Review 2

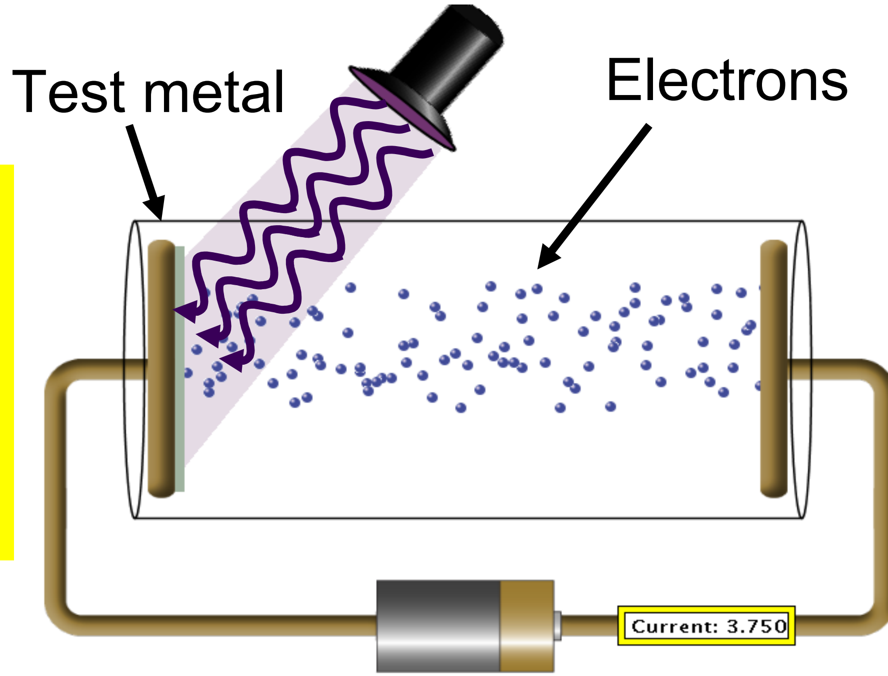
Announcements:

- All homework solutions are available on CULearn
- Final exam is ***tomorrow*** at **1:30pm** in G125 (this room)
- I will be in my office from 2-5 today and also tomorrow morning from 9-12.
- The formula sheet is available on the Exam Info link
- All grades except final exam and HW14 will be up by 4pm.
- Final exam grades should be up Monday but HW14 and final grades won't be available until Thursday.
- Solutions to the final will be on CULearn by Saturday night. You can also pick up your final between 9-5 on Tuesday-Thursday.
- Review slides (including clicker questions) I did not get a chance to cover will be posted in the lecture notes – you should review them. These slides are not necessarily comprehensive.

Final exam

- There are 34 total parts (exam 1 had 22 parts and exam 2 had 21 parts)
  - 7 parts are multiple choice
  - 2 questions ask you to draw
  - 1 question is a short answer
  - Problem order basically follows the order of the class
- Exam is out of 150 points
  - 45 points on relativity
  - 21 points on quantization of light
  - 15 points on wave/particle duality and uncertainty
  - 40 points on quantum stuff (wave function, probability, normalization, square well, tunneling)
  - 29 points on multielectron atoms

Photoelectric effect



Observations didn't match theory:

1. Minimum frequency needed to get current no matter the intensity
2. Current depended on frequency as well as intensity.

1905: Einstein's solution: photons have energy *hf* and only one photon can interact with electron at a time. Minimum frequency leads to minimum energy to eject the electron from the metal (overcome work function). Energy above minimum goes into electron KE.

Conservation of energy: E_{photon} = E to escape metal + electron KE

$$KE_{max} = hf - \phi$$

Clicker question 4 — Set frequency to DA

Calcium has a work function of 2.9 eV. What is the longest wavelength light that can eject electrons from calcium?

A. 230 nm
B. 400 nm
C. 430 nm
D. 620 nm
E. Impossible to tell

$E = hf = 6.626*10^{-34}\ \text{J·s} \cdot f = 4.14*10^{-15}\ \text{eV·s} \cdot f$

$E = hc/\lambda = 1.99*10^{-25}\ \text{J·m} / \lambda = 1240\ \text{eV·nm} / \lambda$

The longest wavelength is the lowest energy so it would leave the electron with zero kinetic energy. That is, the photon energy would just be equal to the work function.

