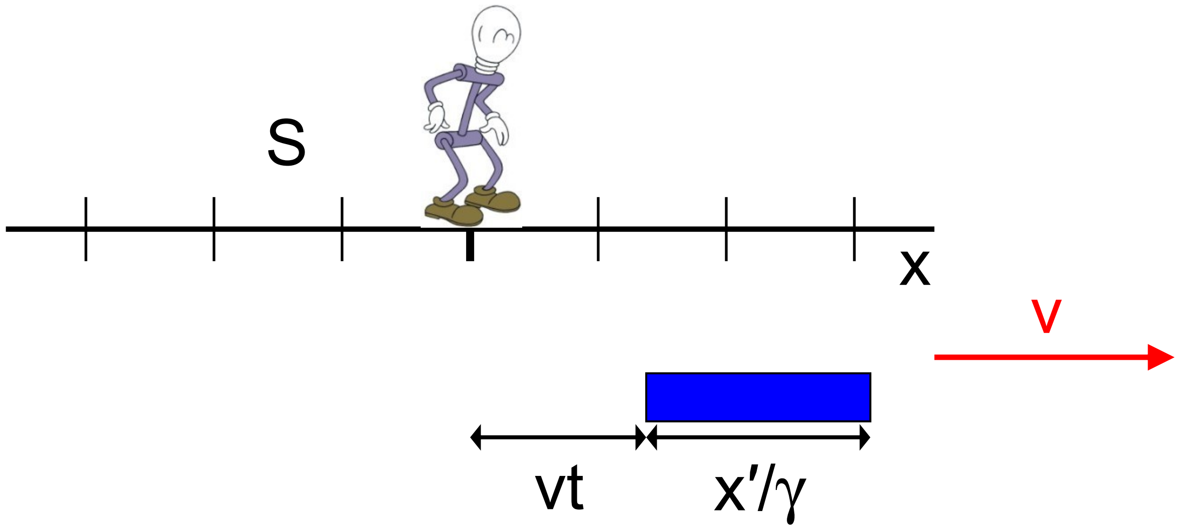


A stick is at rest in S′. Its endpoints are the events (position, c•time) = (0,0) and (x′,0) in S′. S′ is moving to the right with respect to frame S.

Event 1 – left of stick passes origin of S. Its coordinates are (0,0) in S and (0,0) in S′.

Clicker question 2

Set frequency to DA



Q. In the S frame, the stick's length is x'/γ by length contraction. Time *t* passes. According to S, where is the *right* end of the stick?

A. $x = vt$
B. $x = -vt$
C. $x = vt + x'/\gamma$
D. $x = -vt + x'/\gamma$
E. $x = vt - x'/\gamma$

With a little algebra we can rewrite this as $x' = \gamma(x - vt)$

This transformation gives the coordinate in one frame if you have the coordinate (and time) in the other frame.

Transformations

If S′ is moving with speed v in the <u>positive</u> *x* direction relative to S, then the coordinates of the same event in the two frames is related by:

