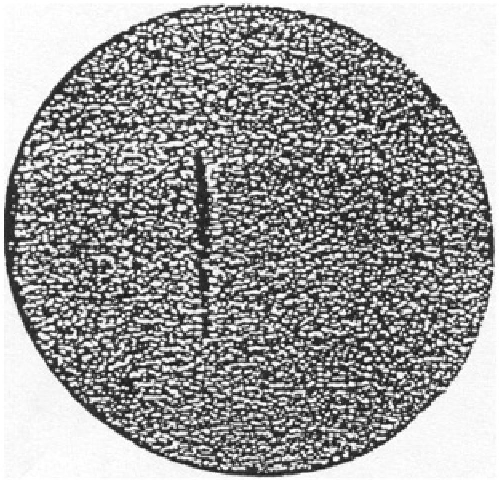
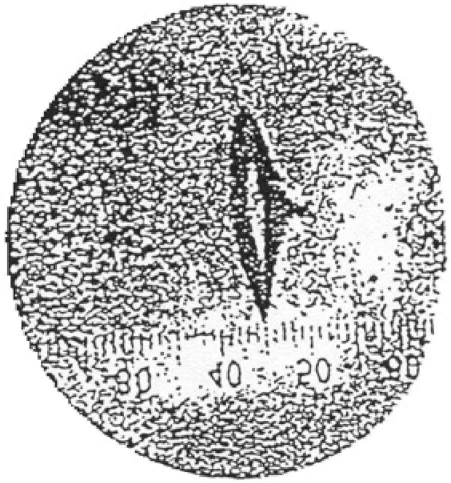
Electron spin

- Homework is due Wednesday at 12:50pm
- Problem solving sessions M3-5 and T3-5.
- FCQs will take place on Wednesday at the beginning of class.
- The last homework set will be out on 4/22 and will be due on Thursday 4/30 (one day later than normal).





Rest of semester

- Investigate hydrogen atom (Wednesday 4/15 and Friday 4/17)
- **Learn about intrinsic angular momentum (spin) of particles like electrons (Monday 4/20)**
- Take a peek at multielectron atoms including the Pauli Exclusion Principle (Wednesday 4/22)
- Describe some of the fundamentals of quantum mechanics (expectation values, eigenstates, superpositions of states, measurements, wave function collapse, etc.) (Friday 4/24 and Monday 4/27)
- Review of semester (Wednesday 4/29 and Friday 5/1)
- Final exam: Saturday 5/2 from 1:30pm-4:00pm in G125 (this room)

Summary of hydrogen wave function

The hydrogen wave function is

$$\psi(r,\theta,\phi) = R_{n\ell}(r)\Theta_{\ell m}(\theta)e^{im\phi} \quad \text{or} \quad \psi(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$$

The quantum numbers are:

