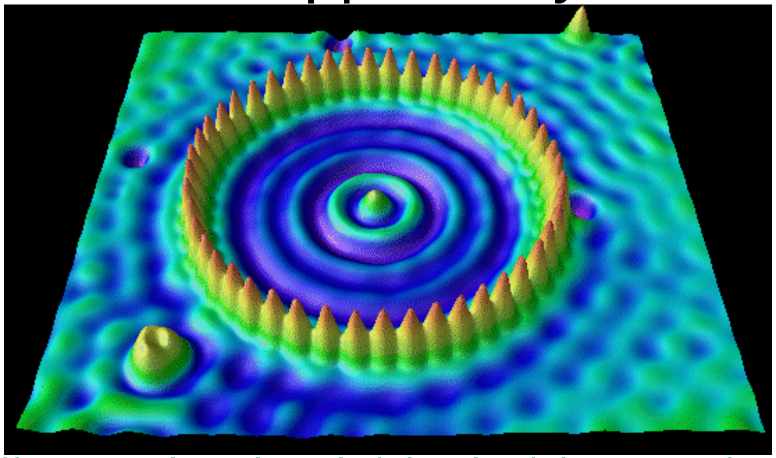
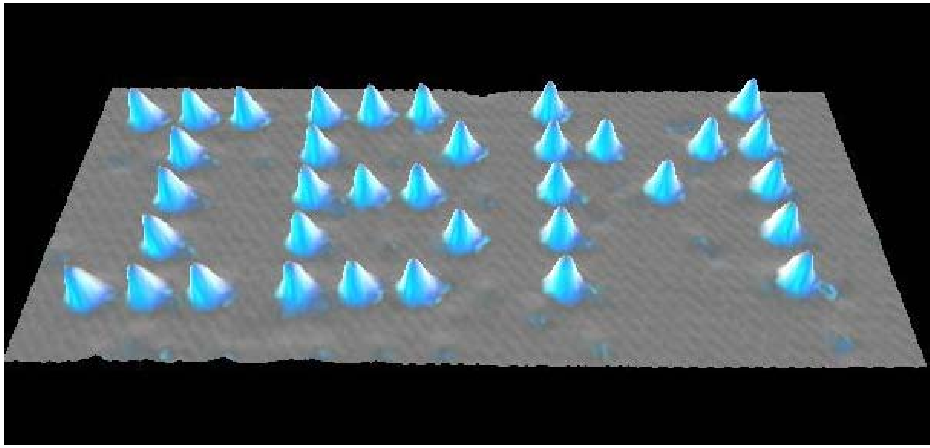
Quantum tunneling: STM & electric shock

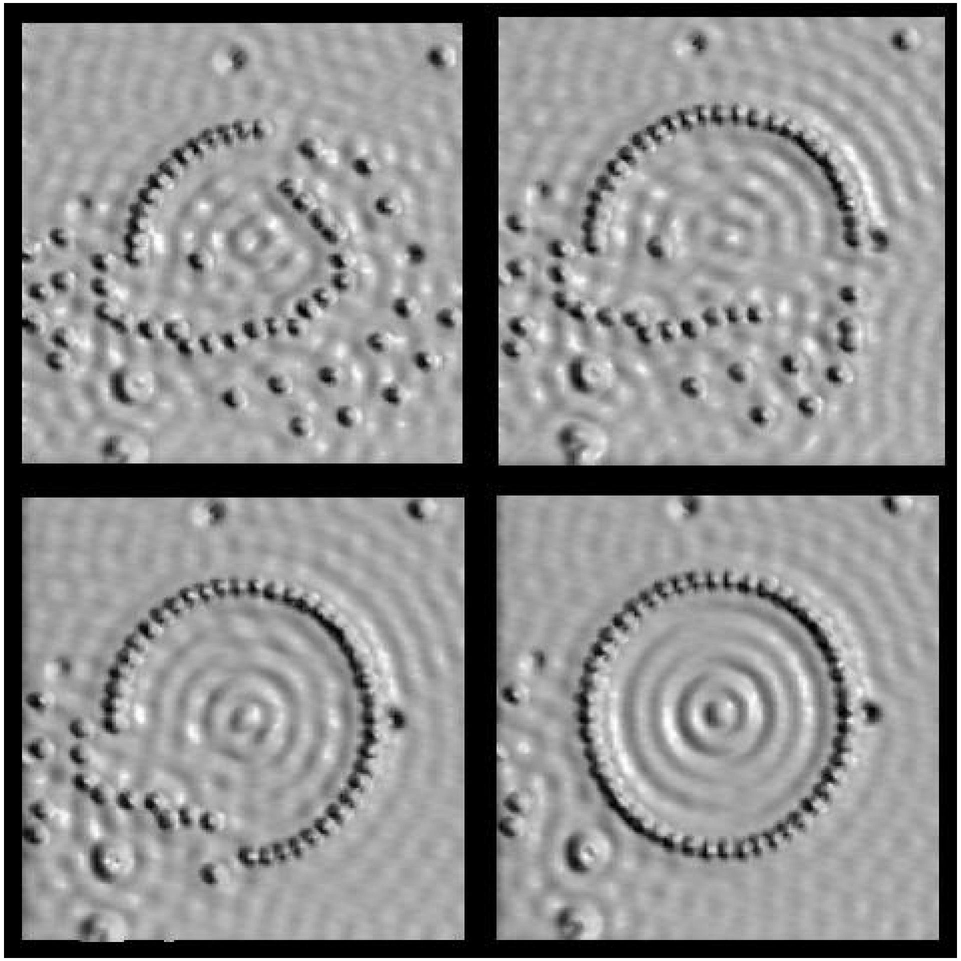
- Homework set 12 is due Wednesday.
- Clicker scores have been entered into CULearn
  - Each Reading Quiz (RQ) is out of 1 (lowest 2 are dropped)
  - Regular clicker questions are out of 100 for each day (lowest 5 are dropped)
- 10 out of 50 points for next weeks homework will be for completing a survey. Full credit will be given for well thought out answers.
- Charles Baily is interested in conducting interviews with students and is willing to pay $15. I will send out email today about the opportunity.