Using the above equation we find: $2.9\,\text{eV} = 1240\,\text{eV} \cdot \text{nm} / \lambda$

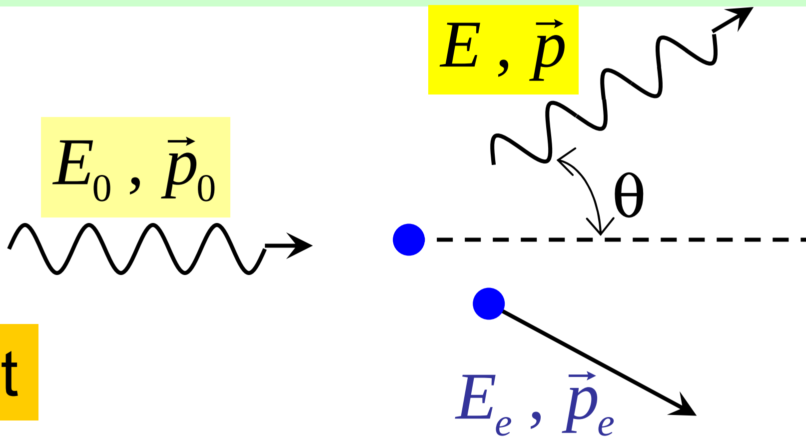
Solving for λ gives: $\lambda = \dfrac{1240\,\text{eV} \cdot \text{nm}}{2.9\,\text{eV}} = 430\,\text{nm}$

End of Chapter 4

Crystals are used as diffraction gratings – Bragg diffraction

The Compton effect showed that X-rays have momentum

X-rays photons hit an atomic electron imparting momentum to the electron; the scattered photon has less energy and momentum



End result: particle-wave duality of light

Light is both a particle and wave!

$$E = hf \qquad p = h / \lambda$$

Wave/particle duality

All matter has wave properties with the particle & wave quantities related by de Broglie relations:

$$p = h/\lambda = \hbar k$$
$$E = hf = \hbar\omega$$

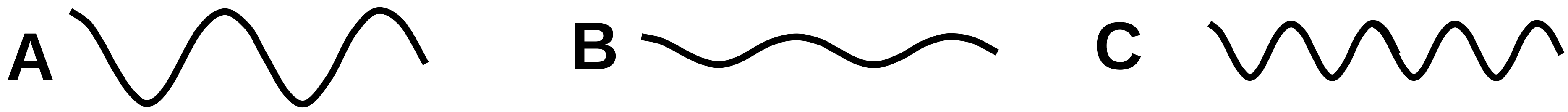
Davisson-Germer scattered electrons of varying energies off a nickel crystal (similar to X-ray diffraction) and found diffraction like X-ray diffraction showing electrons behave like a wave.

The function describing how light waves propagate is the electromagnetic wave function $E(x,t) = E_{\text{max}} \sin(ax - bt)$ with $\text{Intensity} \propto E_{\text{avg}}^2 \propto E_{\text{max}}^2$

Particles also have a wave function $\psi(x)$. In this case, $|\psi(x)|^2$ is the probability density which indicates how likely the particle is to be found at *x* (or what fraction of the particles will be found at *x*).

Clicker question 4

Set frequency to DA



Wave functions for three particles are shown. Rank order the magnitude of momentum.

A. A>C>B
B. A=B>C
C. C>A>B
D. C>A=B
E. None of the above

Momentum is given by

$$p = \frac{h}{\lambda} = \hbar k$$

So smaller wavelength λ gives higher momentum.

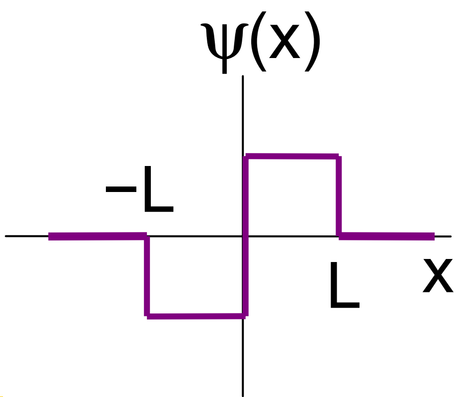
The amplitude does not affect the momentum (or energy) of the particle

Properties of wave functions

Wave functions are complex valued and are ***not*** directly observable.