n = 1, 2, 3, … = principal quantum number

$$E_n = -Z^2 E_R / n^2$$

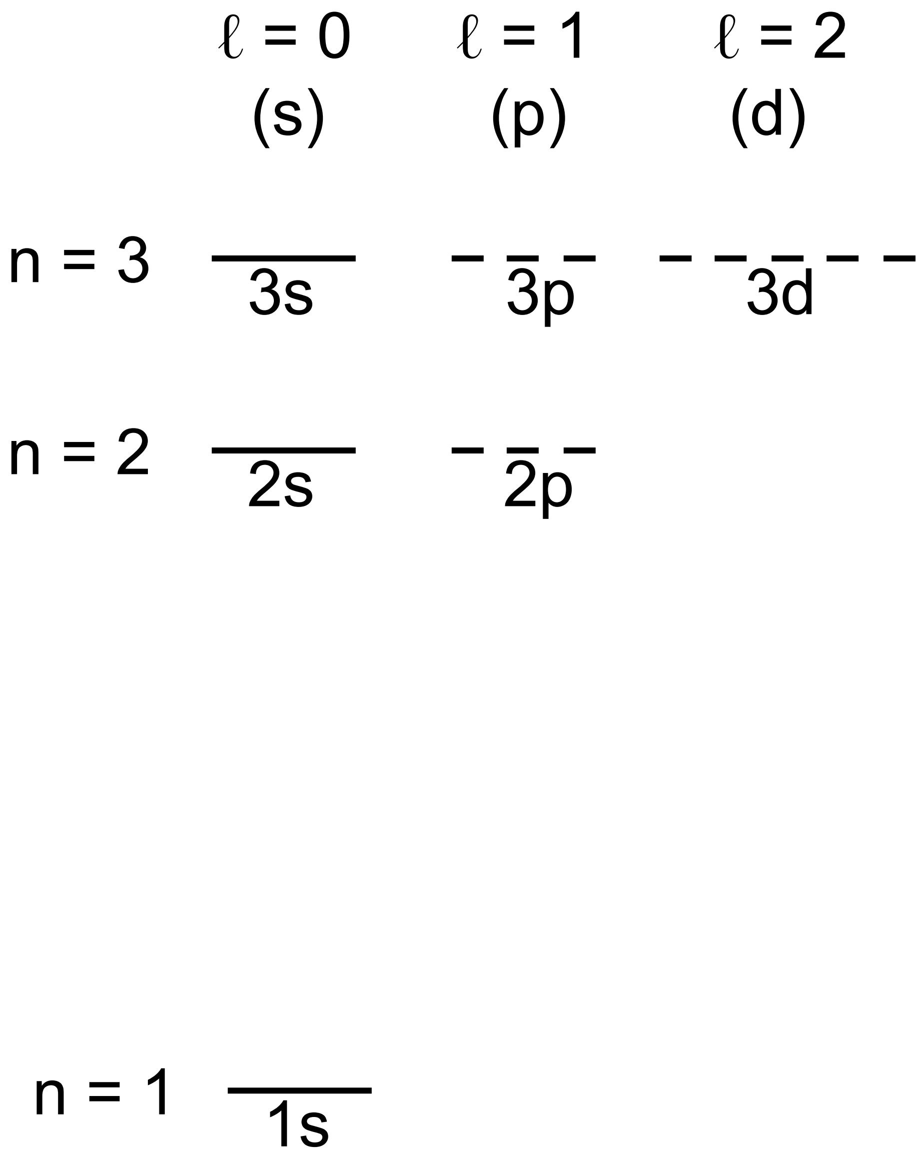
ℓ = 0, 1, 2, … *n*-1 = angular momentum quantum number
= s, p, d, f, …