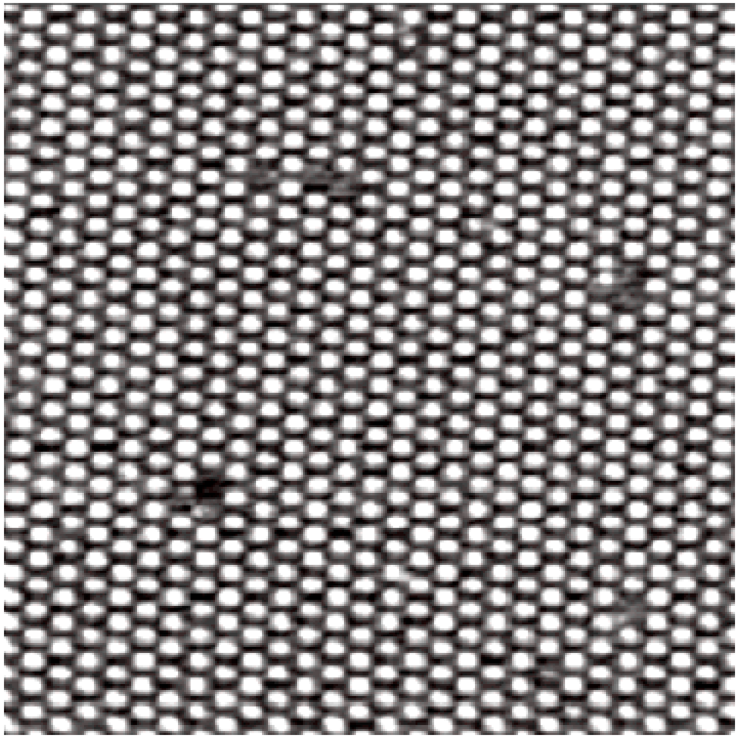




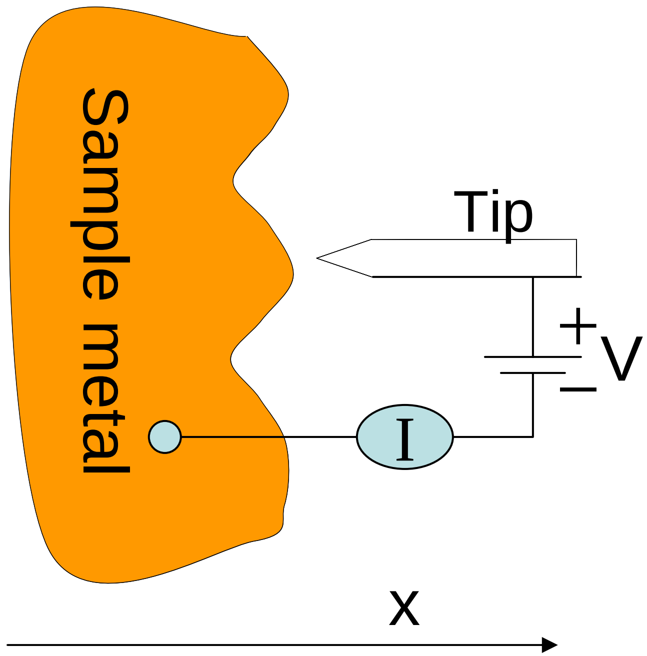
Scanning tunneling microscope

Use tunneling to measure small changes in distance.
Nobel prize winning idea: invention of "scanning tunneling microscope (STM)". Measure atoms on surfaces.