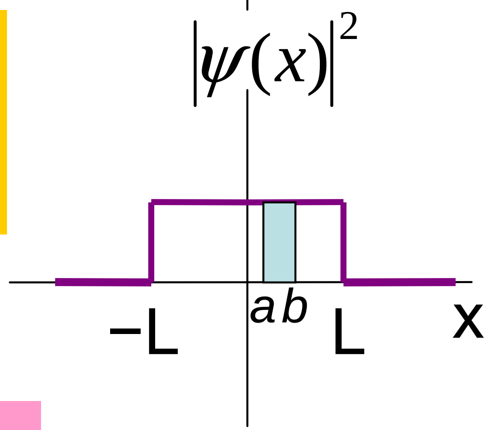
Probability density ***is*** observable: $|\psi(x)|^2 = \psi^*(x)\,\psi(x) = \psi_{\text{real}}^2(x) + \psi_{\text{imag}}^2(x)$

The probability of finding the particle at some infinitesimal distance δx around *x* is: $|\psi(x)|^2 \delta x$



Probability of particle being between *a* and *b* is $\int_a^b |\psi(x)|^2 dx$

The particle must be *somewhere* with a probability of 100%. This is the *normalization condition*: $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$



In 1D, $|\psi(x)|^2$ has dimension of 1/Length

ψ(x) is just the *spatial* part of the wave function.

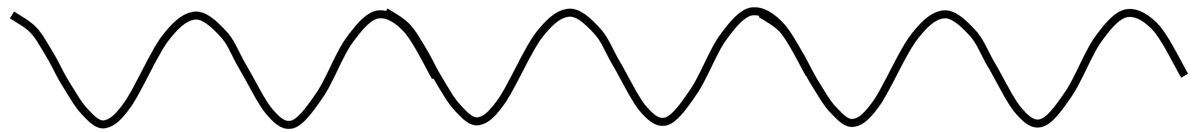
Ψ(x,t) is the full wave function

Clicker question 2

Set frequency to DA

Plane wave:

$$\Psi(x,t) = Ae^{i(kx - \omega t)}$$



Wave packet:

$$\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)}$$

For which wave is the momentum and position most well defined?

A. *p* is well defined for plane wave, *x* is well defined for wave packet

B. *p* is well defined for wave packet, *x* is well defined for plane wave

C. *p* is well defined for one but *x* is equally well defined for both

D. *p* is equally well defined for both but *x* is well defined for one

E. Both *p* and *x* are well defined for both

$$\Delta x\, \Delta p \geq \frac{\hbar}{2}$$

This is how we justified the uncertainty principle

$$\Delta t\, \Delta E \geq \frac{\hbar}{2}$$

1D Schrödinger equation

The wave equation for electromagnetic waves is $\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$

Only works for massless particles with *v*=*c*.

For massive particles, need the time dependent Schrödinger equation (TDSE):

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

Kinetic energy + Potential energy = Total energy

For time independent potentials, V(x): $\Psi(x,t) = \psi(x)\phi(t)$

In this case, the time part of the wave function is: $\phi(t) = e^{-iEt/\hbar}$

The spatial part ψ(x) can be found from the time independent Schrödinger equation (TISE):

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Free particle

When E>V (everywhere) you have a free particle. We deal with the case of V=0 but it can be applied to other cases as well.

Free particles have oscillating solutions

$\psi(x) = A\cos(kx) + B\sin(kx)$ or $\psi(x) = Ce^{ikx} + De^{-ikx}$

To get the full wave function we multiply by the time dependence:

$$\Psi(x,t) = \psi(x)e^{-i\omega t} = Ce^{i(kx-\omega t)} + De^{-i(kx+\omega t)}$$

This is the sum of two waves with momentum $\hbar k$ and energy $\hbar\omega$.

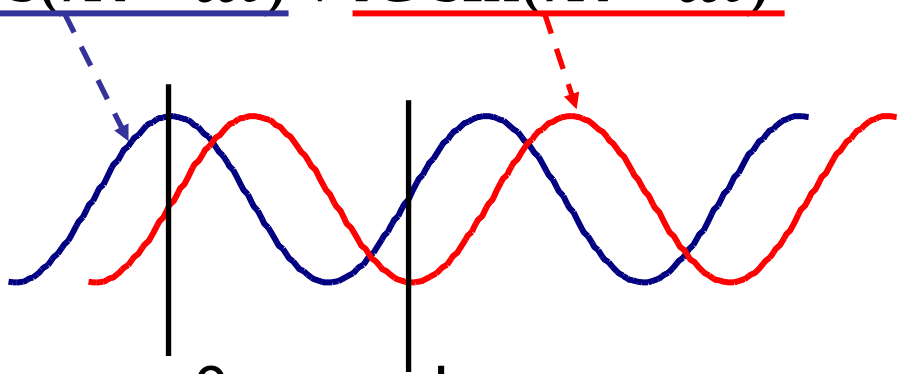
The *C* wave is moving right and the *D* wave is moving left.