$$L = \sqrt{\ell(\ell+1)}\hbar$$

m = 0, ±1, ±2, … ±*ℓ* is the z-component of angular momentum quantum number

$$L_z = m\hbar$$

Hydrogen energy levels



$$E_3 = -E_R / 3^2 = -1.5\text{ eV}$$

$$E_2 = -E_R / 2^2 = -3.4\text{ eV}$$

$$E_1 = -E_R = -13.6\text{ eV}$$

Probability versus radius: $P(r) = |R_{nl}(r)|^2 r^2$

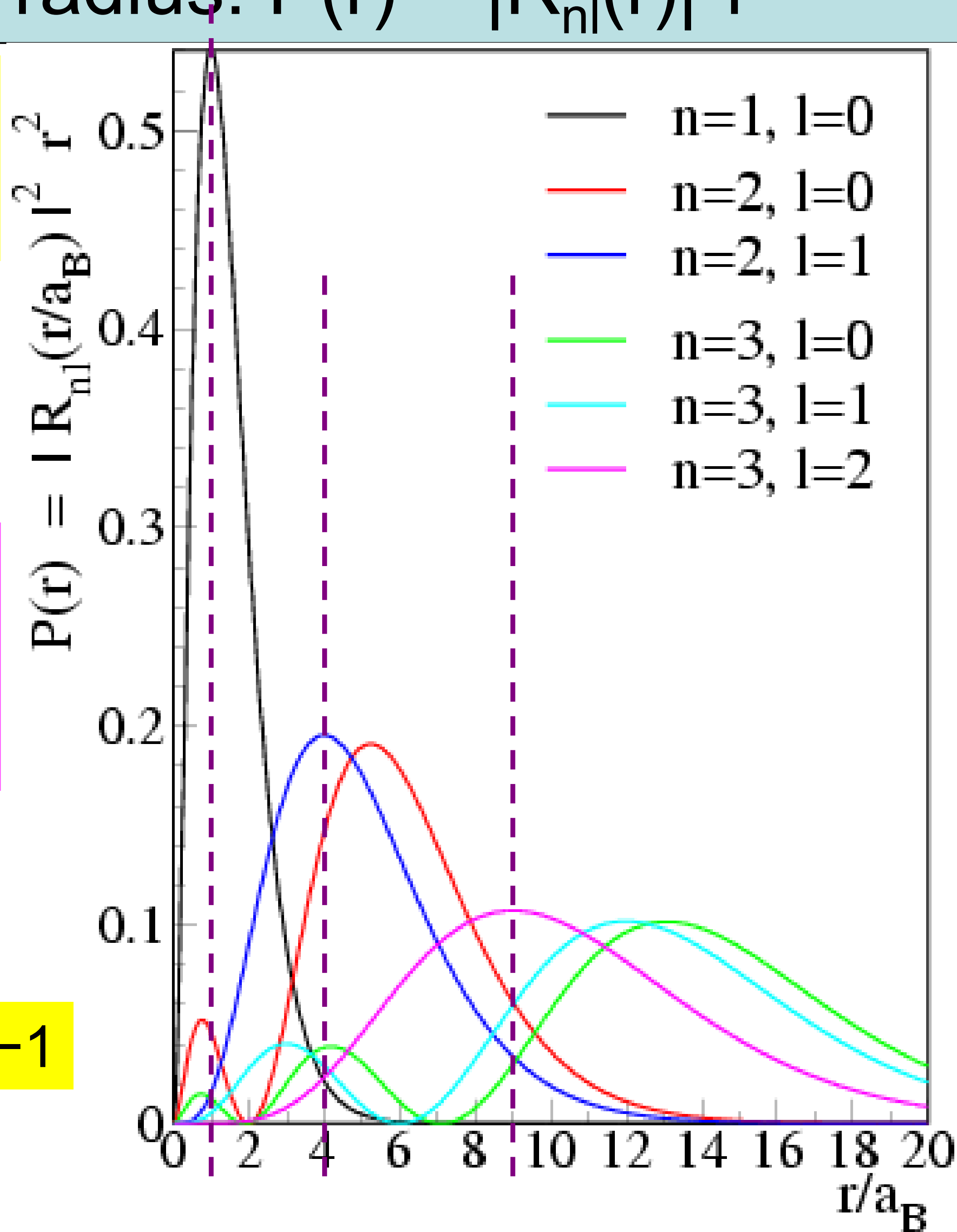
In spherical coordinates, the volume element $\propto r^2$ so probability increases with r^2.

Most probable radius for the n = 1 state is at the Bohr radius a_B.

Most probable radius for all ℓ=n-1 states (those with only one peak) is at the radius predicted by Bohr ($n^2\ a_B$).

Note the average radius increases as *n* increases.