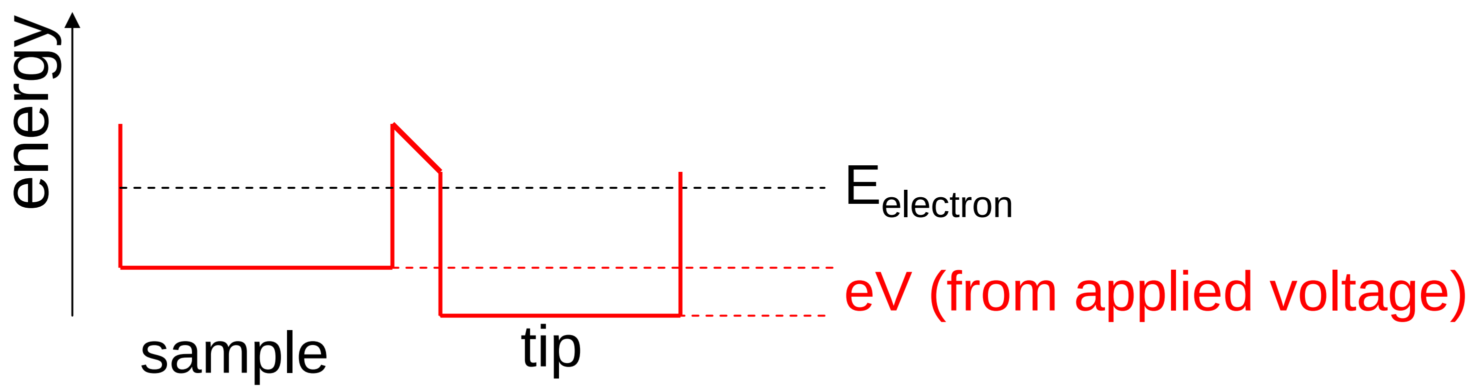


STM potential energy curve



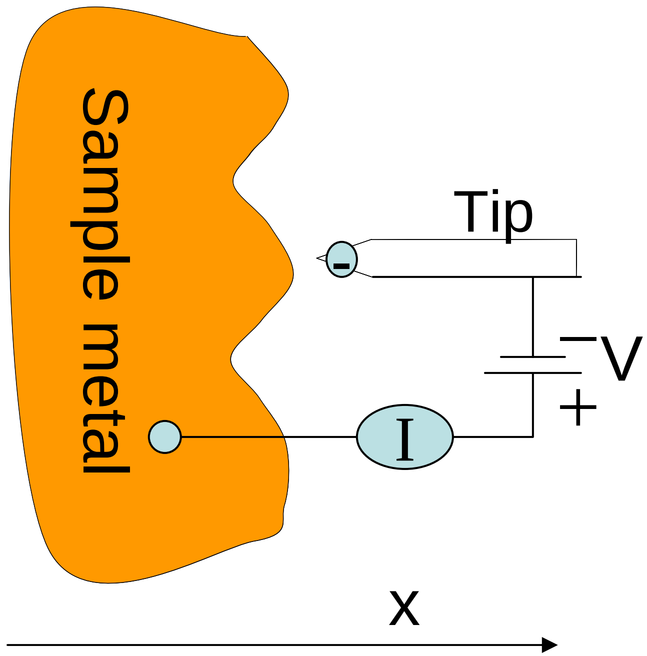
Applying a potential V has two effects

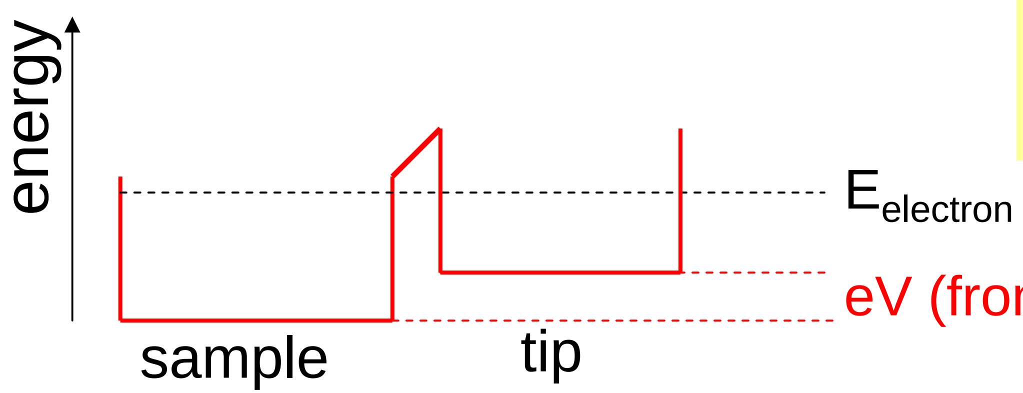
1. Allows a current to flow since electrons will be more likely to tunnel to lower potential
2. Lowers the effective potential barrier making it easier to tunnel



Clicker question 1

Set frequency to DA





If the same voltage is applied in the *opposite* direction how well will this method work?

A. Works just as well

