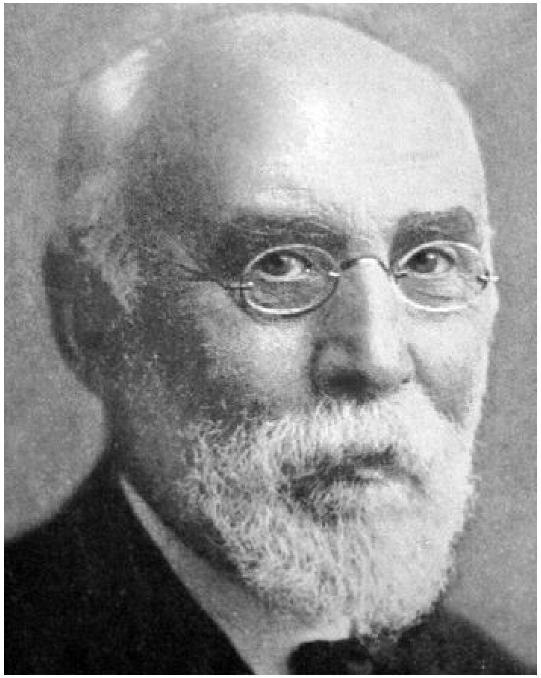
Special relativity

Announcements:

- Homework solutions are on CULearn
- Remember problem solving sessions next week (M3-5 and T3-4,5-6).

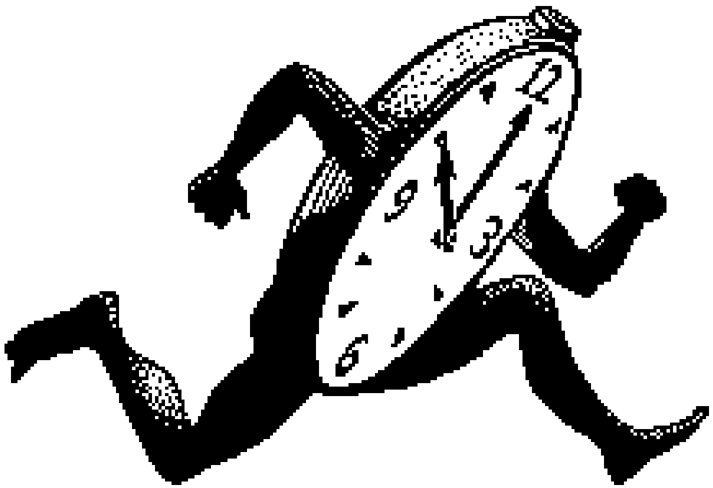


Hendrik Lorentz (1853—1928):

Today we will derive length contraction and Lorentz transformations. Monday will be the twin paradox and velocities.

Summary of Time Dilation

- The proper time Δt_0 is the time between two events in the reference frame where both events take place at the same location.
- So, the proper time of a clock keeping time is the time in the reference frame where the clock is not moving.
- In an inertial reference frame in which the clock is moving, the moving clock will be slower by a factor of γ: $\Delta t = \gamma \cdot \Delta t_0$



where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $\beta = \frac{v}{c}$

Time Dilation

How come we never notice the effect of time dilation?

How much time will your watch lose on a non-stop flight from Los Angeles to Sydney (11 hours at 300 m/s)?