In Galilean relativity

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt)$$

$$y' = y$$

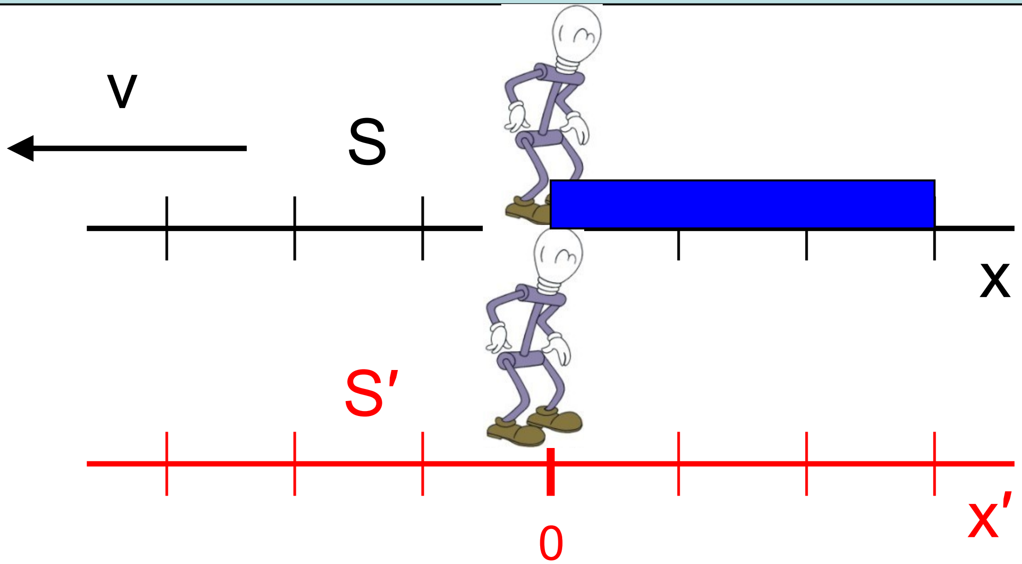
$$z' = z$$

$$t' = \gamma(t - \frac{v}{c^2}x)$$

In a minute…

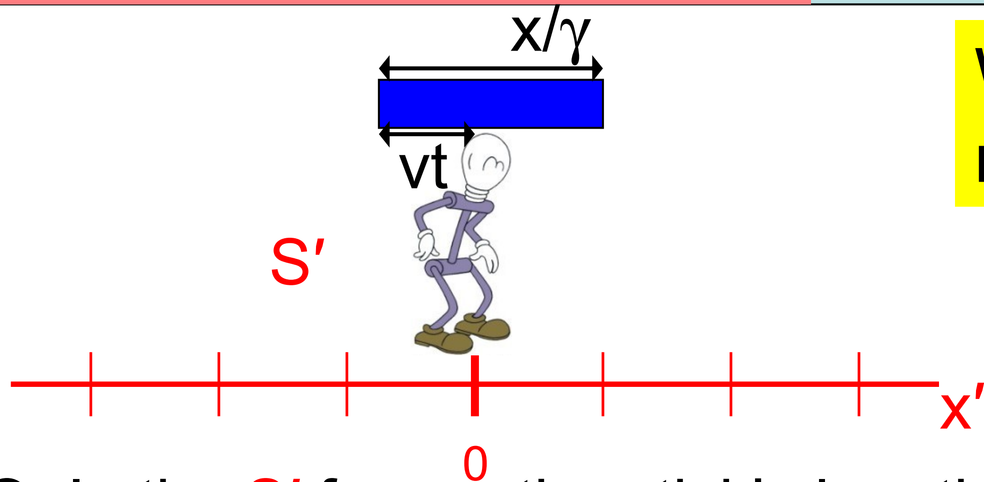
Note that when $v \to 0$, then $\gamma \to 1$ and we recover the Galilean transformations.

The Lorentz transformation



A stick is at rest in S. Its endpoints are the events (position, c•time) = (0,0) and (x,0) in S. S is moving to the left with respect to frame S′.

Event 1 – left of stick passes origin of S′. Its coordinates are (0,0) in S and (0,0) in S′.



With a little algebra we can rewrite this as $x = \gamma(x' + vt')$

Q. In the S′ frame, the stick's length is x/γ by length contraction. Time *t* passes. According to S′, where is the *right* end of the stick?

A. $x' = vt'$
B. $x' = -vt'$
C. $x' = vt' + x/\gamma$
D. $x' = -vt' + x/\gamma$
E. $x' = vt' - x/\gamma$

Can also solve for t′:

$$t' = \frac{x}{v\gamma} - \frac{x'}{v}$$

and put in our x′ transformation to get:

$$t' = \frac{x}{v\gamma} - \frac{\gamma(x - vt)}{v}$$

After some algebra you can get

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Transformations

If S′ is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames is related by:

In Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

This assumes (0,0) is the same event in both frames.

Twin paradox

Vicki

v →

Carol

Vicki stays on Earth and twin Carol departs for the star Sirius, 8 light-years away, traveling at a speed v = 0.8 c (γ = 5/3). We found this journey takes 10 years each way according to Vicki (8 c·years/0.8 c = 10 years). Due to time dilation, only 6 years elapse for Carol each way: $\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{10 \text{ years}}{5/3} = 6 \text{ years}$

As Carol goes to Sirius at 0.8c in her reference frame, the distance to Sirius is $L = L_0 / \gamma = 8\,\text{c} \cdot \text{yrs} / (5/3) = 4.8\,\text{c} \cdot \text{yrs}$ which she covers in 6 years. Since she sees the Earth receding at 0.8c she figures time is running slow there so less than 6 years pass on Earth: $\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{6 \text{ years}}{5/3} = 3.6 \text{ years}$

Twin paradox resolution

Vicki

Carol

Multiplying by 2 for the return trip we would come to following conclusions:

Vicki says she ages 20 years and Carol ages 12 years

Carol says she ages 12 years and Vicki ages 7.2 years

Both statements cannot be true so someone must be wrong!

