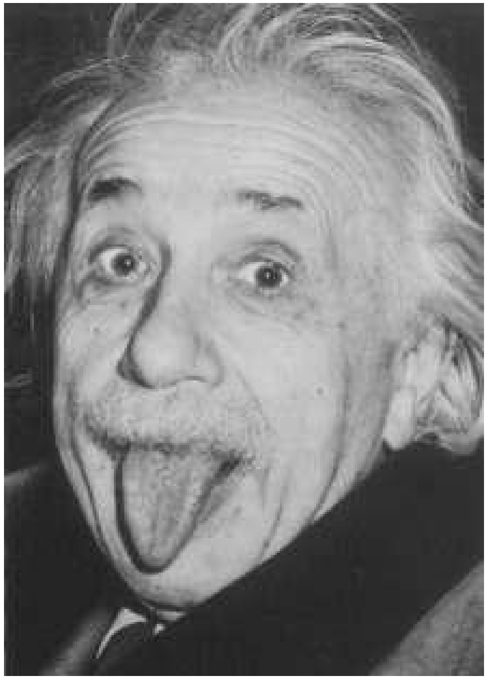
Einstein's theory of special relativity

Announcements:

- Homework was due at 12:50pm today in the wood cabinet just inside the physics help room (G2B90).
- Solutions will be online after class.
- Next homework assignment will be available by 5pm today.
- Clicker questions start counting today.
- Sorry about the confusion on problem 4c



Albert Einstein (1879—1955):

Today we will derive time dilation (and maybe length contraction)

Clicker question 1 | Set frequency to DA

Two events take place at two different positions in the **same** inertial reference frame. One observer, at rest in the same inertial reference frame, observes event #1 occurring before event #2. Which statement is true?

A. Other observers in the same inertial reference frame may not observe the two events being in the same order (even if they account for the finite speed of light).

B. An observer in a different inertial reference frame must observe the events occurring in the same order, although the time between the events may not be the same.

C. An observer in a different inertial reference frame may observe the events occurring in a different order **but** this observer is wrong

D. An observer in a different inertial reference frame may observe the events occurring in a different order **and** both observers are correct.

E. More than one of the above is correct.

Simultaneity is relative!

Given two events located at **different** positions such as:

1) light hits the right end of the train car

2) light hits the left end of the train car

Whether or not these events are simultaneous, or even the **order** in which these events occur depends on which inertial reference frame the observations take place in.

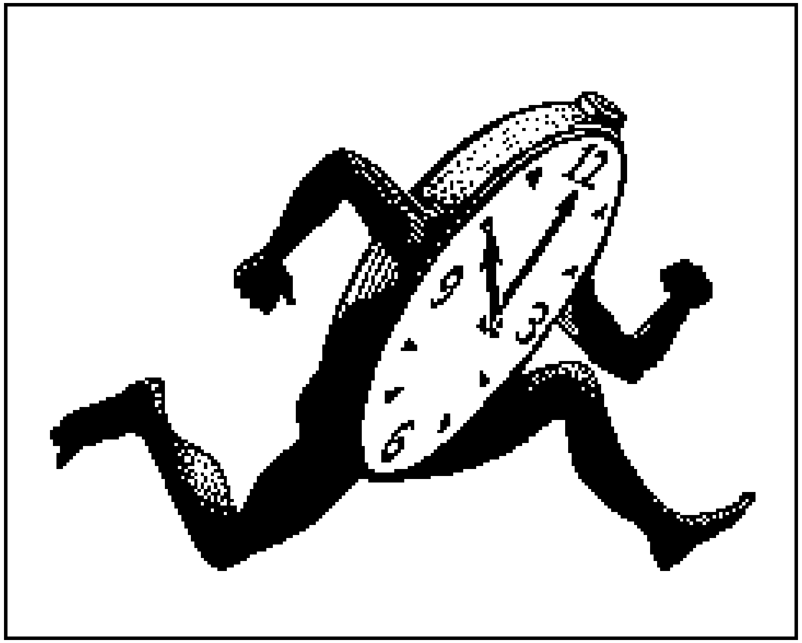
And since there is no preferred inertial reference frame, the answers from all of the inertial reference are all correct!