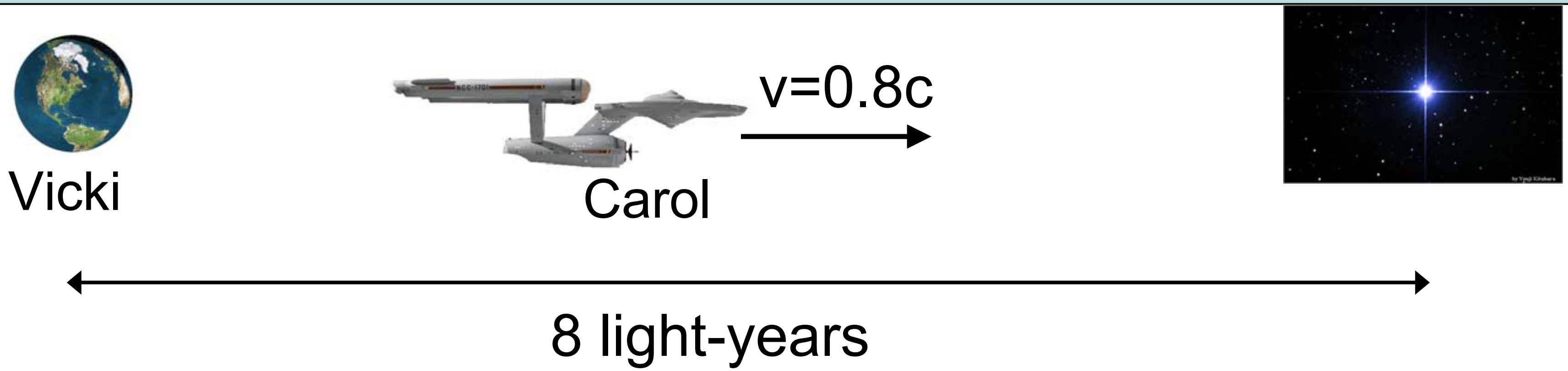
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\left(\frac{3\times10^2\ \text{m/s}}{3\times10^8\ \text{m/s}}\right)^2}} = \frac{1}{\sqrt{1-10^{-12}}} = 1.0000000000005$$

Time dilation is $\Delta t = \gamma \Delta t_0$

If our watch measures 11 hours, the time recorded on the ground will be longer: $\Delta t = \gamma \Delta t_0$ = 11 hours + 20 ns

Not usually noticeable but in fact measured in 1971 by carrying atomic clocks onboard commercial airplanes around the world

Length contraction



We found that only 6 years pass in Carol's reference frame as she travels 8 light-years to Sirius at 0.8 c.

We say that every inertial frame is valid so won't she observe the space ship going faster than the speed of light (8 light years in 6 years!)?

This discrepancy is resolved by length contraction – moving objects are shorter (in the direction of motion) $L = L_0 / \gamma$

Length contraction

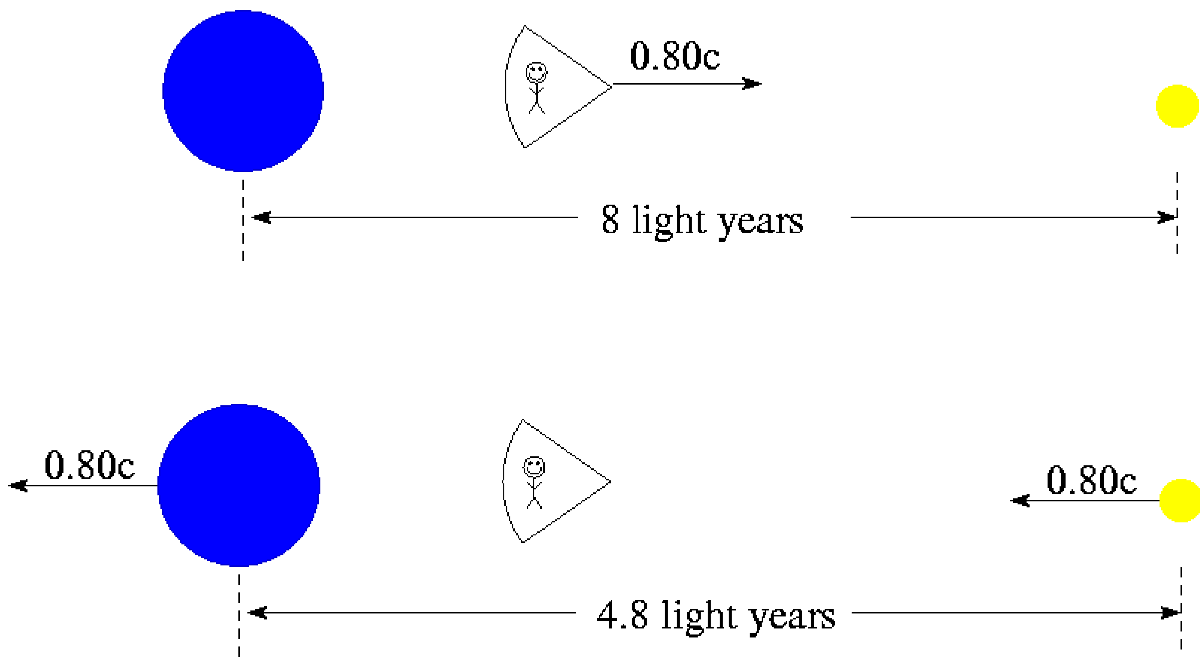
Carol sees a star approaching at 0.80c which gives