Simple answer: Vicki is in a single inertial reference frame during the entire trip. Carol is in at least 2 inertial reference frames during the trip (out and back). Therefore the problem is **not** symmetric. Carol's reasoning is incorrect.

Twin paradox analysis using Lorentz transformations

Vicki

Carol

v →

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{v}{c^2}x)$$

Three inertial reference frames: S=Earth, S′=trip out, S″=return trip

3 events: Leaving Earth (1), reaching Sirius (2), reaching Earth (3)

Let *T* be the time to reach Sirius in Earth frame (10 years).

Then *vT* is the distance to Sirius in the Earth frame.

Event 1 (leaving)

$$(x,t) = (0,0)$$

$$(x',t') = (0,0)$$

Event 2 (Sirius)

$$(x,t) = (vT,T)$$

$$(x',t') = (\gamma[vT - vT], \gamma[T - vvT/c^2])$$

$$= (0, \gamma T[1-\beta^2]) = (0, T/\gamma)$$

$$(x'',t'') = (0, T/\gamma)$$

S″ frame will also have $x''=0$ and start at time $t''=T/\gamma$.

Twin paradox analysis using Lorentz transformations

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

In the S″ frame the transformations will be

$$x'' = \gamma([x - vT] + v[t - T])$$

$$t'' = \frac{T}{\gamma} + \gamma\left([t - T] + \frac{v[x - vT]}{c^2}\right)$$

This is because in Earth frame we are starting at *x=vT* and *t=T* so *x* becomes *x−vT* and *t* becomes *t−T*. Also, at *t=T* we are at *t″=0.*

<u>Event 1 (leaving)</u>

$$(x,t) = (0,0)$$
$$(x',t') = (0,0)$$

<u>Event 2 (Sirius)</u>

$$(x,t) = (vT,T)$$
$$(x',t') = (0,T/\gamma)$$
$$(x'',t'') = (0,T/\gamma)$$

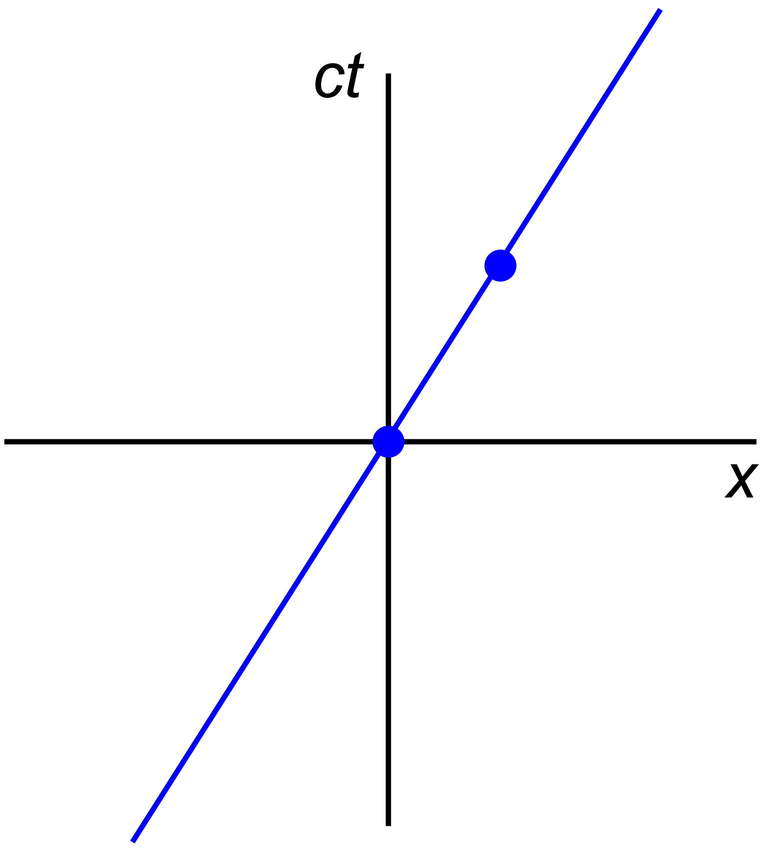
<u>Event 3 (return)</u>

$$(x,t) = (0,2T)$$
$$(x'',t'') = (0,2T/\gamma)$$

$$x'' = \gamma([0 - vT] + v[2T - T]) = \gamma(-vT + vT) = 0$$

$$t'' = \frac{T}{\gamma} + \gamma\left([2T - T] + \frac{v[0 - vT]}{c^2}\right) = \frac{T}{\gamma} + \gamma T\left(1 - \beta^2\right) = \frac{2T}{\gamma}$$