Number of radial nodes = $n-\ell-1$



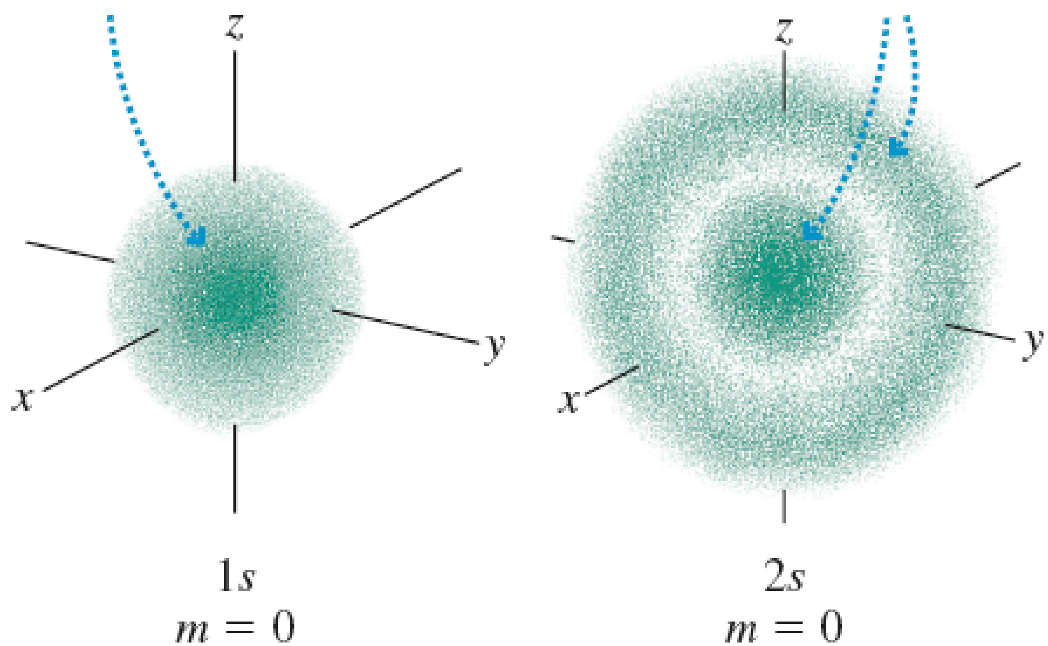
http://www.colorado.edu/physics/phys2170/

Visualizing the hydrogen atom

S states: ℓ = 0

s states have no angular momentum and are thus spherically symmetric.

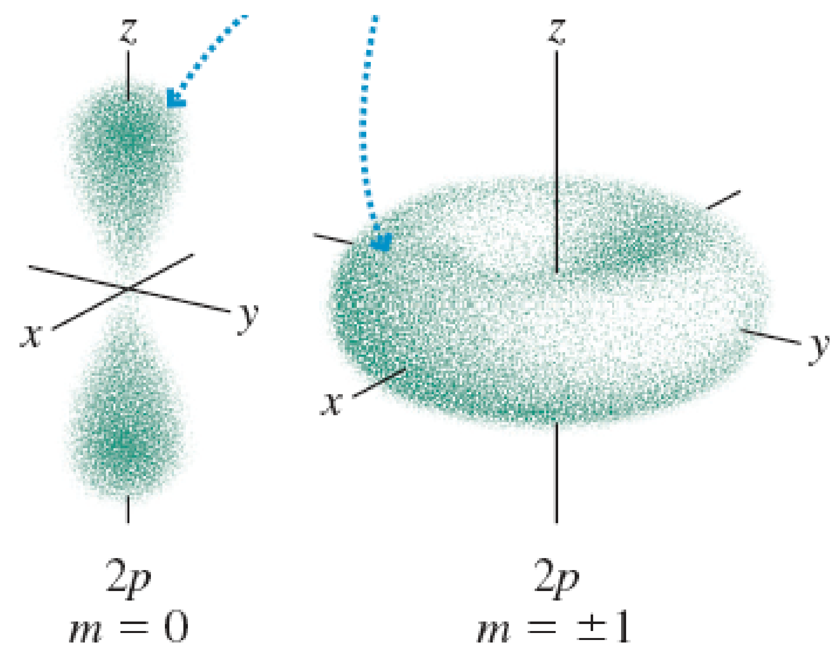
There are n−ℓ−1 radial nodes.
Nodes are where $|\psi|^2 \to 0$.