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-0.8^2}} = \frac{1}{\sqrt{1-.64}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6} = 5/3$$

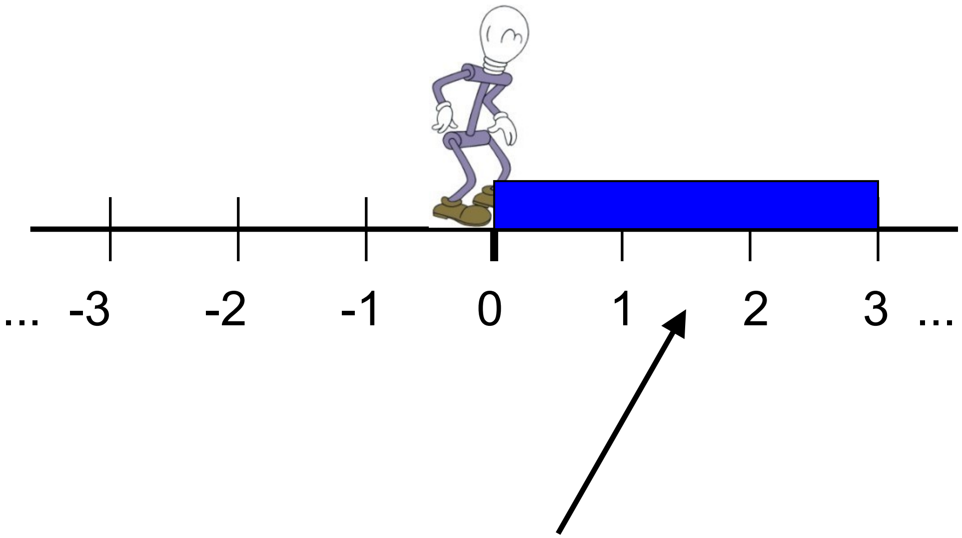
and measures the length between Earth and star as

$$L = L_0 / \gamma = 8\,\mathrm{c}\cdot\mathrm{yrs} / (5/3) = 4.8\,\mathrm{c}\cdot\mathrm{yrs}$$

So in Carol's frame the ship covers 4.8 light years in 6 years which works out to a velocity of 4.8/6 = 0.8c, same as the velocity seen on earth by Vicki.



Length of an object



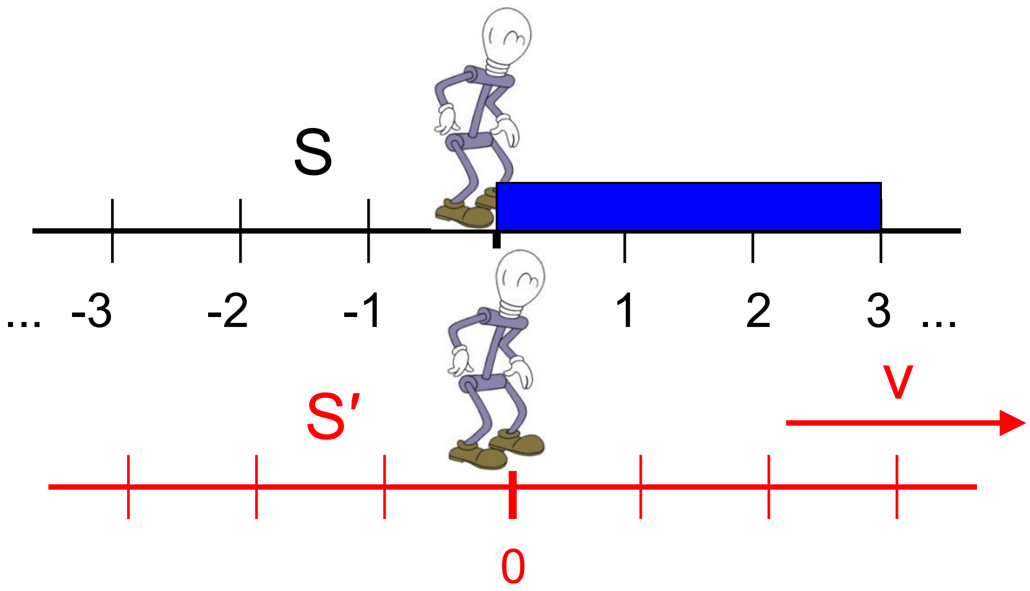
This length, measured in the stick's rest frame, is its **proper length**.