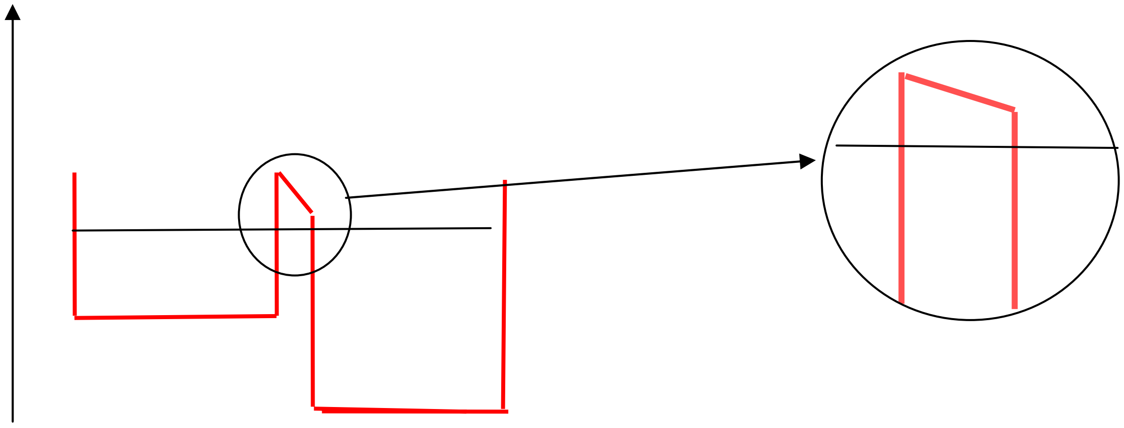
B. Works but not as well

C. Doesn't work at all

The electron will move in the opposite direction so current will be opposite but everything else is the same.

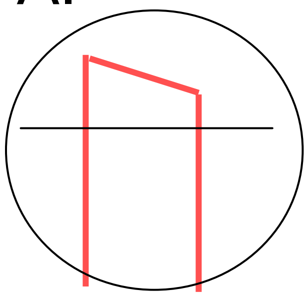
Clicker question 2

Set frequency to DA

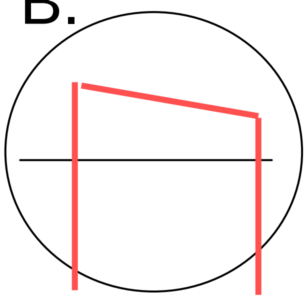


If the tip is moved closer to the sample, what will the new potential graph look like?

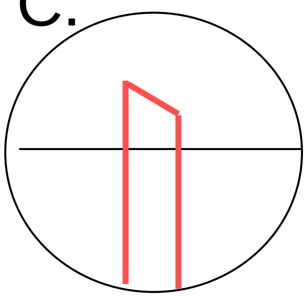
A.



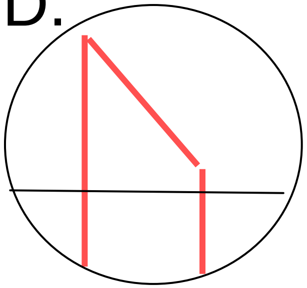
B.



C.



D.



How sensitive is the STM?