P states: ℓ = 1

For *m*=0, L_z=0 so no rotation about the *z* axis

For *m*=±1, L_z=±ħ so there <u>is</u> rotation about the *z* axis (either clockwise or counter clockwise)



Clicker question 1

Set frequency to DA

Schrödinger finds quantization of energy and angular momentum:

n = 1, 2, 3 … ℓ = 0, 1, 2, 3 (restricted to 0, 1, 2 … n-1)

$$E_n = -E_R / n^2 \qquad L = \sqrt{\ell(\ell+1)}\,\hbar$$

How does the Schrödinger result compare to the Bohr result?

I. The energy of the ground state solution is ___same___

II. The angular momentum of the ground state solution is ___different___

III. The location of the electron is ___different___

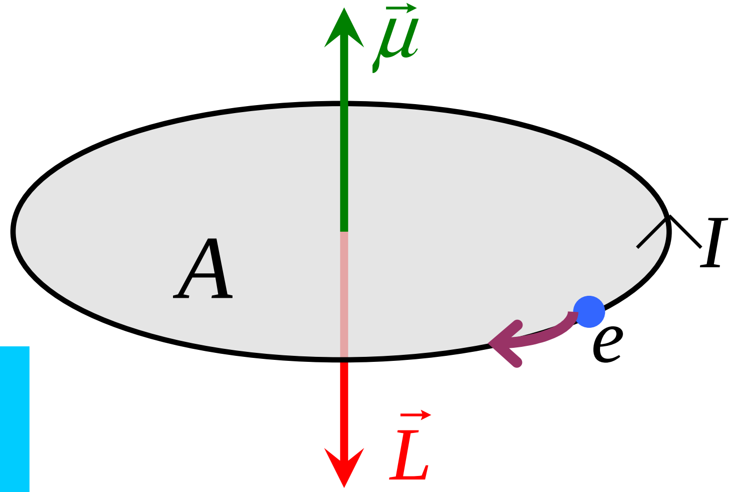
A. same, same, same
B. same, same, different
C. same, different, different
D. different, same, different
E. different, different, different

Bohr got the energy right, but said angular momentum was $L=n\hbar$, and thought the electron was a point particle orbiting around nucleus at a fixed distance.

Magnetic moment

What do you get when you have a current going around in a loop?

Magnetic field which behaves like a magnetic dipole with a magnetic dipole moment of $\vec{\mu} = IA$. Direction is given by the right hand rule.



An orbiting electron creates a current (in the opposite direction) around an area.

The current depends on electron velocity and the area size depends on the orbit radius.