This stick is 3m long. I measure both ends at *the same time* in my frame of reference.

Or not. It doesn't matter, because the stick isn't going anywhere.

But as we know, "at the same time" is relative – it depends on how you're moving.

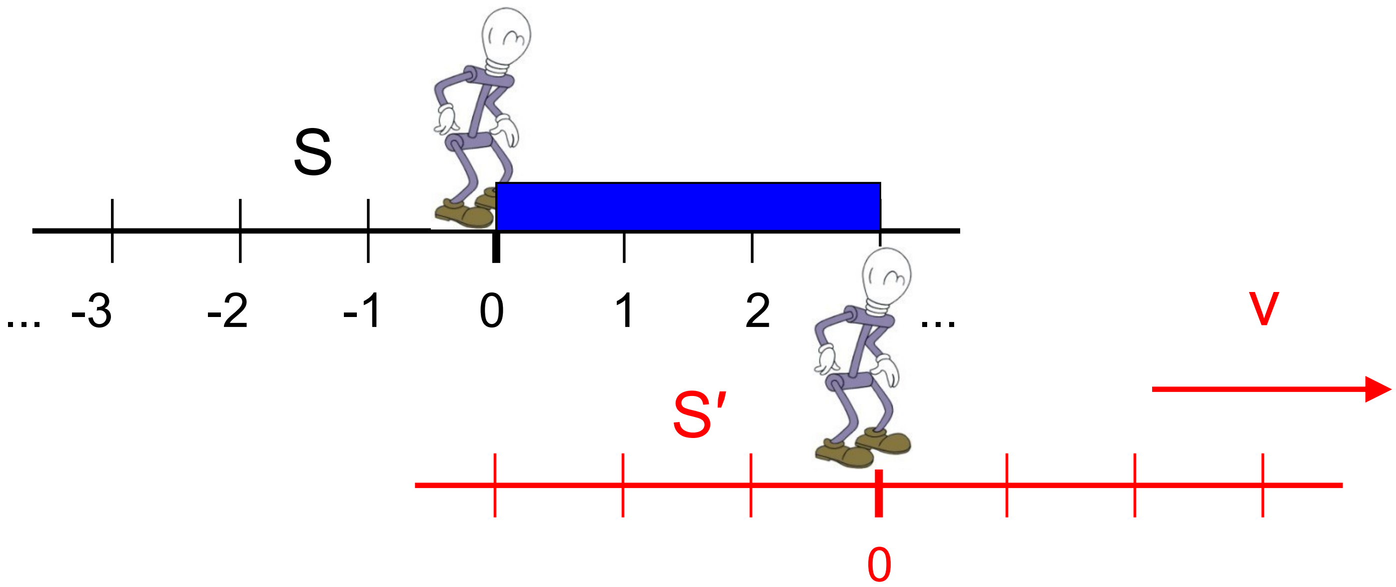
Length of an object



Another observer comes whizzing by at speed v. This observer measures the length of the stick, *and keeps track of the time*.

Event 1 – Origin of S′ passes left end of stick.

Length of an object



Event 1 – Origin of S′ passes left end of stick.

Event 2 – Origin of S′ passes right end of stick.

How many observers are required to measure the time between these two events in reference frame S and S′?

For S: 2 observers at 0 and 3.

For S′: just one at the origin

Clicker question 1 — Set frequency to DA

In frame S:

length of stick = L (this is the proper length)

time between measurements = Δt

speed of frame S′ is $v = L/\Delta t$

In frame S′:

length of stick = L′ (this is what we're looking for)

time between measurements = $\Delta t'$

speed of frame S is $v = L'/\Delta t'$

Q. Which is the correct time dilation formula for this setup?

A. $\Delta t = \gamma \Delta t'$ B. $\Delta t' = \gamma \Delta t$

Follow the proper time!

Length contraction

Speeds are the same (both refer to the relative speed).