Clicker question 3 — Set frequency to DA

Left moving free particle wave function is: $\Psi(x,t) = Ce^{i(kx-\omega t)}$

Can also write as $\Psi(x,t) = \underline{C\cos(kx-\omega t)} + \underline{iC\sin(kx-\omega t)}$

The probability of finding the particle at x=0 is __________ to the probability of finding the particle at x=L.



A. Always larger
B. Always smaller
C. Always equal
D. Oscillates between smaller & larger
E. Depends on other quantities like *k*

Get probability from $|\Psi(x,t)|^2$.

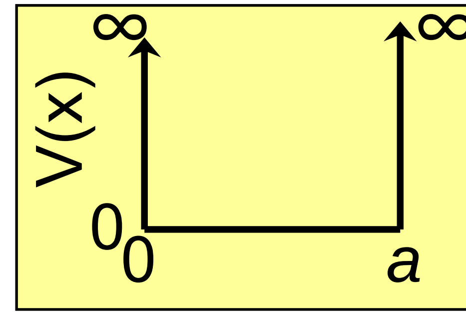
$$\Psi^*\Psi = \left(C\cos(kx-\omega t) - iC\sin(kx-\omega t)\right)\left(C\cos(kx-\omega t) + iC\sin(kx-\omega t)\right)$$
$$= C^2[\cos^2(kx-\omega t) + \sin^2(kx-\omega t)] = C^2$$

or $\Psi^*\Psi = Ce^{-i(kx-\omega t)}Ce^{i(kx-\omega t)} = C^2$

Probability is constant (same everywhere and everywhen)

Infinite square well

Like the free particle, E>V but only in region 0<x<a.



Functional form of solution is also oscillating:

$$\psi(x) = A\cos(kx) + B\sin(kx)$$

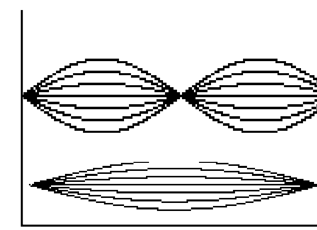
But this time have boundary conditions

Putting in *x* = 0 gives $\psi(0) = A$ so *A* = 0

Putting in *x* = *a* gives $\psi(a) = B\sin(ka)$

To get $\sin(ka) = 0$ requires $ka = n\pi$

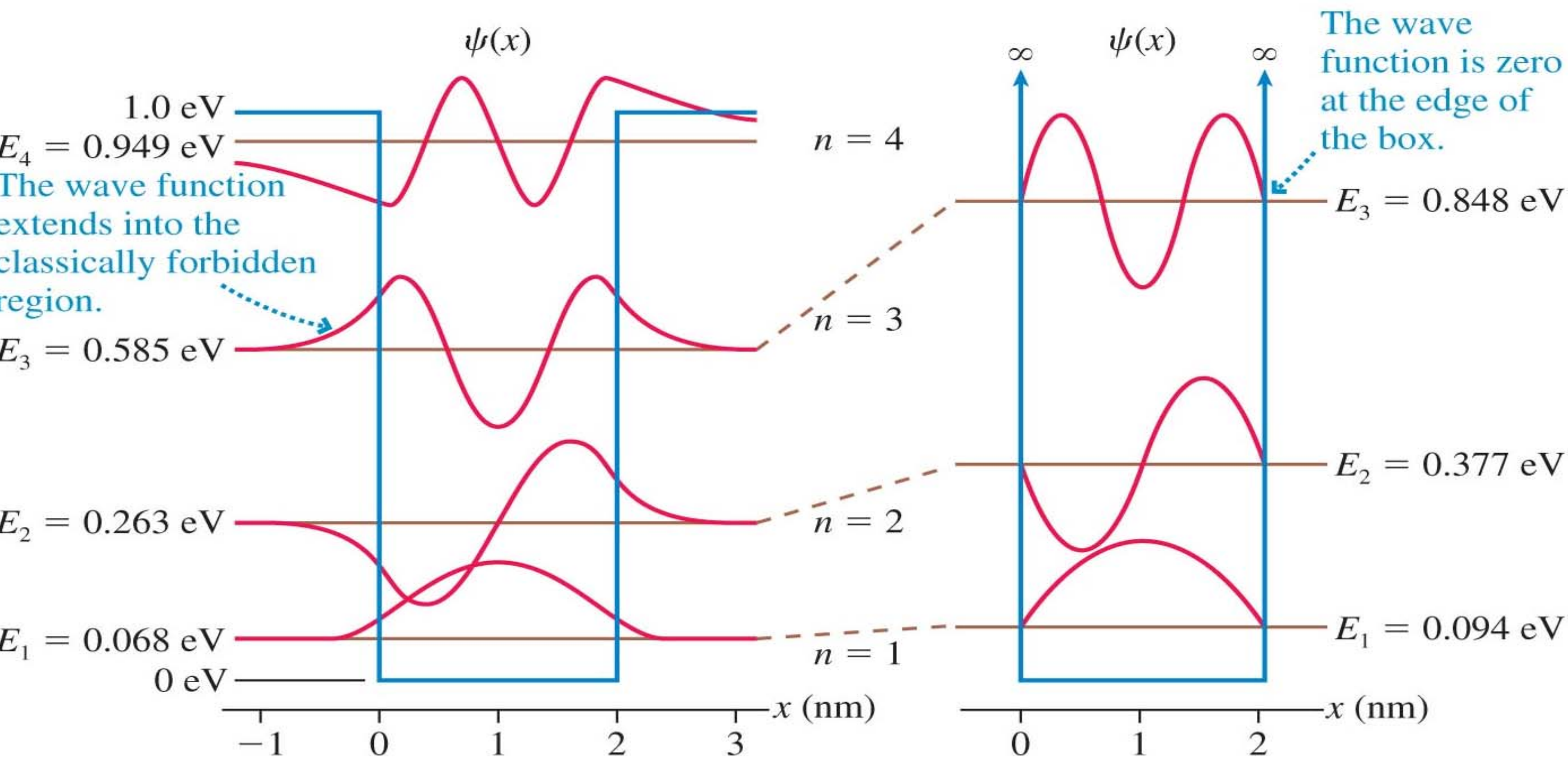
Get the condition $k = \frac{n\pi}{a}$ or $\lambda = \frac{2a}{n}$



Find that energies are quantized: $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

After applying normalization condition we get $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$

Comparison of square wells



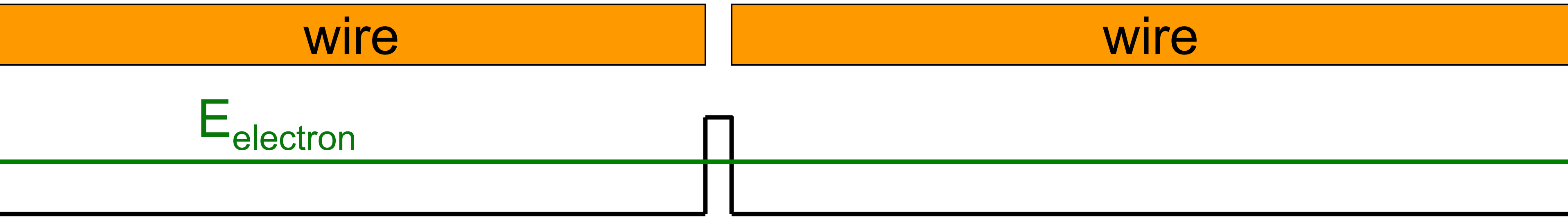
Note that energy level *n* has *n* antinodes

Penetration depth $\lambda = 1/\alpha$ measures how far particle can be found in the classically forbidden region.

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Quantum tunneling through potential barrier

Consider two very long wires separated by a small gap:



This is an example of a *potential barrier.*

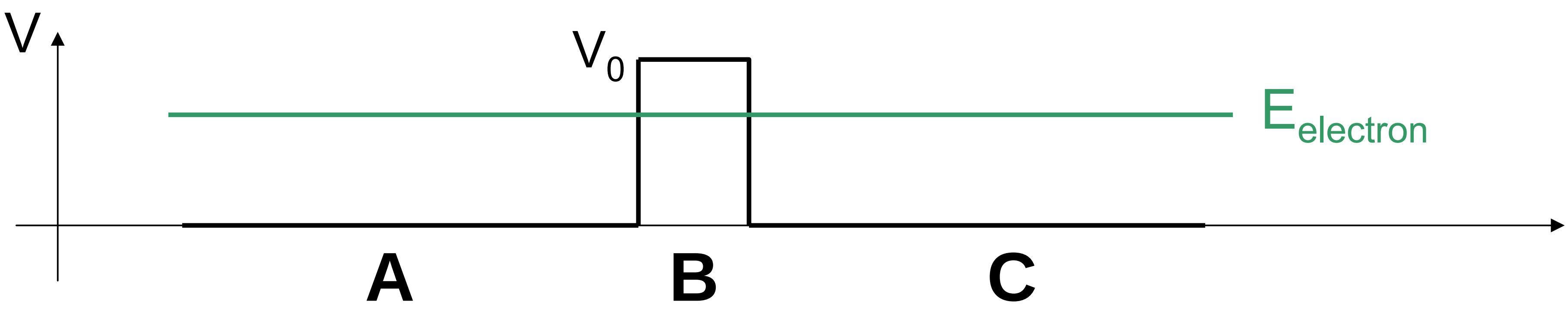
Quantum tunneling occurs when a particle which does not have enough energy to go over the potential barrier somehow gets to the other side of the barrier.

