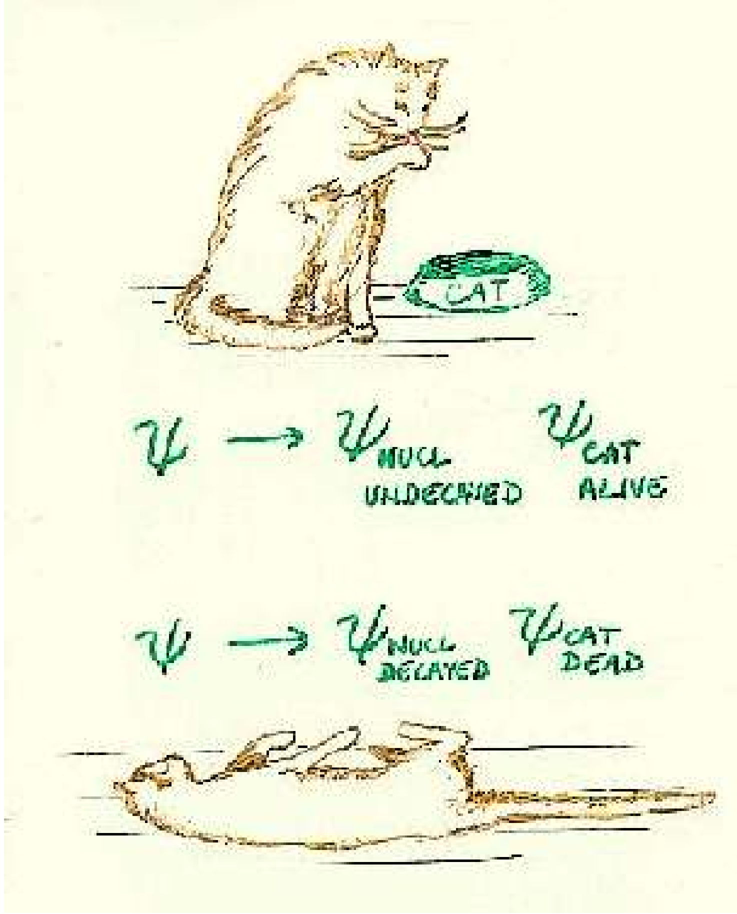
Some interesting aspects of quantum mechanics

- The last homework is due at 12:50pm on Thursday 4/30
- We will have the normal help sessions (M3-5 and T3-5).
- Final is on 5/2 from 1:30pm-4:00pm in G125 (this room)
- Rest of the semester:
  - Today we will talk about some interesting aspects of quantum mechanics.
  - Wednesday and Friday will be review.

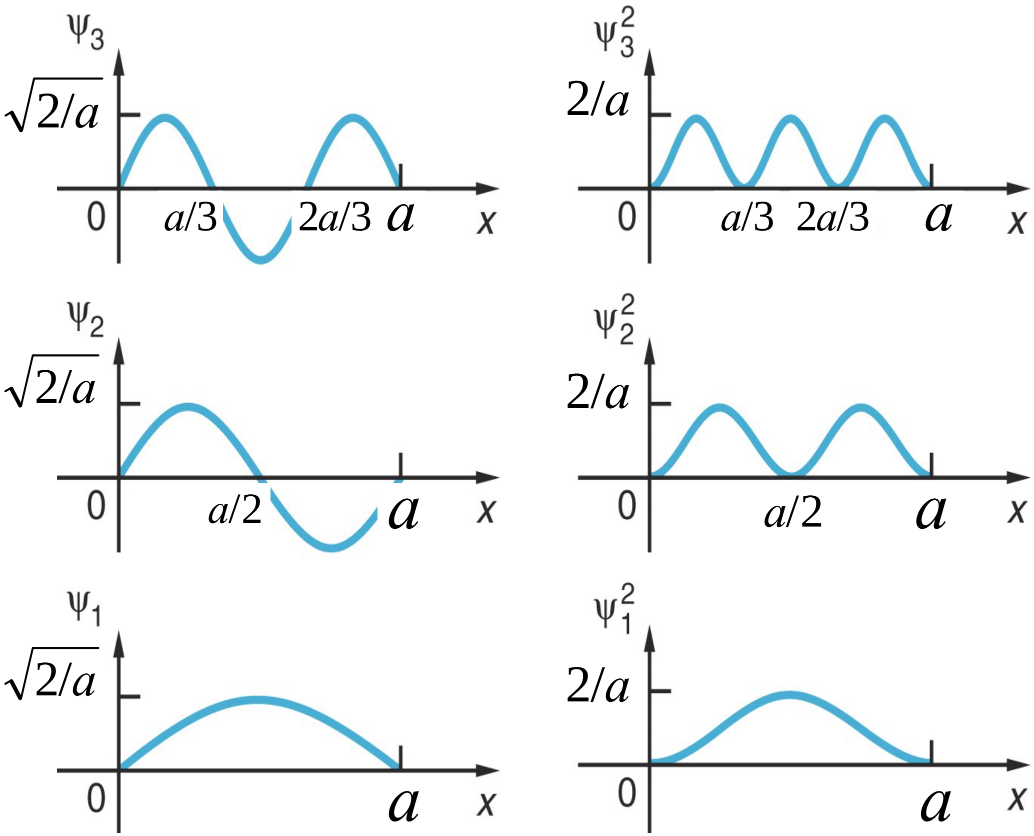


Energy eigenstates

We found that the infinite square well potential has *quantized* energy levels.

