Finite square well and tunneling

Announcements:

- 2^{nd} exam is on Tuesday (April 7) in MUEN 0046 from 7:30 – 9:00 pm.
- Homework solutions and next week's homework will be out late today.
- More practice exams will be posted on CULearn sometime tomorrow
- Friday's class will be a quantum tunneling tutorial. Full clicker points will be given for participation. Also, the tutorial will be part of next week's homework.

Today I will try to answer some questions raised last time, finish up the finite square well, and introduce quantum tunneling.

Lots of interesting talks today and tomorrow

Today at 4pm the Physics Colloquium in G1B20 is given by Tim Donaghy of the Union of Concerned Scientists and is titled “Protecting the Integrity of Federal Government Science” – cookies beforehand.

Today at 6pm Edward Burger of Williams College (standup comedian and Leno joke writer turned math professor) will give a talk in Math 100 titled “Is there a Fourth Dimension? Can we see it?” – pizza and soft drinks follow.

Tomorrow at 4pm a special Physics Colloquium in Humanities 250 is given by David Kaiser of MIT and is titled “How the hippies saved physics” – cookies beforehand.

Tomorrow at 7:30pm the Gamow lecture will be given by Joy Hirsch of Columbia University on “Dialogs with the Specialized Brain.” Talk in Macky, hors d'oeuvres follow in Old Main.

The University of Colorado at Boulder

DEPARTMENT OF MATHEMATICS

Presents

PROFESSOR EDWARD BURGER
Williams College

Is there a Fourth Dimension? Can we see it?

WEDNESDAY APRIL 1, 2009 at 6 pm in MATH 100

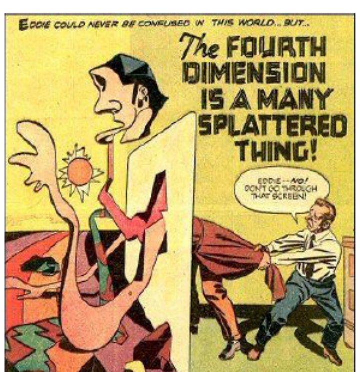
(There will be pizza and soft drinks served after the talk.)

What does it mean to say we live in a "3-dimensional" world? Could there be an extra, fourth dimension? If so, then are we just living in a "thin" slice of space? How can we wrap our minds about worlds that we cannot physically see? Can we use the fourth dimension to find our lost homework or iPod? Can the fourth dimension unlock the secrets to certain "impossible" magical illusions? Here we face these questions, attempt some daring feats of dimension, and delve into a vast, mysterious, and invisible universe. Along the way, we'll see how embracing life lessons from mathematics allows us to see our world—and even the world of art—in an entirely new way.

If you hate mathematics, this talk is for you; if the sight of an equation makes you ill, this talk is for you; if you never thought you'd ever go to a math lecture, this talk is for you!

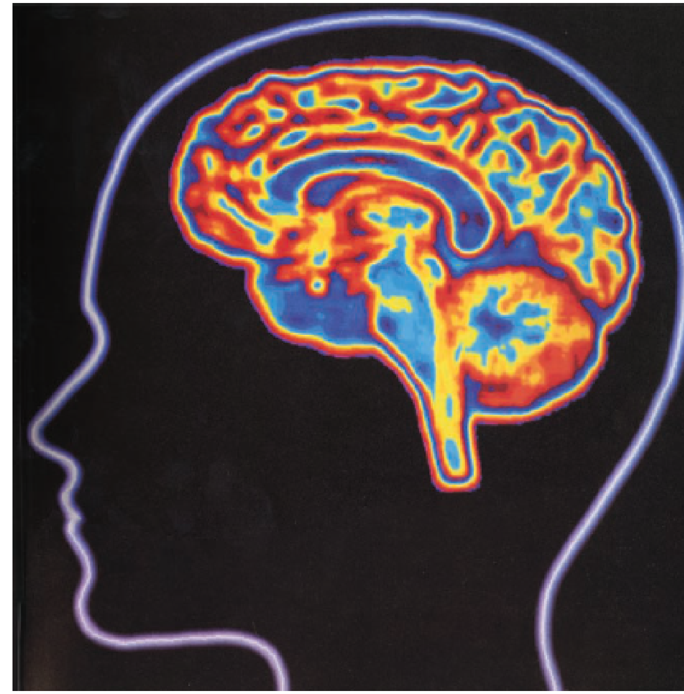
Not even one formula or equation will be uttered. Math-fans and math-phobes of all ages are encouraged to attend this lively event.