This is due to the particle being able to penetrate into the *classically forbidden region*.

If it can penetrate far enough (the barrier is thin enough) it can come out the other side.

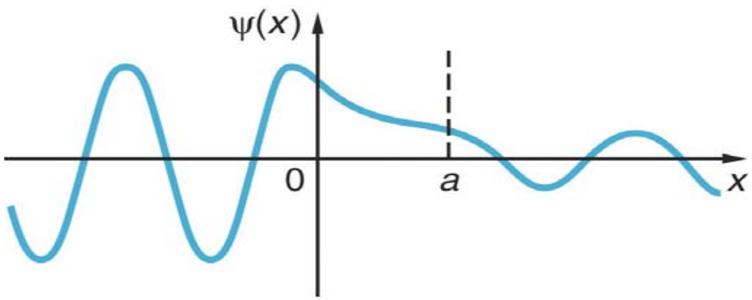
Clicker question 4

Set frequency to DA



As in the quantum tunneling tutorial, an electron is moving from left to right when it encounters a potential barrier. Given what is drawn, rank the kinetic energy in the three regions.

A. A>B>C
B. A>C>B
C. A=C>B
D. A=B>C
E. None of the above



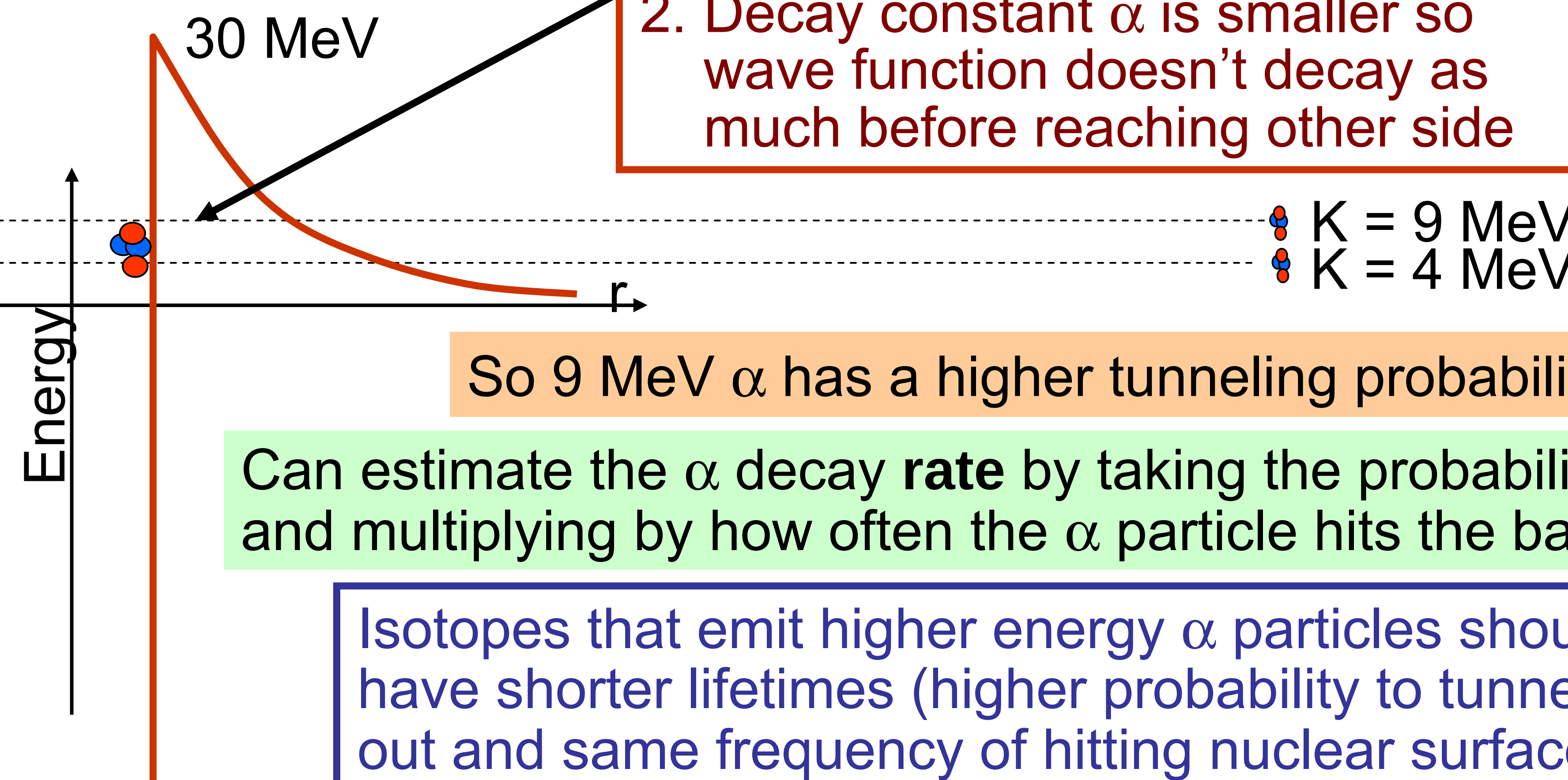
Electrons in B have negative kinetic energy