And that's the relativity of simultaneity.

Time Dilation

Today:

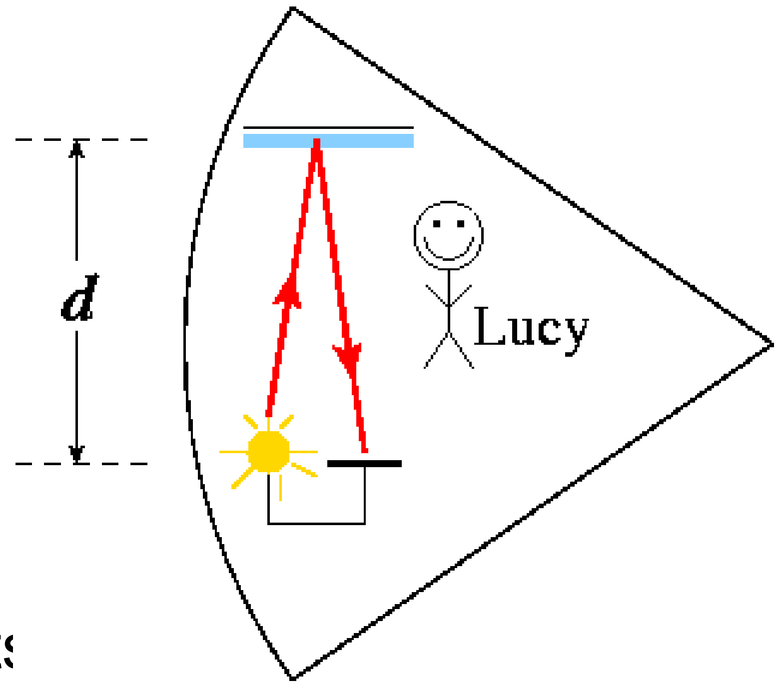
- Clocks run at different rates in different reference frames
- By how much?
- Proper time
- We are going to ditch trains and move on to space ships because they are much cooler.



Clicker question 2

Set frequency to DA

We will use a "light clock." A light flash starts a tick and when the light reflects off the mirror and back down to a detector it fires off another light flash. So a light clock "ticks" each time light makes a complete circuit: up and down.



For Lucy, how long does each tick take?