These energy levels have an associated wave function ψ_n and quantum number *n*.

After the energy is measured, the quantum system has a definite energy. We say that it is in an energy *eigenstate*.



If we haven't yet measured the energy it is possible for the particle to be in a mixture (superposition) of energy eigenstates.

Clicker question 1

We might guess that a particle in a 50/50 mixture of *n*=1 and *n*=2 would have $\psi(x) = \frac{1}{2}\psi_1(x) + \frac{1}{2}\psi_2(x)$

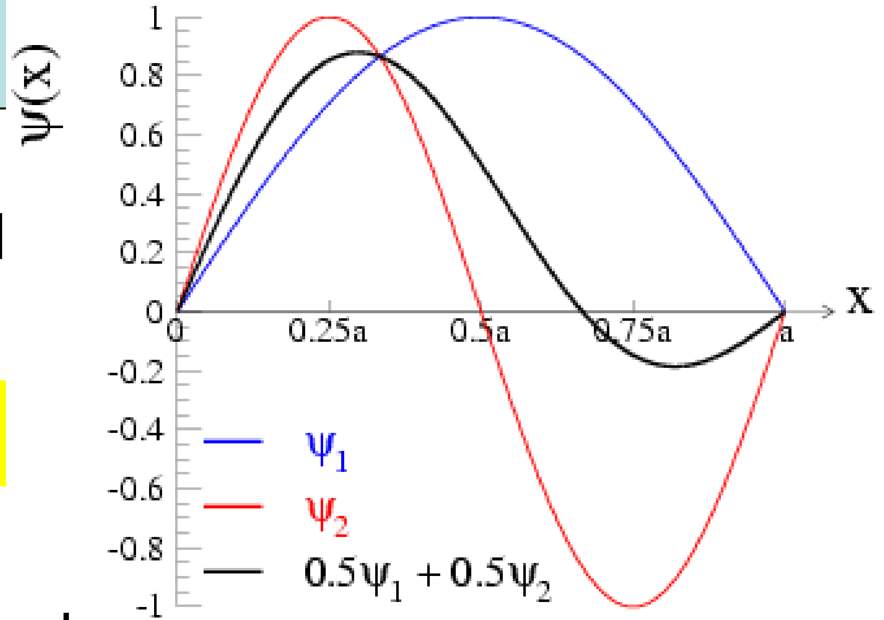
$|\psi|^2$ gives us the probability density.

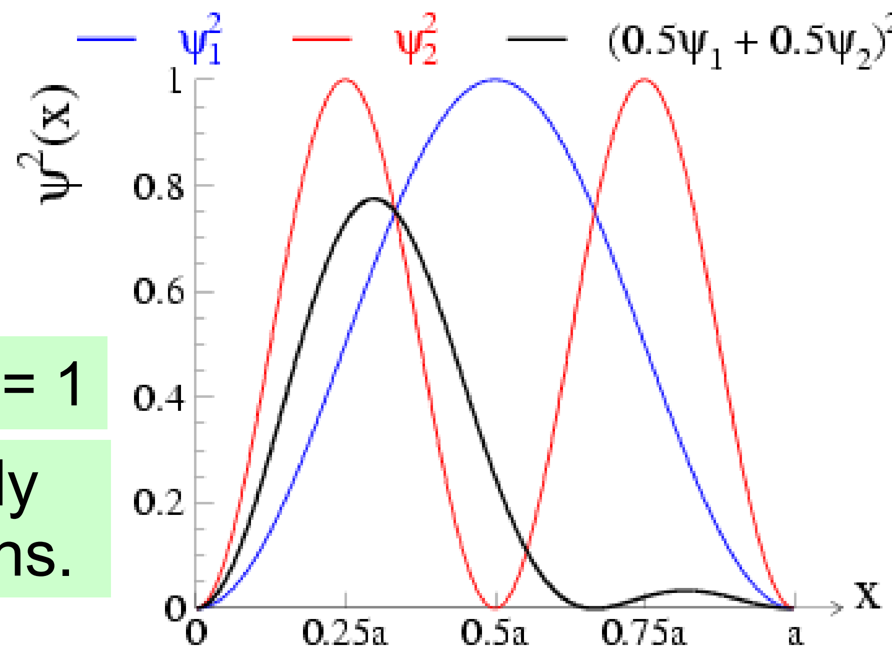
Q. Is this ψ(x) properly normalized?
Note: ψ_1 and ψ_2 *are* properly normalized.