Remember tunneling probability is $P \approx e^{-2\alpha L}$ with $\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$

For work function of 4 eV $\alpha = \frac{\sqrt{2m(V-E)}}{\hbar} \approx 10\,\text{nm}^{-1}$

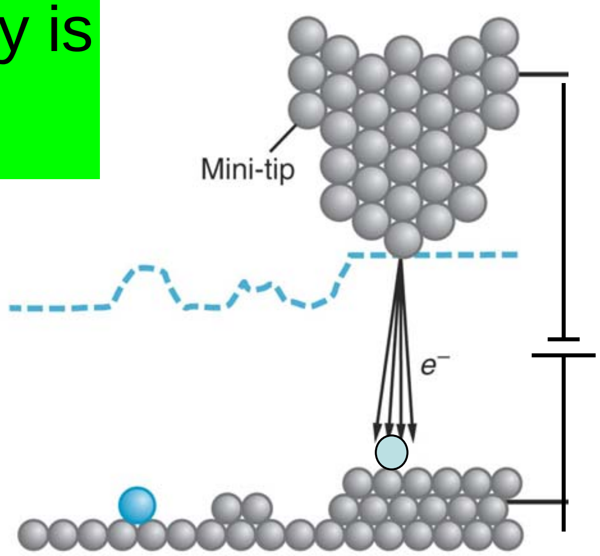
Note this corresponds to a penetration depth of $\lambda = 1/\alpha = 0.1\,\text{nm}$

If probe is 0.3 nm away (*L=0.3* nm), probability is

$$e^{-2\alpha L} = e^{-2(10\,\text{nm}^{-1})(0.3\,\text{nm})} = e^{-6} = 0.0025$$

An extra atom on top decreases the distance by 0.1 nm so *L* = 0.2 nm giving a tunneling probability of

$$e^{-2\alpha L} = e^{-2(10\,\text{nm}^{-1})(0.2\,\text{nm})} = e^{-4} = 0.018$$

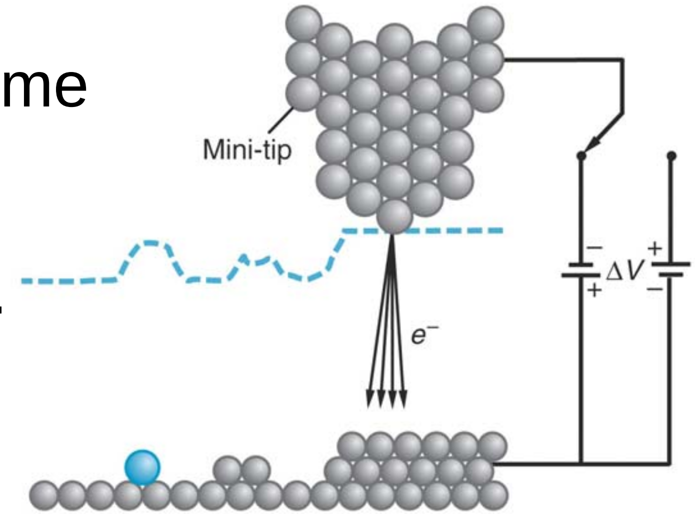


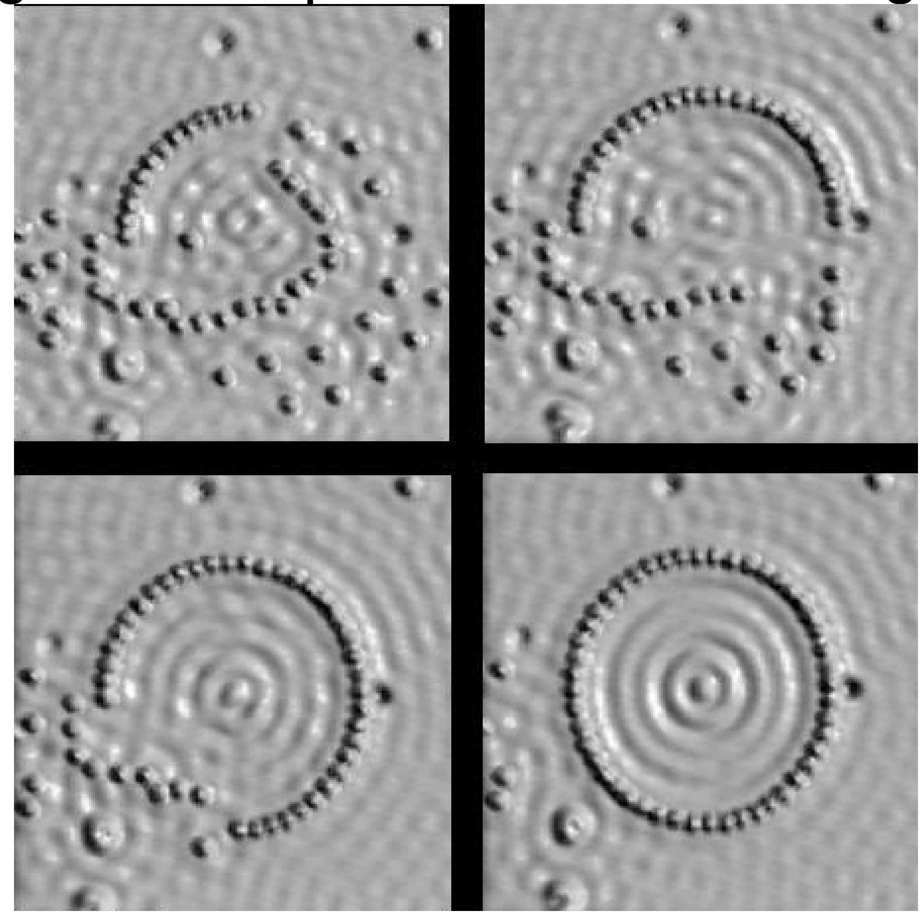
Current is proportional to the probability of an electron tunneling.