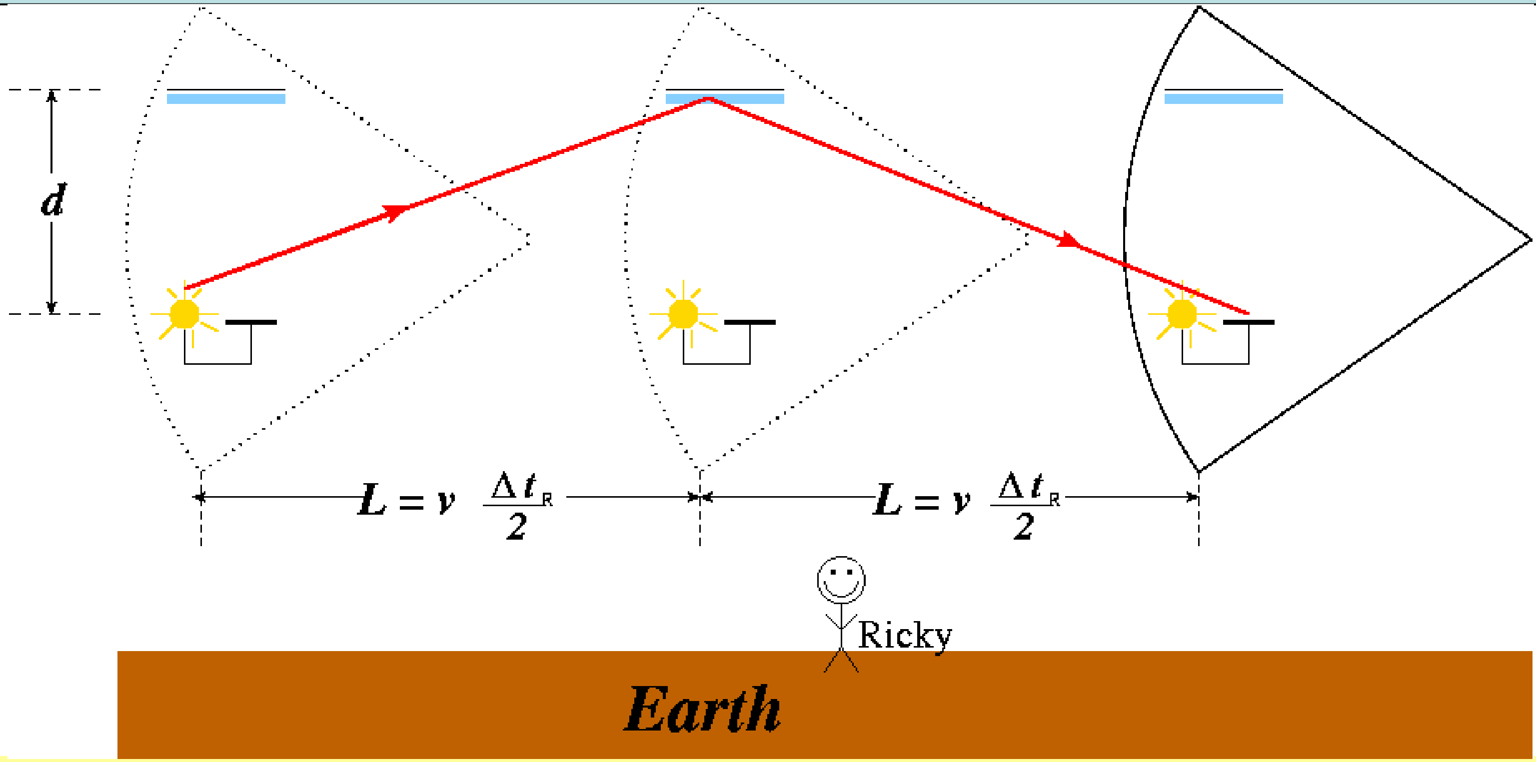
A. $\Delta t_L = 0$
B. $\Delta t_L = d/c$
C. $\Delta t_L = 2d/c$
D. $\Delta t_L = d/2c$
E. $\Delta t_L = c/2d$

This is the time between two events:

Event 1: The light flashes

Event 2: The reflected light reaches the detector

Lucy's spaceship flies by Earth at speed *v*

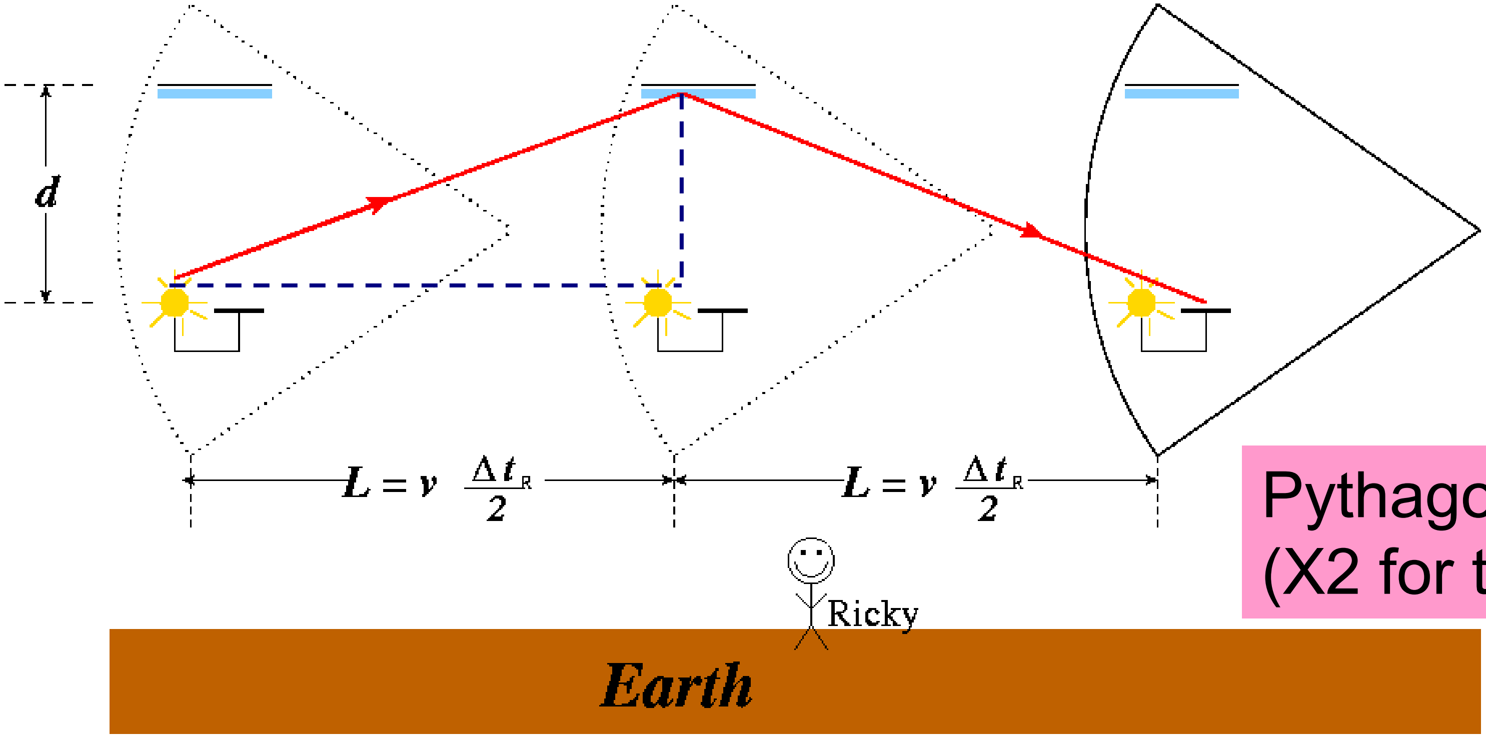


Ricky is on Earth watching Lucy's spaceship fly by at a speed *v*.

Ricky has observers set up at the location of the two events. Using their synchronized clocks he finds that a time of Δt_R passes between the light flash and when the reflected light reaches the detector.

Clicker question 3

Set frequency to DA



Pythagorean theorem (X2 for two triangles)

According to Ricky, how **far** does the light beam travel between the two events (light flash and reflected light hitting the detector)?

A. d

B. $2d$

C. $\sqrt{d^2 + (v \cdot \Delta t_R / 2)^2}$

D. $2\sqrt{d^2 + (v \cdot \Delta t_R / 2)^2}$

E. None of A-D

Note, this length is also equal to $c \cdot \Delta t_R$

Algebra to give us the time dilation formula

Ricky sees light travel a distance of $2\sqrt{d^2 + (v \cdot \Delta t_R / 2)^2}$

The distance traveled by light can also be written:

$$c \cdot \Delta t_R = 2\sqrt{d^2 + (v \cdot \Delta t_R / 2)^2}$$

Squaring both sides gives

$$c^2 \cdot \Delta t_R^2 = 4d^2 + v^2 \cdot \Delta t_R^2$$

Collecting Δt_R^2 terms gives

$$(c^2 - v^2)\Delta t_R^2 = 4d^2$$

Dividing both sides by c^2 gives

$$\left(1 - \frac{v^2}{c^2}\right)\Delta t_R^2 = \frac{4d^2}{c^2}$$

Dividing by $1 - \frac{v^2}{c^2}$ and taking the square root gives

$$\Delta t_R = \frac{2d}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Note the time measured by Lucy is $\Delta t_L = \frac{2d}{c}$