A. Yes
B. No
C. Need the infinite square well wave functions to determine

Normalized means area under curve = 1

Area is clearly less than the properly normalized ψ_1 and ψ_2 wave functions.





Superpositions

Turns out eigenstates are orthogonal (like different dimensions) so that if

$$\psi = c_1\psi_1 + c_2\psi_2 + \cdots + c_n\psi_n \quad \text{then}$$

$$|\psi|^2 = c_1^2\psi_1^2 + c_2^2\psi_2^2 + \cdots + c_n^2\psi_n^2$$

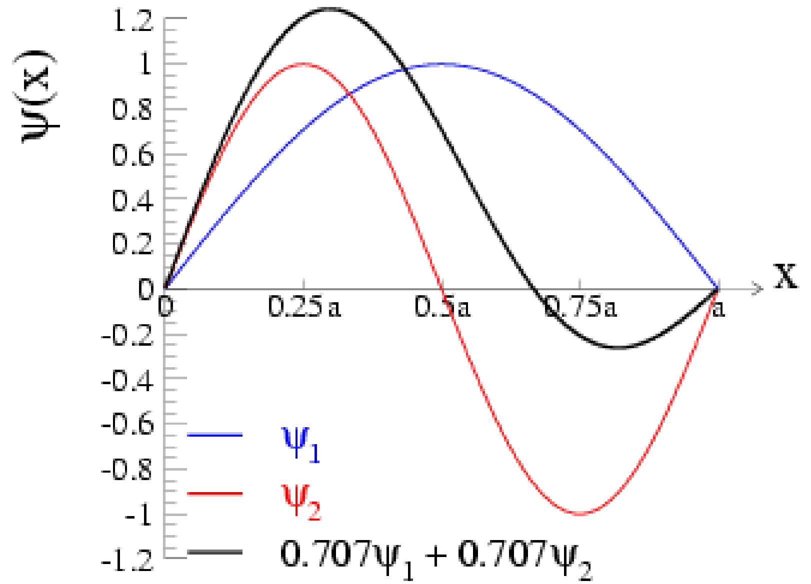
Furthermore, the probability of being in the state ψ_i^2 is just c_i^2

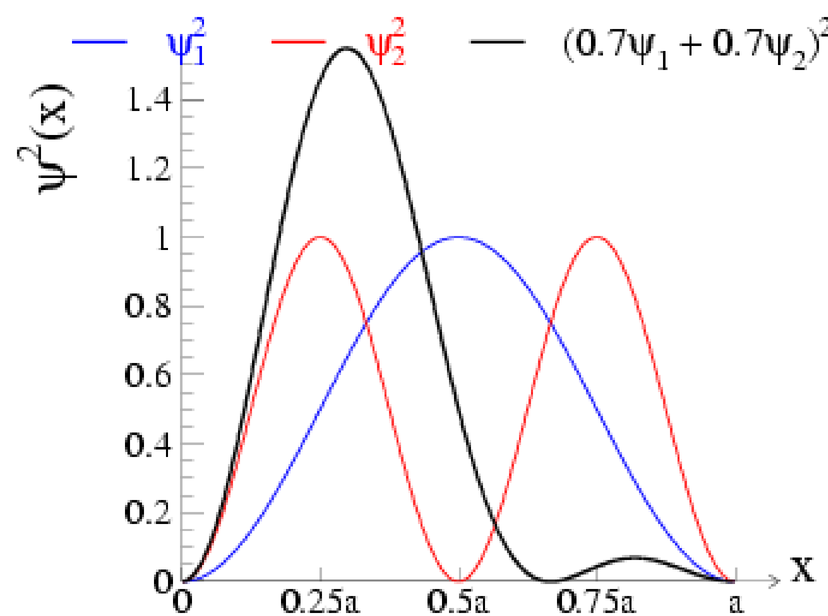
So an equal mixture of ψ_1 and ψ_2 is

$$\psi = \sqrt{\tfrac{1}{2}}\psi_1 + \sqrt{\tfrac{1}{2}}\psi_2$$

which gives a probability density of

$$|\psi|^2 = \tfrac{1}{2}\psi_1^2 + \tfrac{1}{2}\psi_2^2$$





But why would the particle prefer one side?