And so $v = \dfrac{L}{\Delta t} = \dfrac{L'}{\Delta t'}$ which gives $L'\Delta t = L\Delta t'$

Using $\Delta t = \gamma \Delta t'$ to replace Δt′ we arrive at $L' = \dfrac{L}{\gamma}$

and identifying L as the *proper length* we get the length contraction formula

$$L = \frac{L_0}{\gamma}$$

Length in moving frame

Length in stick's rest frame (proper length)

Length contraction is a consequence of time dilation (and vice-versa).

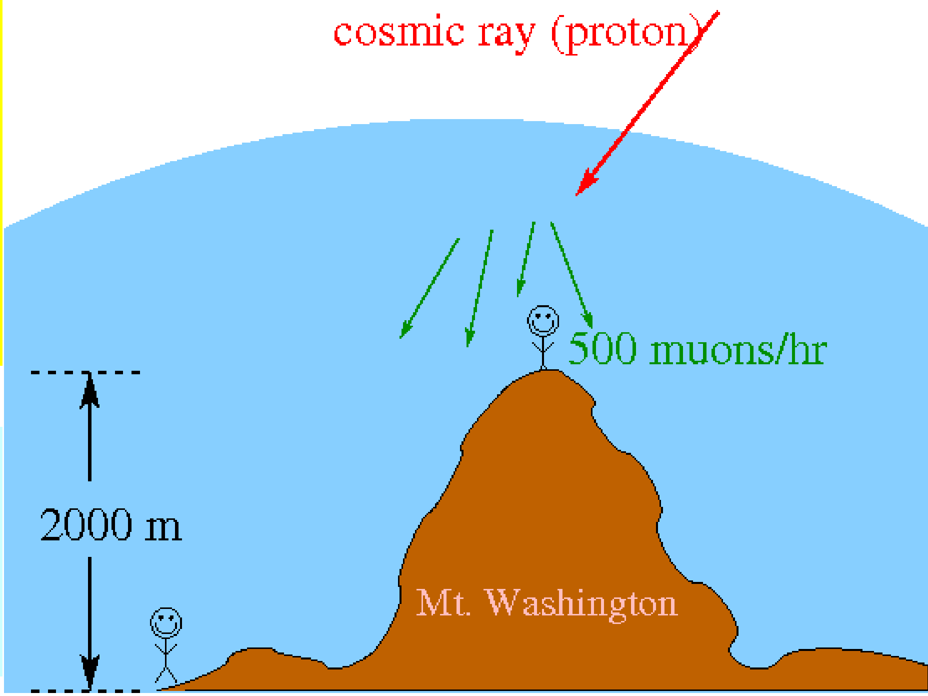
Experimental tests of time dilation & length contraction

Muons are elementary particles similar to electrons but they decay (into other particles) in about 2.2 μs (at rest).

Muons can be produced from cosmic rays interacting in the upper atmosphere.

Detectors to count muons with v=0.99c were placed on top of Mt. Washington (2000 m above sea level) and 563 muons/hour were observed.

The detectors were then moved to sea level to see what fraction of the muons survived.



Experimental tests of time dilation & length contraction

Down the mountain at 0.99c takes t=d/v=2000m/(0.99•3•10^{8}m/s)=6.7μs.
Muons live ~2.2μs so might expect very few at sea level (~25 per hr).

However, from special relativity, the muon clock is slow and the 6.7μs observed on Earth (Δt) will be $\Delta t_0 = \Delta t / \gamma$. Calculating we find

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.99)^2}} = 7.1$$ which gives $$\Delta t_0 = 6.7\,\mu s / 7.1 = 0.94\,\mu s$$

Since 0.94μs is less than 2.2μs, most should survive (about 400 muons/hr).

Experimentally confirmed in 1963

Run	On Mt. Washington	At Cambridge
1	568	412
2	554	403
3	582	436
4	527	395
5	588	393
6	559	...
Av hourly rate	563±10	408±9

Clicker question 2

Set frequency to DA

Suppose the experimenters now measure muons traveling at 0.8c (instead of 0.99c) and measure from a mountain 0.8•2000m=1600m high.