Therefore, the relation between the time observed by Ricky (on Earth) and Lucy (in the spaceship) is:

$$\Delta t_R = \frac{\Delta t_L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The relationship between the time between two events in Ricky's frame (on Earth) and the time between the same two events in Lucy's frame (on spaceship) is:

$$\Delta t_R = \frac{\Delta t_L}{\sqrt{1-\frac{v^2}{c^2}}}$$

Define $\beta = \frac{v}{c}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ so then $\Delta t_R = \gamma\,\Delta t_L$

Q. According to Ricky, the time between the two events (light flash and reflected light reaching detector) is

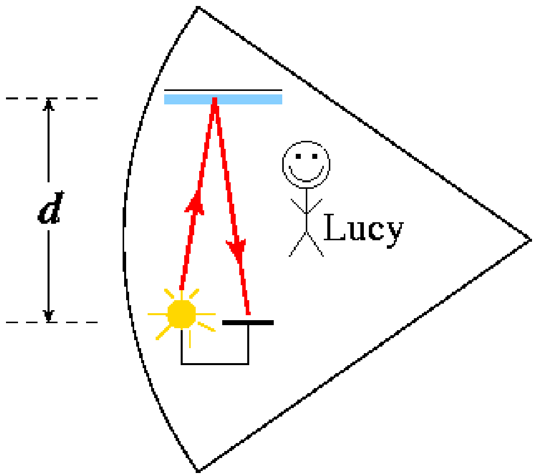
A. greater than what is observed by Lucy
B. less than what is observed by Lucy
C. equal to what is observed by Lucy

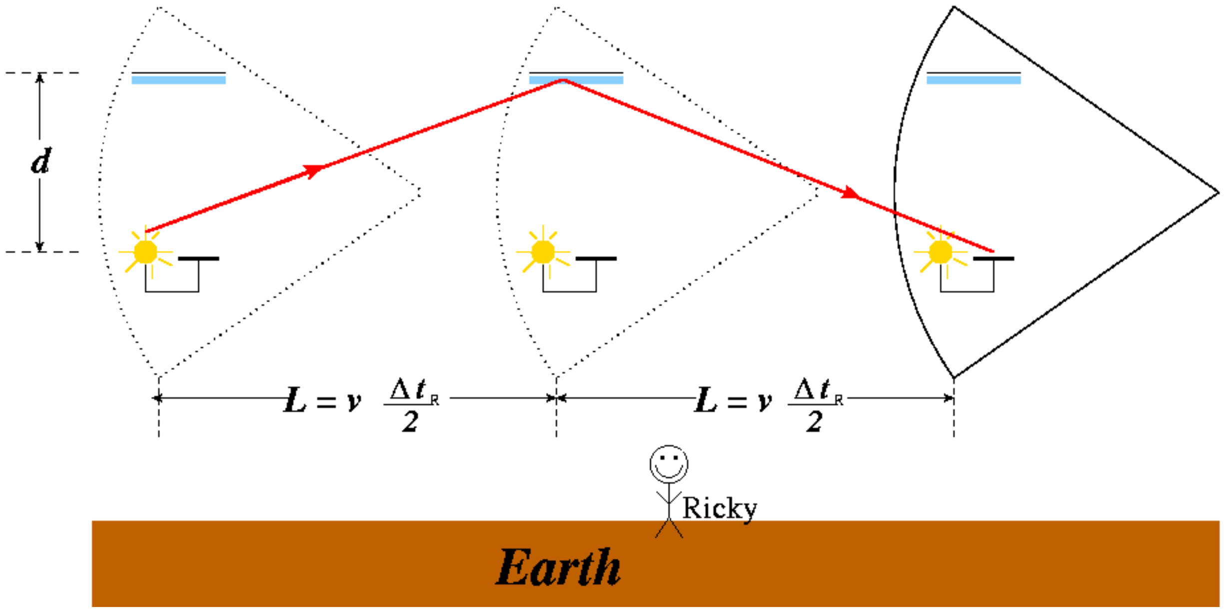
$0 \le \frac{v}{c} < 1$ and therefore $\gamma \ge 1$

So $\Delta t_R \ge \Delta t_L$. They are equal only when $v = 0$.