Time dependence revisited

The time dependence for an energy eigenstate is $e^{-iEt/\hbar}$ and for this part of the wave function, $|\psi|^2=\psi^*\psi$ gives $e^{iEt/\hbar}e^{-iEt/\hbar}=1$

Energy eigenstates are sometimes called stationary states because the probability density does not depend on time

For a *superposition* of energy eigenstates, this is no longer true.

$$\Psi(x,t)=\sqrt{\tfrac{1}{2}}\psi_1(x)e^{-iE_1t/\hbar}+\sqrt{\tfrac{1}{2}}\psi_2(x)e^{-iE_2t/\hbar}$$

gives a probability density of

$$|\Psi(x,t)|^2=\tfrac{1}{2}|\psi_1(x)|^2+\tfrac{1}{2}|\psi_2(x)|^2+\overbrace{\psi_1(x)\psi_2(x)\sin\left([E_2-E_1]t/\hbar\right)}^{\text{Interference term}}$$

The interference term gives a time dependence to the superposition of energy eigenstates.

Can be seen in Quantum Bound States simulation

Clicker question 2

Set frequency to DA

A particle in an infinite square well is in a superposition of two eigenstates with a wave function of $\psi = \sqrt{\frac{2}{5}}\psi_1 + \sqrt{\frac{3}{5}}\psi_2$

What happens if we measure the energy of this particle?

A. Energy will be a weighted average of $0.4E_1+0.6E_2$

B. Energy will be either E_1 (probability 50%) or E_2 (probability 50%)

C. Energy will be either E_1 (probability 40%) or E_2 (probability 60%)

D. Energy will be a straight average of $(E_1 + E_2)/2$

E. Depends on the time at which you measure the energy.

Measuring the energy causes a *collapse of the wave function*.

A single state is somehow picked out. We cannot say which one will be chosen. We can only calculate the probabilities.

Other eigenstates

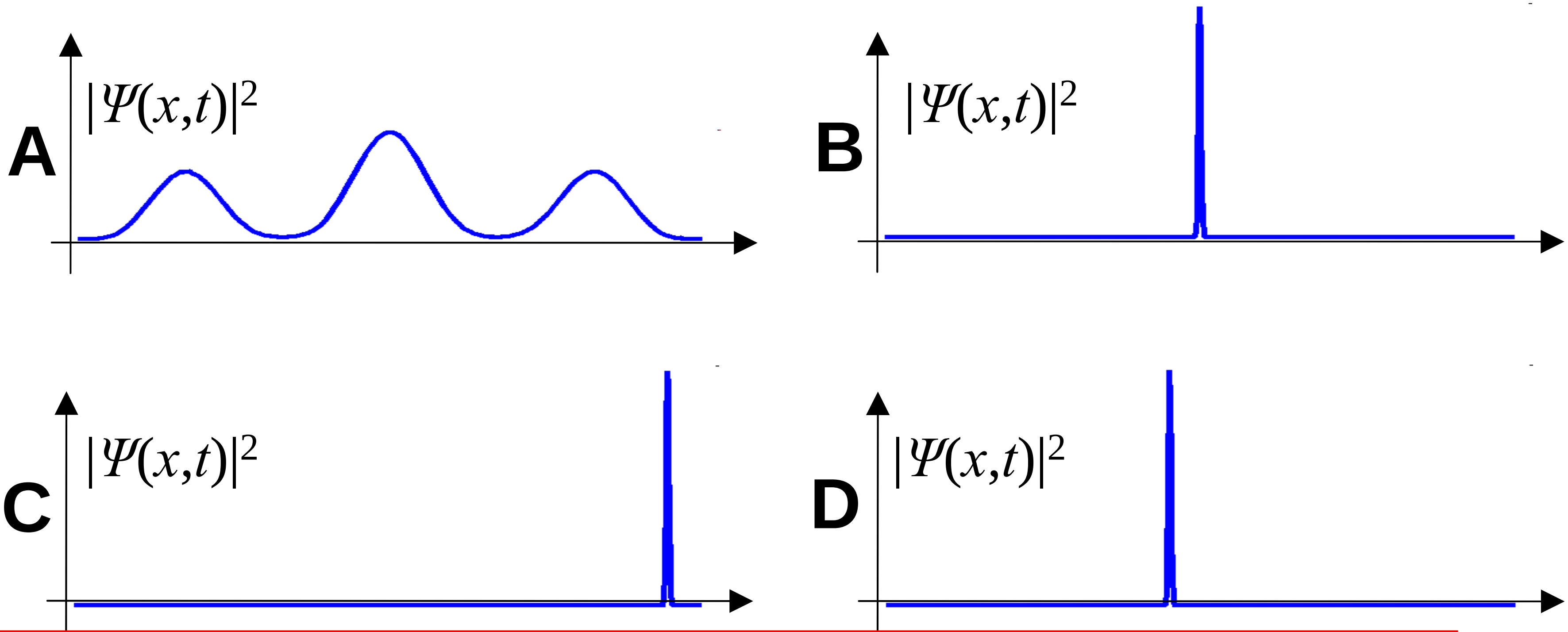
There are actually eigenstates of any measurable quantity; not just energy.