One atom increases current by 0.018/0.0025 = 7 times!

STM details

Actual STM uses feedback to keep the current (and therefore the distance) the same by moving the tip up or down and keeping track of how far it needed to move. This gives a map of the surface being scanned.





STM's can also be used to slide atoms around as shown.

Another manifestation of quantum tunneling

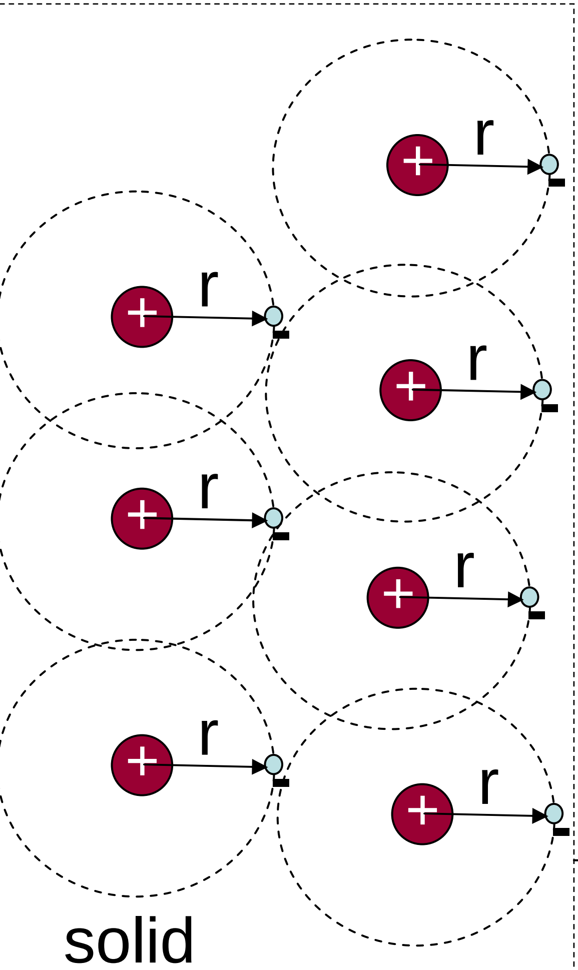
What electric field is needed to pull an electron out of a solid if we ignore quantum tunneling?

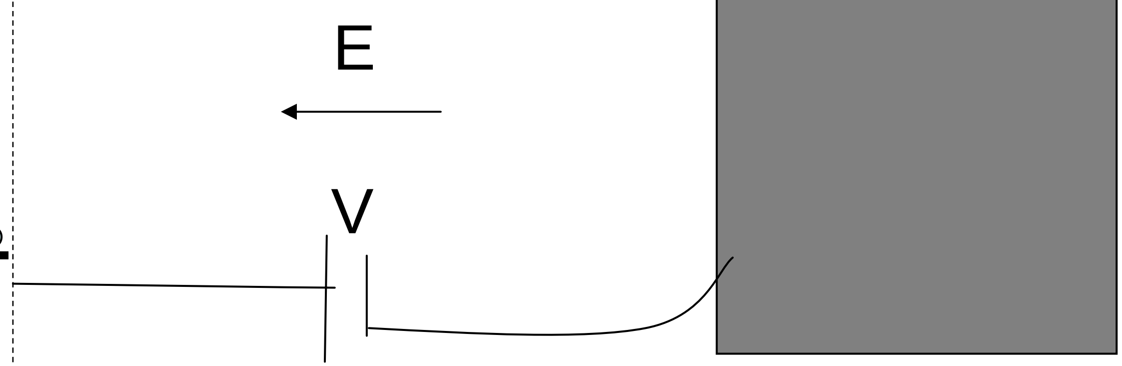
Applied force on the electron must be larger than the force by the nucleus.