Same quantities go into angular momentum: $\vec{L} = m\vec{r} \times \vec{v}$

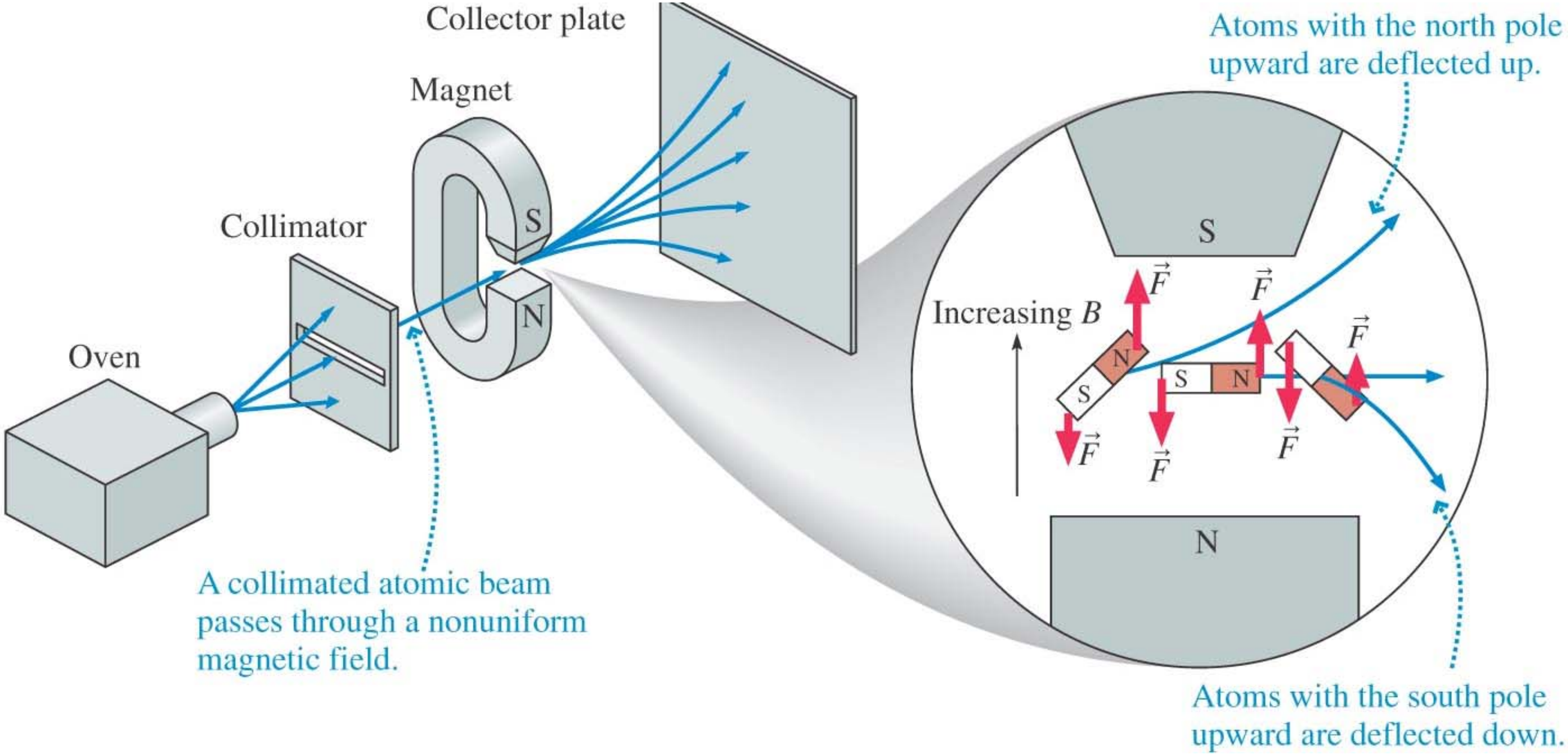
Turns out we can write the magnetic moment of an atom in terms of the electron's angular momentum:

$$\vec{\mu} = -\frac{e}{2m_e}\vec{L}$$

Stern-Gerlach experiment

Placing a magnetic dipole in an external *uniform* magnetic field $\vec{B}$ causes a torque on the dipole $\vec{\tau} = \vec{\mu} \times \vec{B}$ but **no** net force.

A Stern-Gerlach experiment sends atoms through a *nonuniform* magnetic field which *can* exert a net force on a magnetic dipole.



Clicker question 2 — Set frequency to DA

The Stern-Gerlach magnet is oriented so deflections occur in the z direction. Based on what we know so far, if the atoms passing through have no angular momentum $(\ell = 0 \text{ so } L = 0)$ what will happen?

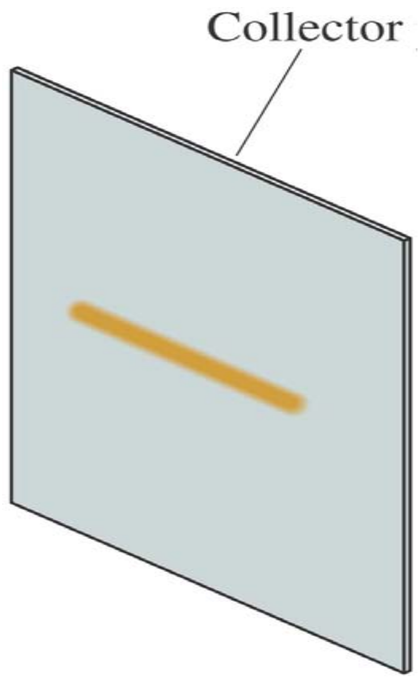
A. Atoms will be deflected in z direction