Q. Relative to the survival fraction observed in the initial experiment, will the fraction of muons that survive to sea level

A. Increase
B. Decrease
C. Remain the same
D. Impossible to tell

The transit time is the same 1600m/0.8c = 6.7μs

But $\gamma = \frac{1}{\sqrt{1-(0.8)^2}} = 1.7$ so less time dilation: $\Delta t_0 = 6.7\mu s / 1.7 = 3.9\ \mu s$

For the muon, t=3.9μs which is longer than before (0.94μs) so less muons survive (more decay)

Clicker question 3

Set frequency to DA

Suppose you were piggybacking on the back of a muon traveling at 0.99c toward the Earth

Q. What would you measure for the height of Mt. Washington (which is observed by an Earthling to be 2000 m)?

A. 2000 m

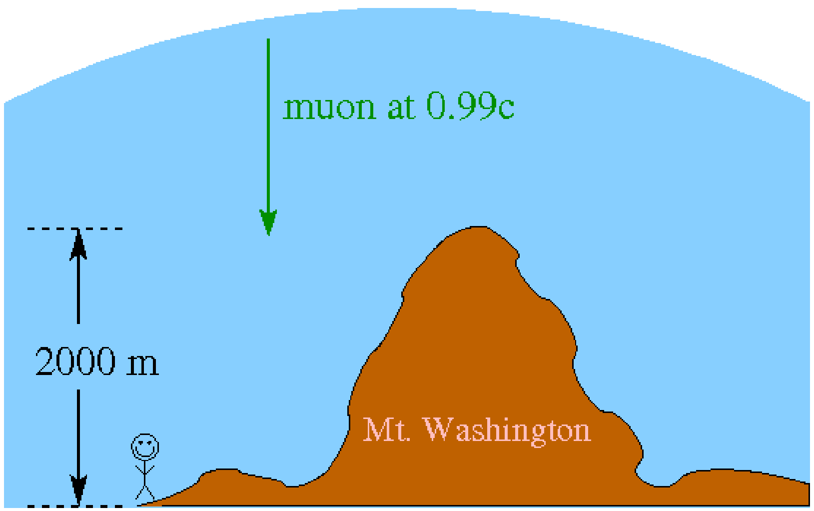
B. Less than 2000 m

C. Greater than 2000 m

D. Impossible to tell

Comparison of Earth and muon frames

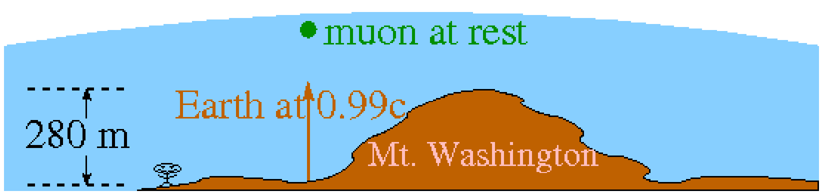
Earth observer sees 2000 m high mountain with 0.99c muon coming down and taking 6.7μs to make the trip



Muon sees Earth approaching at 0.99c and measures the height of Mt. Washington to be

$$L = L_0 / \gamma = 2000\ \text{m} / 7.1 = 280\ \text{m}$$

Getting down the mountain takes t = d/v = 280m/0.99c = 0.94μs in muon frame (same as expected by time dilation)



Summary of time dilation and length contraction

Factor of gamma always shows up: $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$

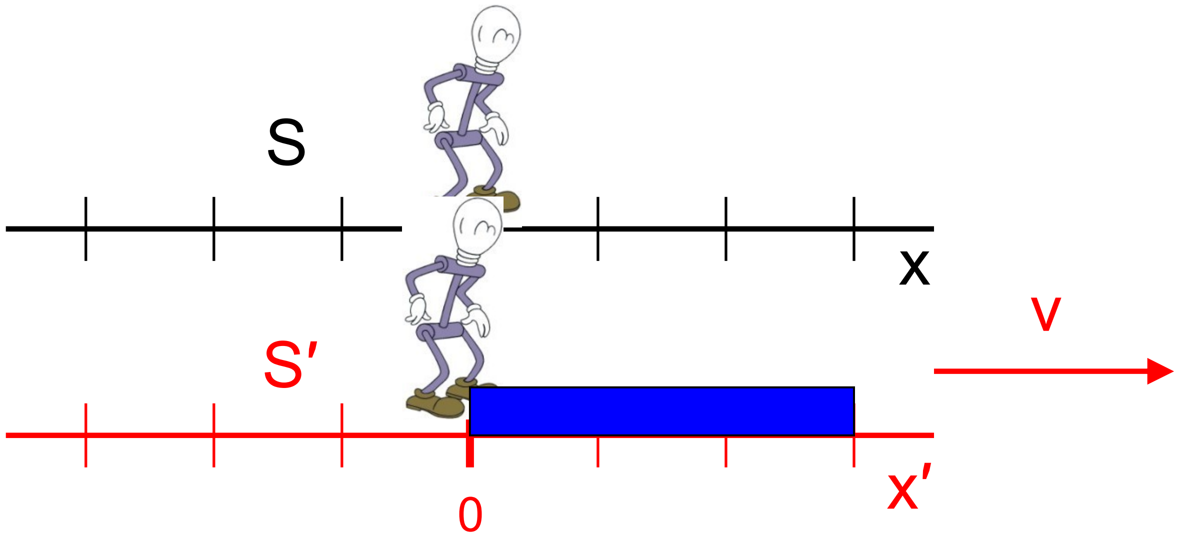
Time dilation – moving clocks run slower: $\Delta t = \gamma \Delta t_0$