Assume we are dealing with hydrogen.

Since F=qE, the applied electric field E must exceed nucleus electric field E_{nuc}.

$$E_{nuc} = \frac{kq}{r^2} = \frac{9\times10^9\ \mathrm{Nm^2/C^2}\cdot 1.6\times10^{-19}\ \mathrm{C}}{(0.053\ \mathrm{nm})^2}$$
$$= 5\times10^{11}\ \mathrm{V/m}$$



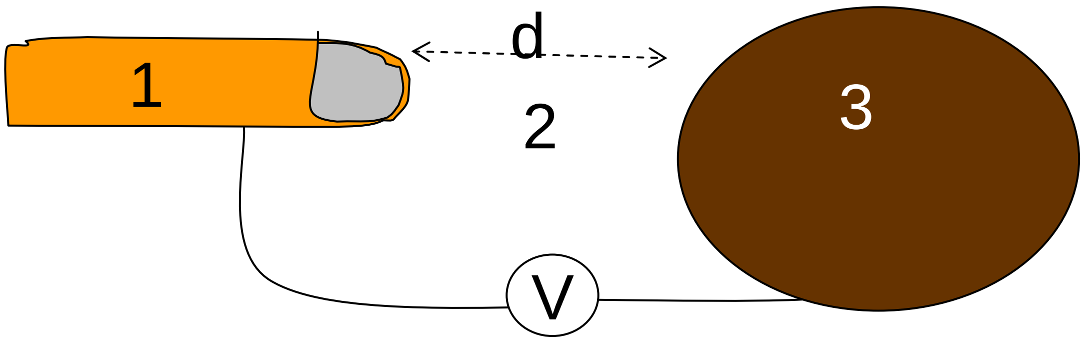


Clicker question 3 — Set frequency to DA

E = 5×10^{11} V/m means need 1 billion volts for a 2 mm long spark

Do we get a billion volts by rubbing feet on rug?

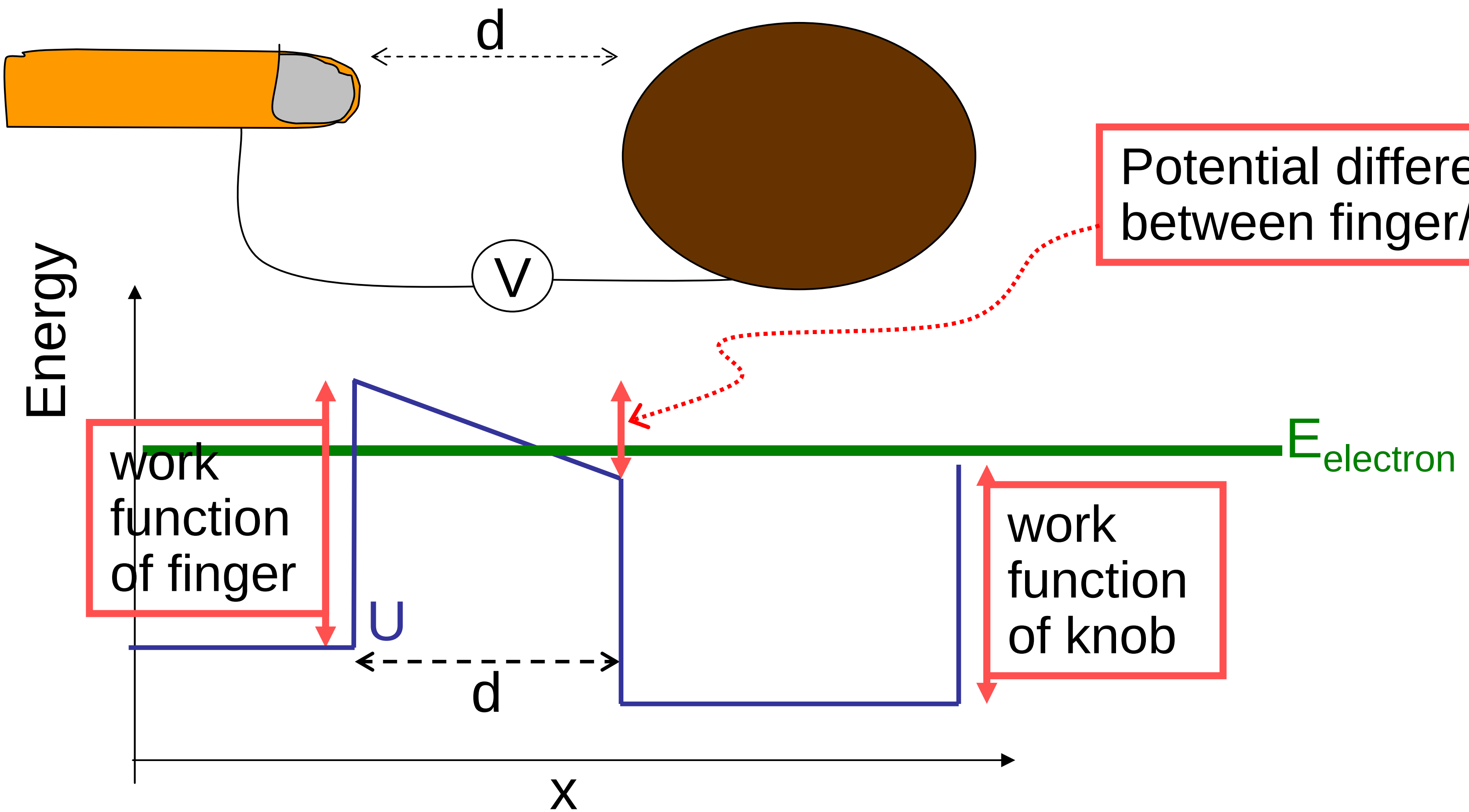
NO! Electrons tunnel out at much lower voltage.