B. Atoms will be deflected in x direction

C. Atoms will be deflected in y direction

D. Atoms will not be deflected

E. Need quantum number *m* to tell



Atoms with no angular momentum have no magnetic dipole moment

$$\vec{\mu} = -\frac{e}{2m_e}\vec{L}$$

Therefore, they are not affected by the magnet (no torque or force)

Clicker question 3

Set frequency to DA

The Stern-Gerlach magnet is oriented so deflections occur in the z direction. If the atoms passing through have angular momentum of $\ell = 1$ so $L = \sqrt{2}\hbar$ but the z-component $L_z = m\hbar$ is unknown, how many possibilities are there for deflection?

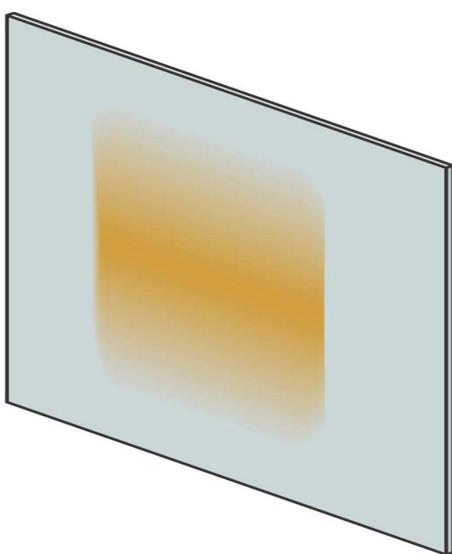
A. 0
B. 1
C. 2
D. 3
E. Infinite

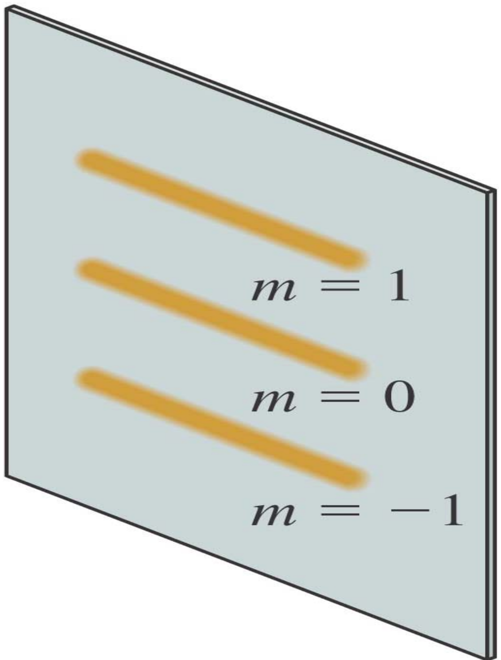
When ℓ=1, the quantum number *m* can only have three possible values (−1, 0, 1) so $L_z = -\hbar, 0, \text{or } \hbar$

The *m* = −1 and *m* = 1 atoms are deflected in opposite directions and the *m* = 0 atoms are not deflected at all.

Classical result

Classically, an atom with $L = \sqrt{2}\hbar$ can have any value of L_z as long as $|L_z| \le \sqrt{2}\hbar$





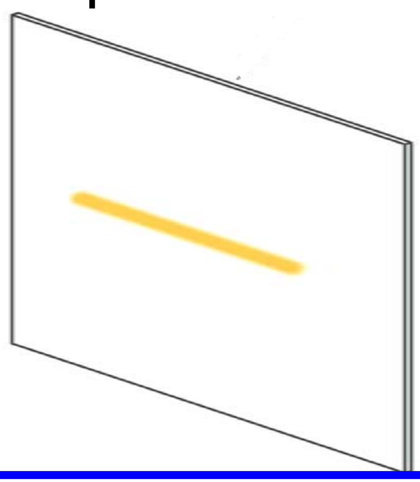
Please answer this question on your own.
No discussion until after.