Space-time (Minkowski diagrams)



Remember events are defined by a coordinate giving position and time. Can write as (*x*,*y*,*z*,*ct*).

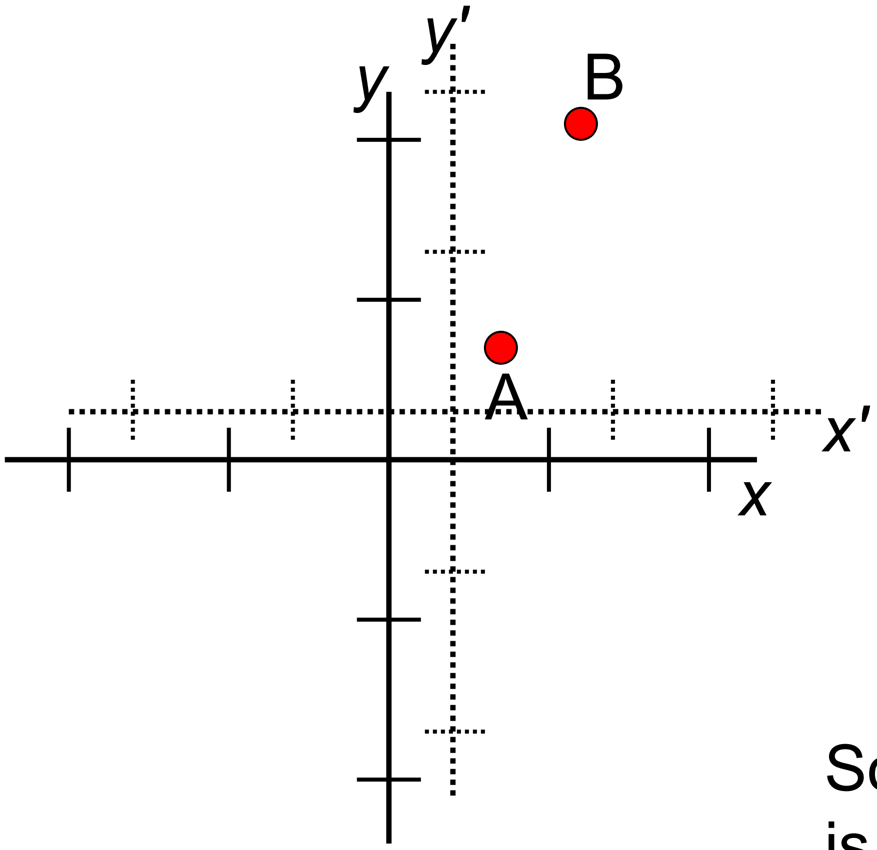
Suppose something is moving to the right in frame S.

It starts at (*x*,*t*) = (0,0).

It moves to positive *x* at positive time.

Connect the dots – this is the **worldline**.

Spacetime



Consider two points, **A** and **B**, in regular geometry as shown.