Clicker question 5

Set frequency to DA





Q. Is there something special about the two events in Lucy's frame? (Be prepared to back up your answer!)

A. Yes
B. No

Both events occur at the same location in Lucy's frame.

Proper time and time dilation

If two events occur at the same location, then the time between them can be measured by a *single observer* with a *single clock* (the "Lucy time" in our example). The time measured between these types of events is called the **proper time**, Δt_0

Example: any given clock never moves with respect to itself. It keeps proper time in its own frame.

Any observer moving with respect to this clock sees it run slowly (i.e., time intervals are longer). This is **time dilation**: $\Delta t = \gamma \Delta t_0$

Clicker question 6

Set frequency to DA

Vicki

Carol

v →

Q. Carol and Vicki are identical twins. While Vicki stays on Earth, Carol departs for the star Sirius, 8 light-years away, traveling at a speed v = 0.8c (Note $\gamma = 5/3$). According to observers in Vicki's frame, how long does the trip take?

A. 6 years B. 8 years **C. 10 years** D. 16.67 years

The distance is 8 light-years and the speed is 0.8 c so we get

$$t = d / v = (8\,\mathrm{c} \cdot \mathrm{years}) / 0.80\,\mathrm{c} = 10\ \mathrm{years}$$

Clicker question 7

Set frequency to DA

Vicki

Carol

v →

Q. Vicki stays on Earth and Carol departs for the star Sirius, 8 light-years away, traveling at a speed v = 0.8 c (γ = 5/3). According to Carol, how long does the trip take?

A. 6 years   B. 8 years   C. 10 years   D. 16.67 years

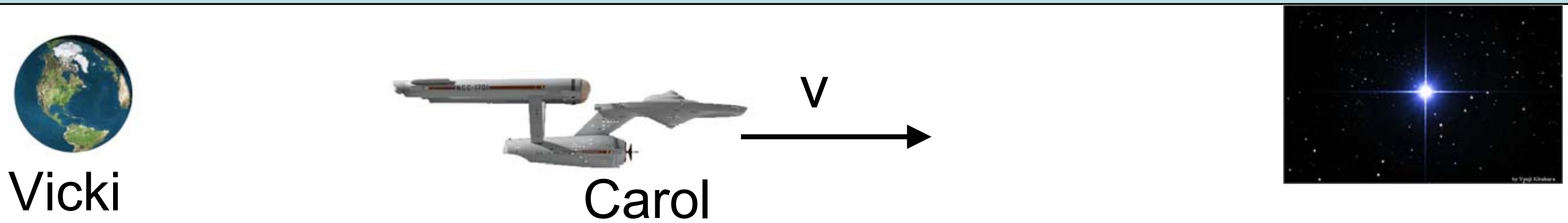
Carol's clock is the only one present at both events (leaving Earth and arriving at Sirius). Therefore, she is measuring the **proper time!** This tells us which time is which in the time dilation formula.

Solving $\Delta t = \gamma \Delta t_0$ for the proper time gives

$$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{10 \text{ years}}{5/3} = 6 \text{ years}$$

Follow the proper time!

Length contraction



We found that only 6 years pass in Carol's reference frame as she travels 8 light-years to Sirius at 0.8 c.

We say that every inertial frame is valid so won't she observe the space ship going faster than the speed of light (8 light years in 6 years!)?

This discrepancy is resolved by length contraction – moving objects are shorter (in the direction of motion) $L = L_0 / \gamma$