Q. The spin quantum number for the electron *s* is…

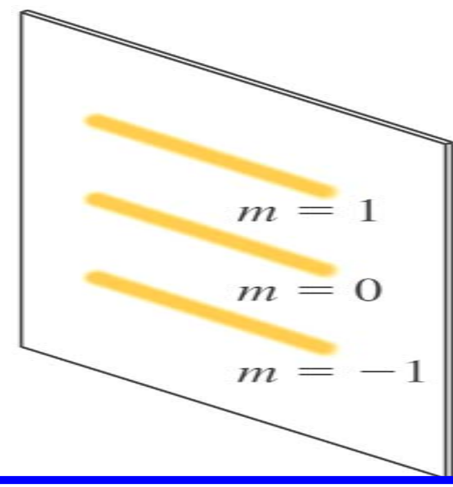
A. 0
B. ½
C. 1
D. Can be more than one of the above
E. None of the above

Result of Stern-Gerlach

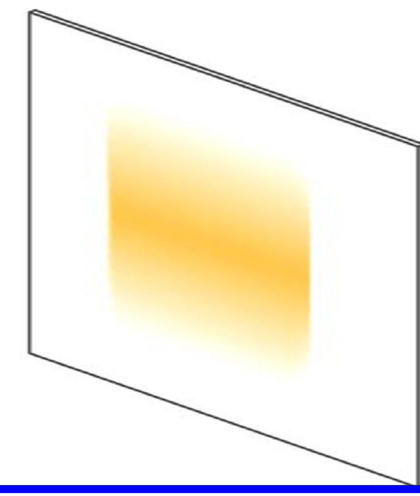
Sending in (ground state) hydrogen atoms which were believed to have ℓ=0, one expects no deflection.



If ℓ≠0, would find 2ℓ+1 bands (odd number)

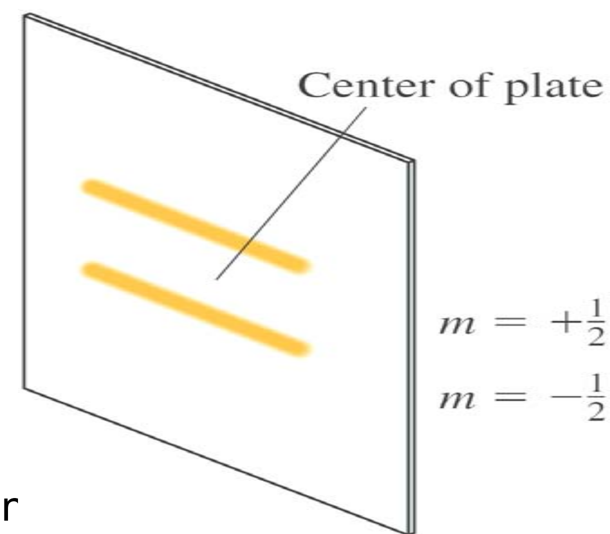


Classically, one would see a broad band



Doing the experiment gave two lines.

Interpretation: ℓ=0 but the electron itself has some *intrinsic* angular momentum which can either be −ħ/2 or ħ/2.



Electron spin

ℓ = 0, 1, 2, … n-1 = *orbital* angular momentum quantum number

$$L = \sqrt{\ell(\ell+1)}\hbar$$

m = 0, ±1, ±2, … ±ℓ is the z-component of *orbital* angular momentum

$$L_z = m\hbar$$

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s = *spin* (or *intrinsic*) angular momentum quantum number. The actual spin angular momentum is

$$S = \sqrt{s(s+1)}\hbar$$

Electrons are s = ½ (spin one-half) particles.
Since this never changes, it is often not specified.

m_s = z-component of *spin* angular momentum and can have values of m_s = −s, −s+1, … s−1, s. The actual z-component of spin angular momentum is

$$S_z = m_s\hbar$$

For an electron only two possibilities: $m_s = \pm s = \pm ½$

An electron with m_s = +½ is called spin-up or ↑

An electron with m_s = −½ is called spin-down or ↓