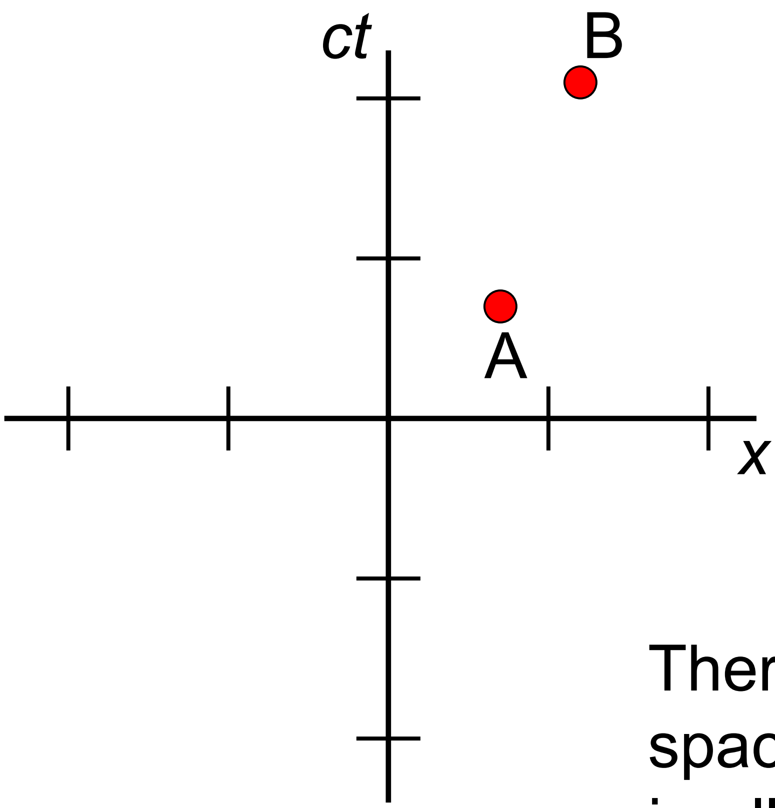
Changing coordinate systems will change the position of each point.

But the distance between them Δr is constant.

So the distance between two points is an **invariant** in regular geometry.

Due to length contraction, this is not true in relativity. Different inertial frames will not find the same distance between two points.

Spacetime



Now consider two events, **A** and **B**, in spacetime as shown.

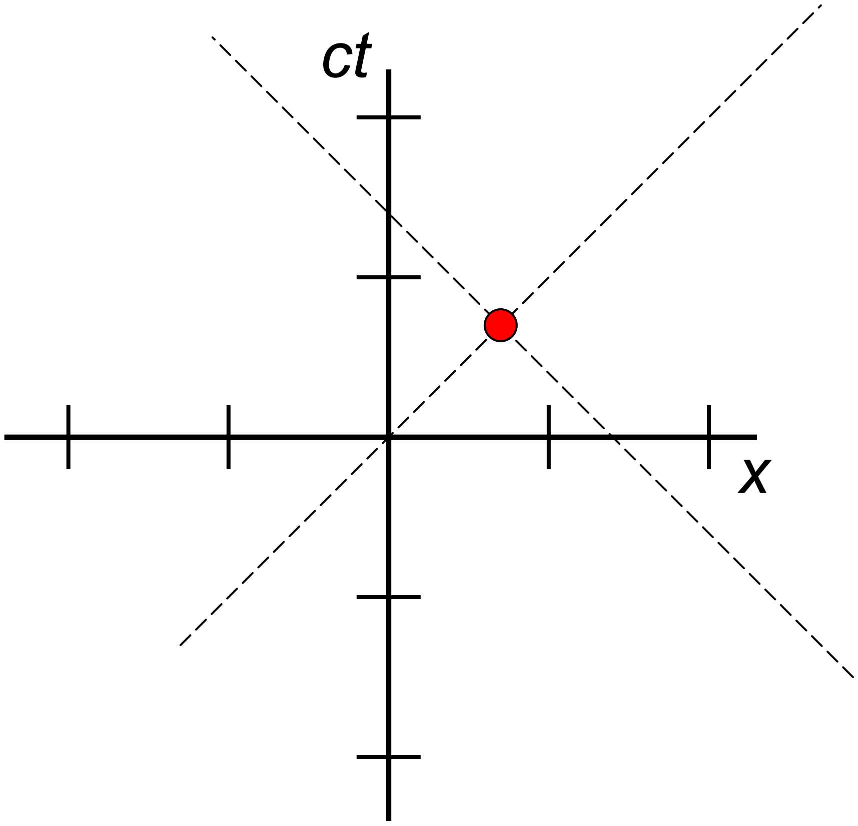
The 3-D distance between them may be different in different inertial frames due to length contraction.

There is a 4-D distance, called the spacetime interval which **is** an invariant in all inertial reference frames.

$$\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

If you apply the Lorentz transformations to this quantity you get back the same result.

Spacetime



Here is an event in spacetime.

Any light signal that passes through this event has the dashed world lines. These identify the *light cone* of this event.

Light cones are 45° lines on spacetime diagrams.