Energy is conserved and since V=0 at A and C, any electrons that tunnel will have the same kinetic energy at C as at A.

α decay probability

Probability of tunneling is $P \approx e^{-2\alpha L}$ where $\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$

1. Less distance to tunnel (smaller *L*)
2. Decay constant α is smaller so wave function doesn't decay as much before reaching other side



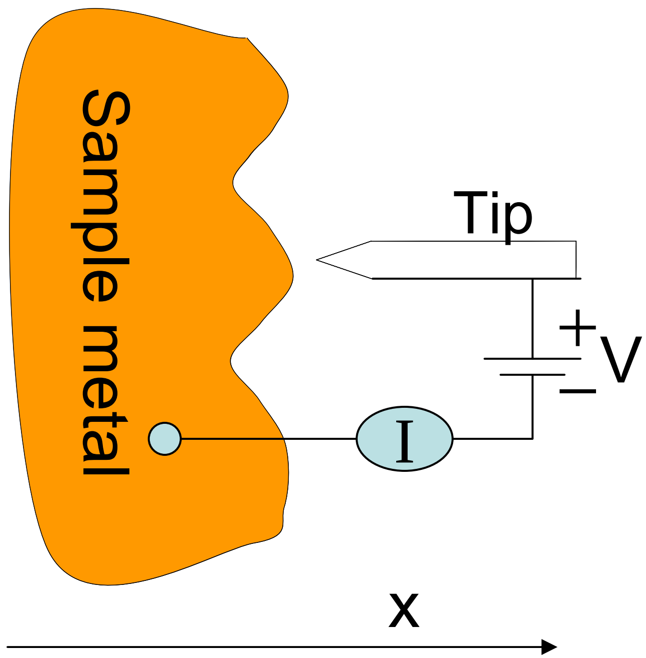
So 9 MeV α has a higher tunneling probability

Can estimate the α decay **rate** by taking the probability and multiplying by how often the α particle hits the barrier

Isotopes that emit higher energy α particles should have shorter lifetimes (higher probability to tunnel out and same frequency of hitting nuclear surface)

Experimentally confirmed! See text for details.

Scanning tunneling microscope (STM)



What does the potential at the tip look like?