The rest frame time (proper time) is Δt_0 and is the time in the moving system rest frame. It is shorter than the time measured in the frame where the system is moving.

Length contraction – moving objects are shorter (in the direction of motion): $L = L_0 / \gamma$

The length of an object in motion (L) is less than the rest frame length (proper length) (L_0).

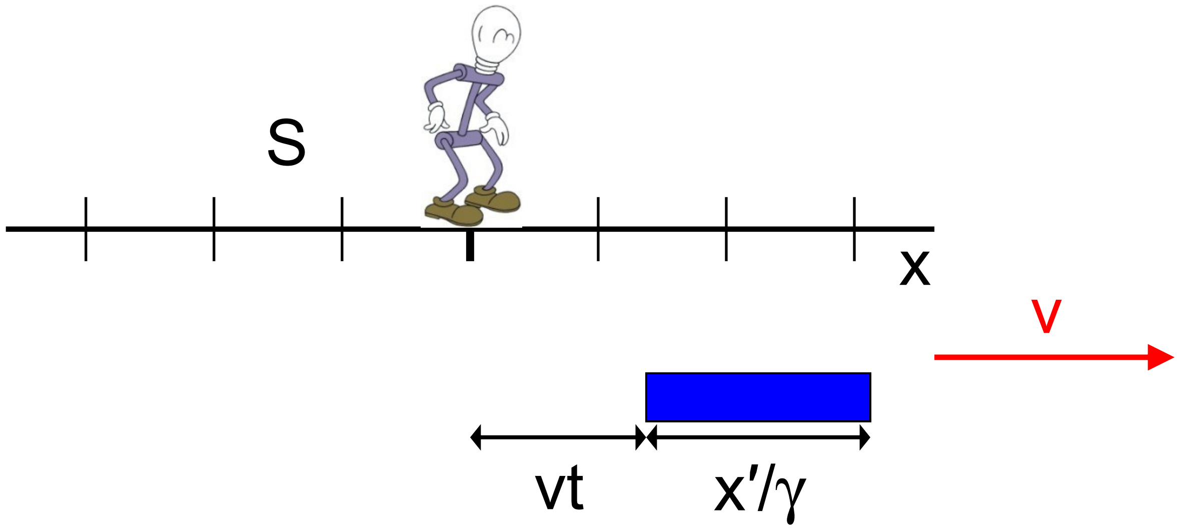
The Lorentz transformation



A stick is at rest in S′. Its endpoints are the events (position, c•time) = (0,0) and (x′,0) in S′. S′ is moving to the right with respect to frame S.

Event 1 – left of stick passes origin of S. Its coordinates are (0,0) in S and (0,0) in S′.

Clicker question 4



Q. In the S frame, the stick's length is x'/γ by length contraction. Time *t* passes. According to S, where is the *right* end of the stick?

A. $x = vt$
B. $x = -vt$
C. $x = vt + x'/\gamma$
D. $x = -vt + x'/\gamma$
E. $x = vt - x'/\gamma$

With a little algebra we can rewrite this as $x' = \gamma(x - vt)$

This transformation gives the coordinate in the S frame if you have the coordinate (and time) in the S′ frame.

Transformations

If S′ is moving with speed v in the positive *x* direction relative to S, then the coordinates of the same event in the two frames is related by:

In Galilean relativity

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{v}{c^2}x)$$

In a minute…

Remark: this assumes (0,0) is the same event in both frames.