If you measure a quantity other than energy (like position, momentum, angular momentum, …) then the particle will be in an eigenstate of that quantity (and not necessarily of energy).

If we measure the position in an infinite square well the wave function collapses to a state of definite position (instead of definite energy).

Clicker question 3 — Set frequency to DA

A particle in a box initially has a wave function which has the probability distribution shown in **A**. Immediately after the position is measured, what does the probability distribution look like?



E Could be B, C, or D, depending on where you found it.

B, C, & D are eigenstates of position (definite position).

Other eigenstates

Remember that the probability density for energy eigenstates does not depend on time (stationary state).

Measuring the energy puts the particle in an energy eigenstate and it stays there until disturbed (for example by a position measurement)

This is not generally true for other eigenstates.

The position eigenstate is *not* a stationary state.

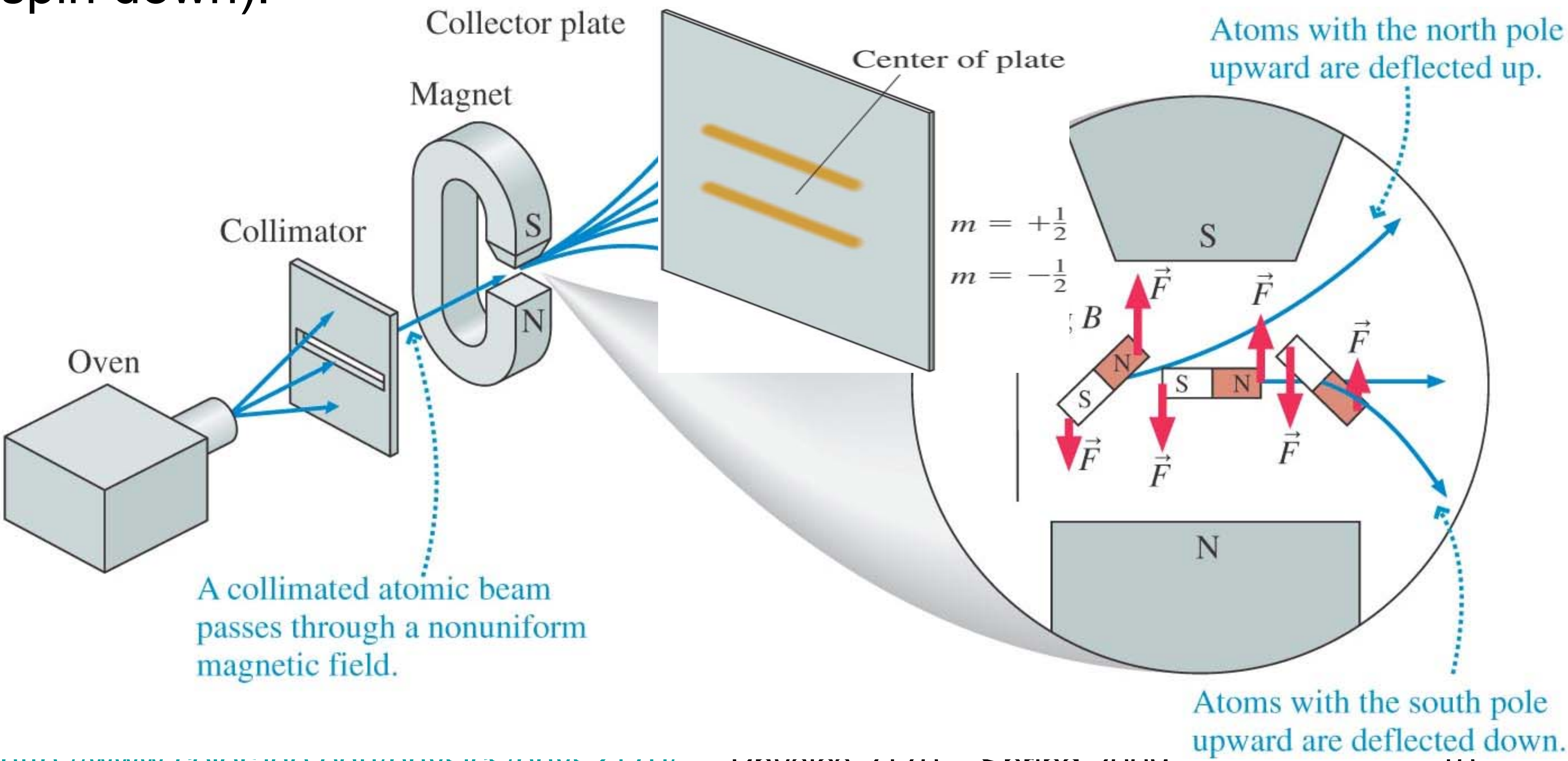
So a short time after measuring the position, the particle is no longer in a position eigenstate

Can be seen in quantum tunneling simulation.