What is the minimum info needed to find the tunneling probability?

A. only d
B. only V
C. V and d
D. V, d, and work functions of finger and doorknob
E. none of the above, need additional information

Potential energy for electric shock from door knob



Tunneling probability:

$$P \approx e^{-2\alpha L}$$

Can effectively shorten *L* by moving finger closer **or** by increasing voltage

Rest of semester

- Investigate hydrogen atom skipping most of how we get the solutions to find out what the solutions mean (Wednesday 4/15 and Friday 4/17)
- Learn about intrinsic angular momentum (spin) of particles like electrons (Monday 4/20)
- Take a peak at multielectron atoms including the Pauli Exclusion Principle (Wednesday 4/22)
- Describe some of the fundamentals of quantum mechanics (expectation values, eigenstates, superpositions of states, measurements, wave function collapse, etc.) (Friday 4/24 and Monday 4/27)
- Review of semester (Wednesday 4/29 and Friday 5/1)
- Final exam: Saturday 5/2 from 1:30pm-4:00pm

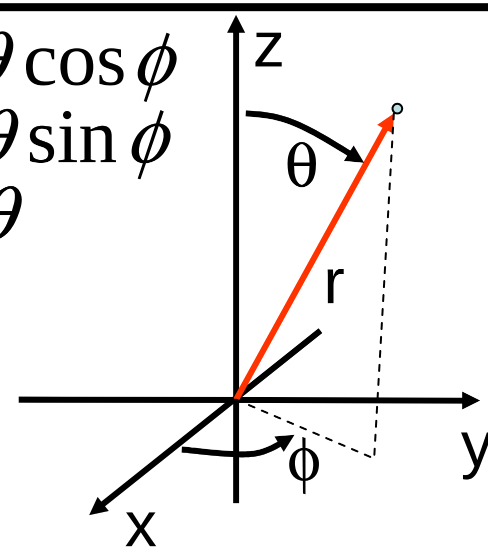
3-D central force problems

The hydrogen atom is an example of a 3D central force problem.

The potential energy depends only on the distance from a point (spherically symmetric)

Spherical coordinates is the natural coordinate system for this problem.

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$



Engineering & math types sometimes swap θ and ϕ.

General potential: V(r,θ,ϕ)
Central force potential: V(r)

The Time Independent Schrödinger Equation (TISE) becomes:

$$-\frac{\hbar^2}{2m_e}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}\right]+V(r)\psi=E\psi$$

We can use separation of variables so $\psi(r,\theta,\phi)=R(r)\Theta(\theta)\Phi(\phi)$