EDWARD BURGER is Professor of Mathematics at Williams College. He is the author of over 30 research articles and 12 books. He has won numerous awards including the 2001 Mathematical Association of America's Deborah and Franklin Tepper Haimo National Award for Distinguished Teaching of Mathematics, the Chauvenet Prize, and the Lester R. Ford Prize. The MAA also named him a Polya Lecturer. In 2003 he received a Residence Life Teaching Award from The University of Colorado at Boulder. In 2007 Williams College awarded him the Nelson Bushnell Prize for Scholarship and Teaching and this year the College named him the Gaudino Scholar. Burger is an associate editor of the *American Mathematical Monthly* and *Math Horizons*. In 2006, Reader's Digest listed Burger in their annual "100 Best of America" as America's Best Math Teacher. In a former life Burger had an unsuccessful short-lived "career" as a stand-up comedian and briefly was an independent joke writer for Jay Leno.



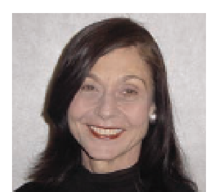
44th George Gamow Memorial Lecture

"DIALOGS WITHIN THE SPECIALIZED BRAIN"



JOY HIRSCH

Professor of Functional Neuroradiology, Neuroscience, and Psychology
Director of the Program for Imaging & Cognitive Sciences, PICS
Columbia University



Thursday, April 2, 2009 • 7:30 p.m.

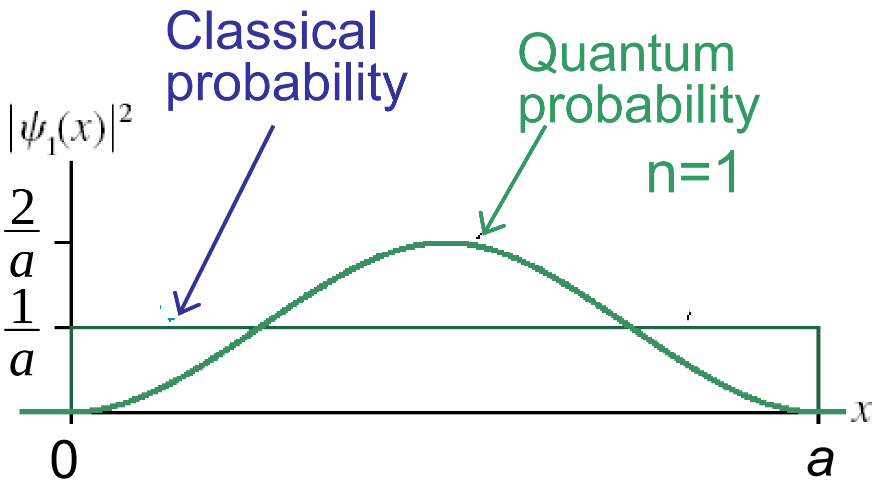
Macky Auditorium • University of Colorado at Boulder
Free and open to the public