Stern-Gerlach experiment

A Stern-Gerlach experiment sends atoms through a *nonuniform* magnetic field which exerts a net force on a magnetic dipole.

Sending in hydrogen atoms with total angular momentum just from the electron (½) splits atoms in two samples (spin up and spin down).



Result of Stern-Gerlach

The Stern-Gerlach experiment can be viewed as separating atoms according to their angular momentum direction.

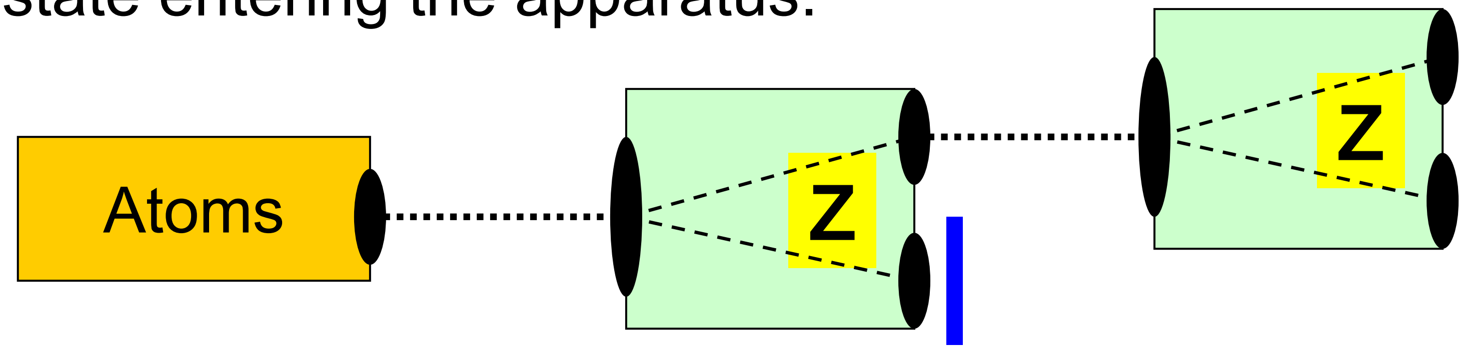
Assume our atoms start out in a random spin state. What fraction of the atoms will emerge from the top/bottom hole?

50% will go through each hole as +z or –z states are selected.

Suppose we block the −z spin atoms and pass the +z spin atoms through another SG system. One important point: in free space, angular momentum is independent of time (like energy).

What fraction of the atoms will emerge from the top/bottom hole?

100% will now come out of the top hole since it is in a +z spin eigenstate entering the apparatus.

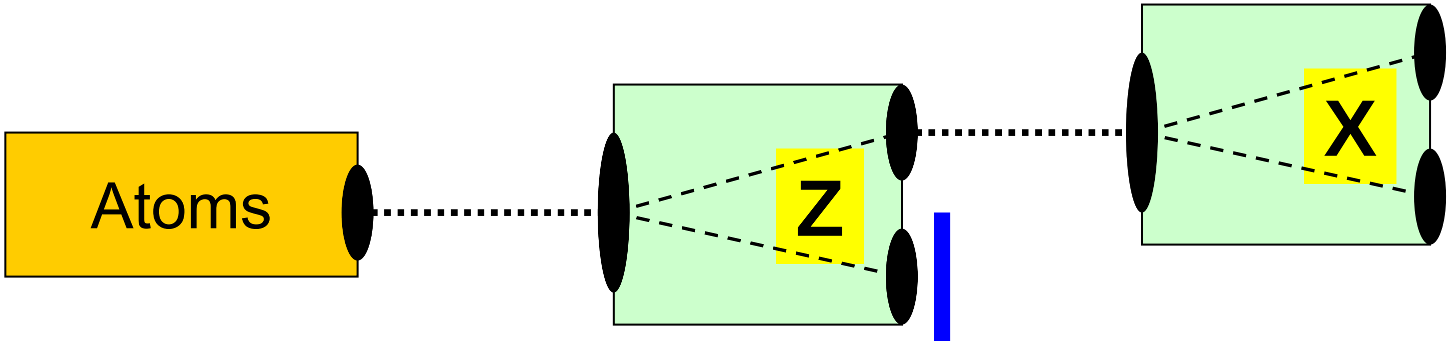


Result of Stern-Gerlach

Suppose we pass the +z spin atoms through a SG system which is oriented in **x** instead of z. One important point: Like position and momentum, we cannot know both the x-spin and z-spin at the same time.

What fraction of the atoms will emerge from the top/bottom hole?