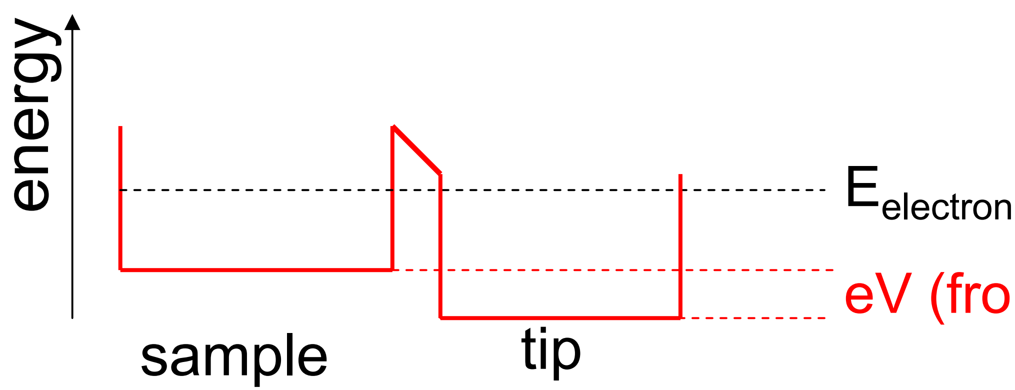
A. higher than the sample

B. same as the sample

C. lower than the sample

D. tilts downward from left to right

E. tilts upward from left to right



Atomic wavefunctions and quantum numbers

Each atomic electron can be identified by four quantum numbers:

n = 0, 1, 2, … = principal quantum number

ℓ gives total orbital angular momentum: $L = \sqrt{\ell(\ell+1)}\hbar$

m gives z-component of orbital angular momentum: $L_z = m\hbar$

m_s = ±½ gives the z-component of spin: $S_z = m_s\hbar$

The atom itself has angular momentum which is the vector sum of orbital and intrinsic angular momenta of the electrons. $\vec{J} = \vec{L} + \vec{S}$

Thus, the Stern-Gerlach experiment actually measures the z-component of the total angular momentum: $J_z = L_z + S_z$

Clicker question 6

Set frequency to DA

n = 1, 2, 3, … = Principal Quantum Number

$$E_n = -Z^2 E_R / n^2$$

ℓ = 0, 1, 2, … n-1 = orbital angular momentum quantum number
= s, p, d, f, …

$$L = \sqrt{\ell(\ell+1)}\hbar$$

m = 0, ±1, ±2, … ±ℓ is the z-component of orbital angular momentum

$$L_z = m\hbar$$

m_s = ±½ gives the z-component of spin: $S_z = m_s \hbar$

A hydrogen atom electron is excited to an energy of −13.6/4 eV. How many different quantum states could the electron be in?

A. 2
B. 3
C. 4
D. 8
E. more than 8

E = −13.6/4 eV means $n^2 = 4$ so $n = 2$

For $n = 2$, ℓ = 0 or ℓ = 1.

For ℓ = 0, $m = 0$.

For ℓ = 1, $m = -1, 0$, or 1

For any of these four states m_s can be ±½, doubling the number of states.

Multielectron atoms

Using an approximation for Z which we call Z_{eff}, we can use our hydrogen results to get

$$E_n = -Z_{\mathrm{eff}}^2 E_R / n^2$$

$$r_{\mathrm{mp}} \approx n^2 a_B / Z_{\mathrm{eff}}$$

$Z_{\mathrm{eff}} \approx 1$ for an outermost electron alone in an energy shell (fully screened by inner electrons) and $Z_{\mathrm{eff}} \approx Z$ for inner electrons (no screening).

Using this approximation gives the energy levels which tells us how the energy levels fill up with electrons.

Pauli Exclusion Principle does not allow two electrons to have the same quantum numbers

Each orbital is defined by quantum numbers *nℓm* and can have two electrons since $m_s = \pm ½$ (spin up or spin down).

This explains why electrons are in the energy levels they are and explains the periodic table of the elements.

Fundamentals of quantum mechanics

A quantum system can be in a superposition of states such as a superposition of energy states $\psi = c_1\psi_1 + c_2\psi_2 + \cdots + c_n\psi_n$

The states are orthogonal so $|\psi|^2 = c_1^2\psi_1^2 + c_2^2\psi_2^2 + \cdots + c_n^2\psi_n^2$

When a quantity (like energy) is measured the wave function collapses into a definite state ψ_i is simply c_i^2

An energy eigenstate is special because it does not change in time (why it is also called a stationary state)

Other eigenstates like a positions eigenstates do change in time; a particle initially in a position eigenstate does not remain in that state

Clicker question 7 — Set frequency to DA

Which of these is a true statement about quantum mechanics?

A. Energies are always quantized
B. Nothing can be known precisely
C. A quantum system may not have a precisely defined energy
D. More than one of the above
E. None of the above

A. The energy of a particle is only quantized when the particle is confined (infinite square well, finite square well, atom, simple harmonic oscillator). It comes from boundary conditions.

B. If you know a particle is in an energy eigenstate you know the energy precisely.

C. A particle can be in a superposition of energy eigenstates so it has a mixture of energies