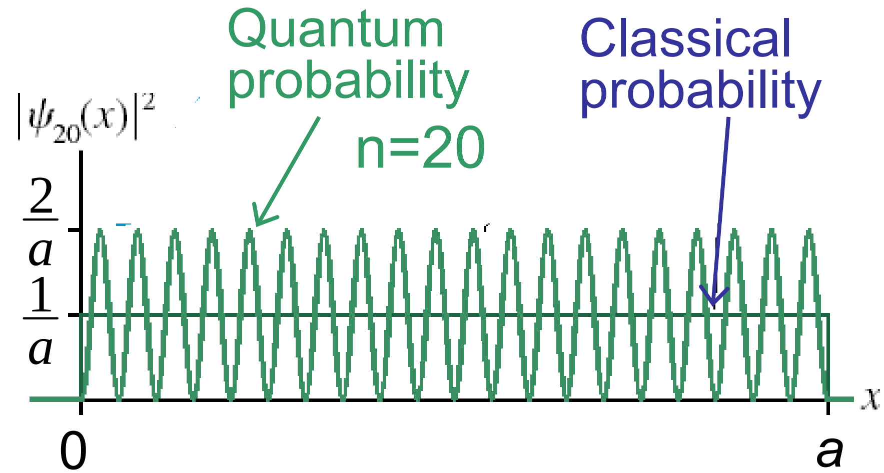
Meet Professor Hirsch at a public reception following the lecture
CU Heritage Center, third floor of Old Main
University of Colorado at Boulder

For more information about Joy Hirsch, visit www.fmri.org
For more information about George Gamow and the Gamow lec
visit www.colorado.edu/physics/Web/Gamow/lecture_2009.html
or call 303-492-1440

Correspondence principle

Proposed by Bohr: Quantum physics results should match classical physics results in the appropriate regions (large quantum number *n*).





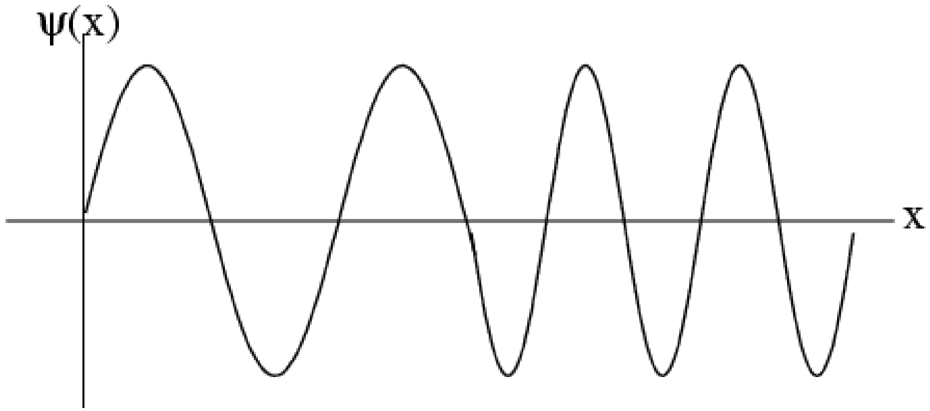
As *n* increases, the quantum probability averages out to flat across the well. This is exactly what is predicted by classical physics.

In HW 4d you found millions of levels between ground state and average thermal energy for a normal piece of wire. This basically means the energy levels form a continuum as in classical physics.

Only really tiny wires had quantum effects at thermal energies

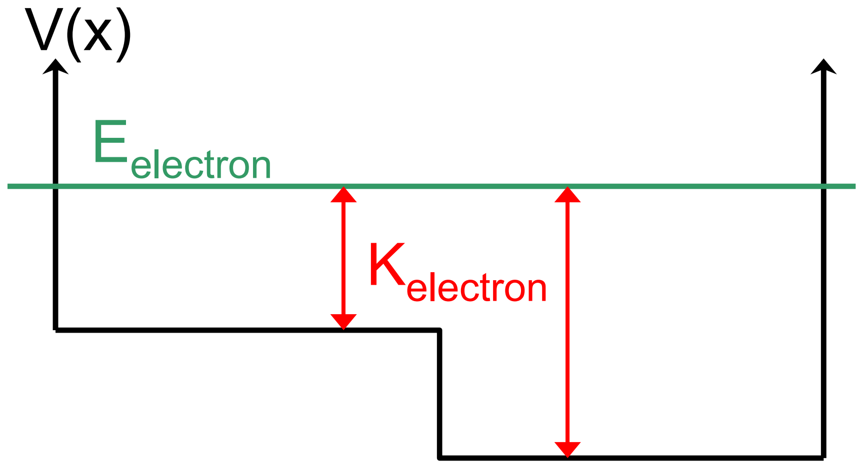
Modifications of square well potential

Wave function from question 6 of homework set 10.