It will be a 50/50 mix. A wave function of +z spin contains no information about the x spin so measuring the spin is just as likely to get +½ as –½.



Result of Stern-Gerlach

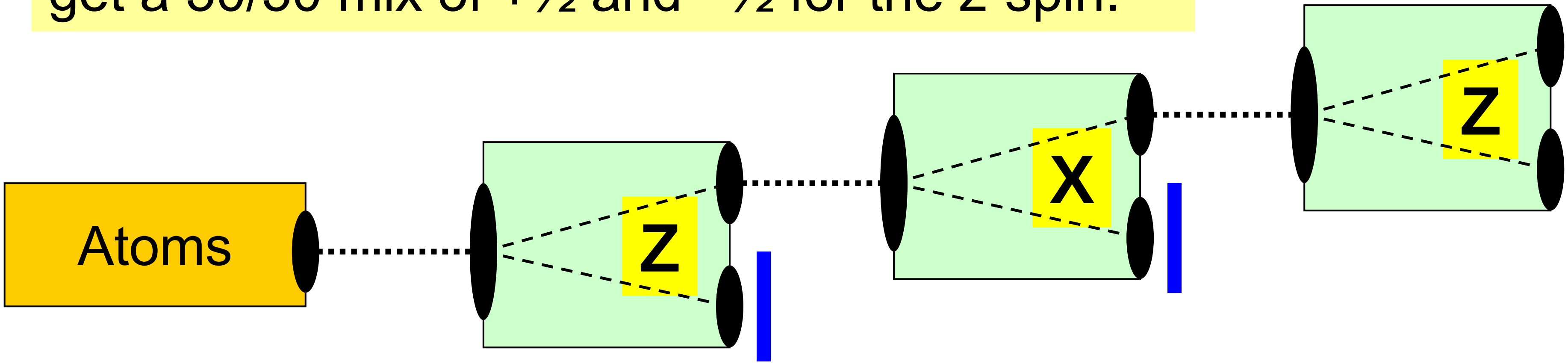
Suppose we now pass the +x spin atoms through a SG system which is oriented again in z.

What fraction of the atoms will emerge from the top/bottom hole?

It will be a 50/50 mix.

After the first magnet we only had +z spin atoms but measuring the x spin caused all knowledge of the z spin to be destroyed.

The wave function for an electron with +x spin contains no information on the z spin so we get a 50/50 mix of +½ and –½ for the z-spin.



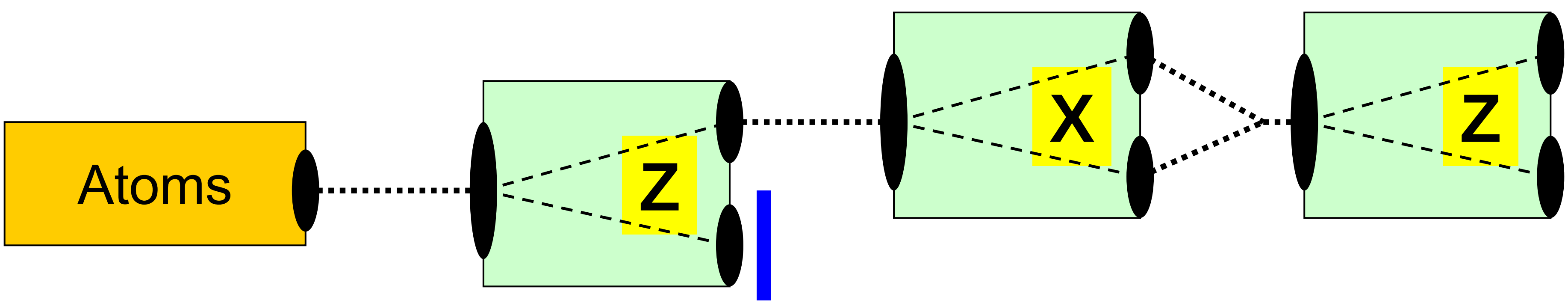
Result of Stern-Gerlach

What if we pass *both* the +x spin and –x spin atoms through the last magnet?

What fraction of the atoms will emerge from the top/bottom hole?

It will be a 100% +z atoms!

If we are careful not to actually *measure* the x spin then the wave function does not collapse to +x or –x. So we preserve the +z spin state from before!

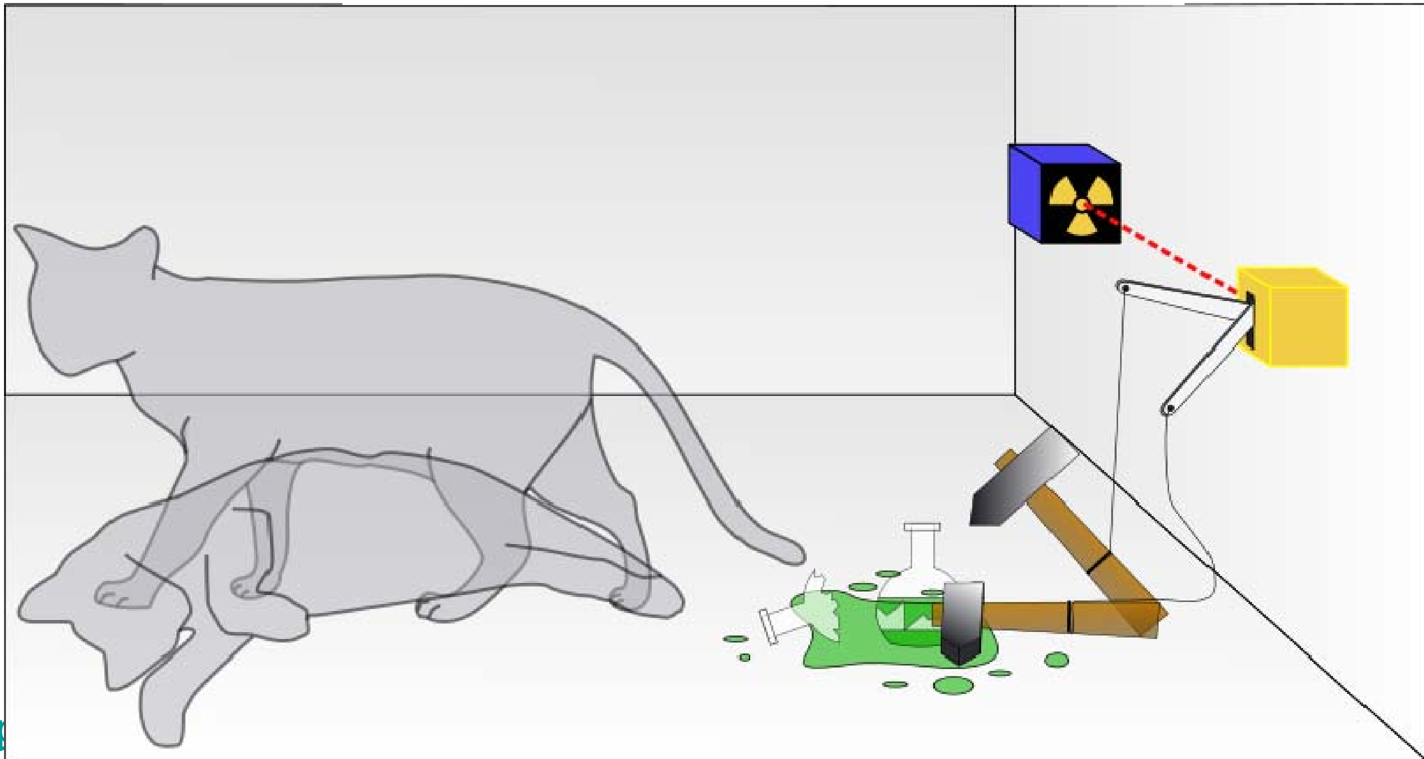


Schrödinger's Cat

A radioactive sample has a 50% chance of emitting an alpha particle. If it decays, a Geiger detector triggers the release of poison killing a cat in the box. Before opening the box, the cat is in a superposition of wave functions: $\psi = \sqrt{\frac{1}{2}}\psi_{\text{dead}} + \sqrt{\frac{1}{2}}\psi_{\text{alive}}$

When does the wave function collapse to either dead or alive?

No clear agreement. Interesting physics/philosophical question.



http://www.colorado.edu/p

Quantum entanglement/teleportation

Suppose a particle with angular momentum of 0 decays into two electrons. By conservation of angular momentum, the sum of the two electrons angular momentum must also be 0.

If we measure the z-spin of one electron to be +½ then we know that the other electron must have a z-spin of –½.

But before we measure the first electron, it is in a mixture of +½ and –½ spin states. The act of measuring causes the electron to have a definite spin.

We can separate the two electrons, measure the 1st electron and then measure the 2nd electron before any possible (light speed) signal can reach the 2nd electron. And yet the 2nd electron always has the correct spin.

Einstein called this “spooky action at a distance”

Quantum entanglement/teleportation

In 1935 Einstein, Podolsky, and Rosen wrote a paper attempting to show that quantum mechanics results in a paradox (now called the EPR paradox).

They proposed a way out: Electrons actually always <u>know</u> their spin in every direction but experiments can only get the limited knowledge allowed by quantum mechanics. A better theory would allow one to get access to this information.

This is called a *hidden variable* theory.

In 1964, J.S Bell proved that local hidden variable theories would give a different result in some cases than quantum mechanics.

Experiments in the 70s & 80s confirmed that quantum mechanics was correct and local hidden variable theories don't work.