In question 1 you determined that closer spaced waves in *x* means higher wave number *k*.

From deBroglie, $p=\hbar k$. Kinetic energy = $K = p^2/2m = \hbar^2k^2/2m$, so higher *k* means higher *K*.

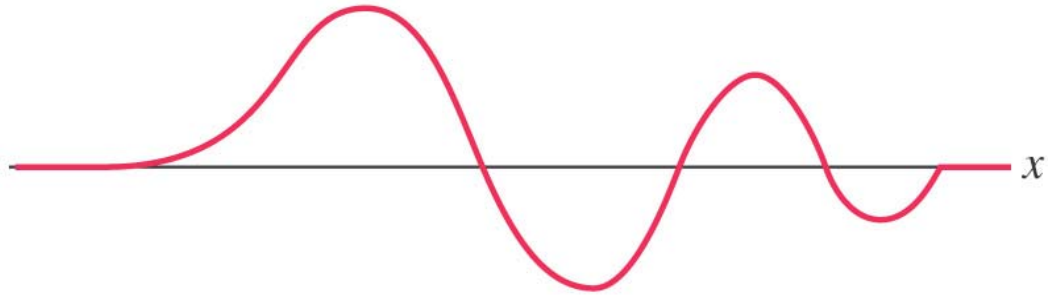


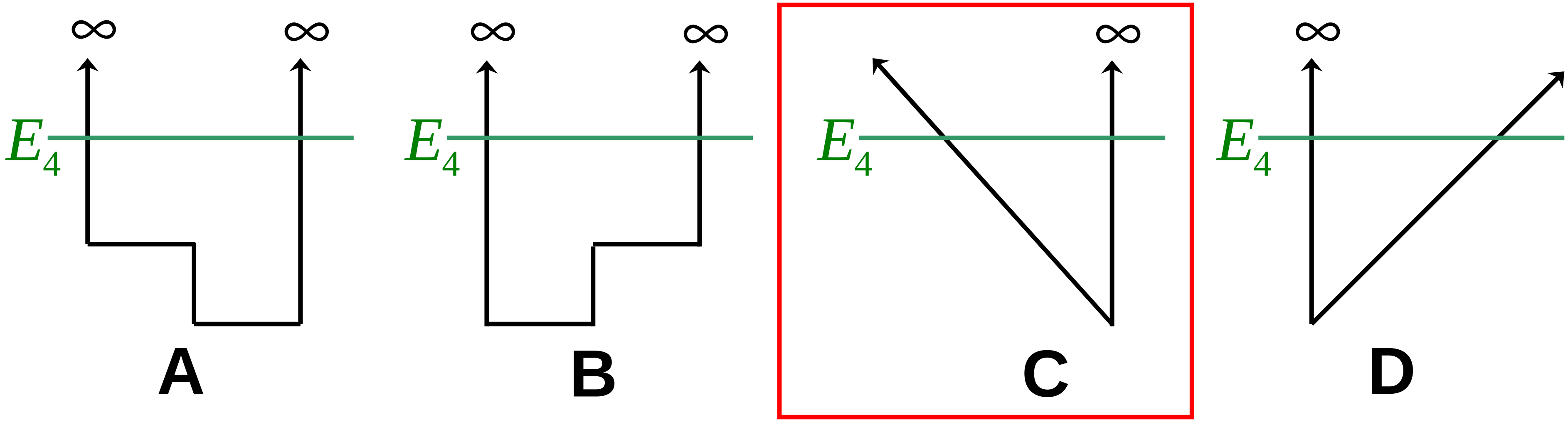
Since there are no outside forces, total energy is conserved.

If kinetic energy goes up, potential energy must go down.

Clicker question 1 — Set frequency to DA

Q. For which of the potentials below is the curve in red a possible n=4 wave function?





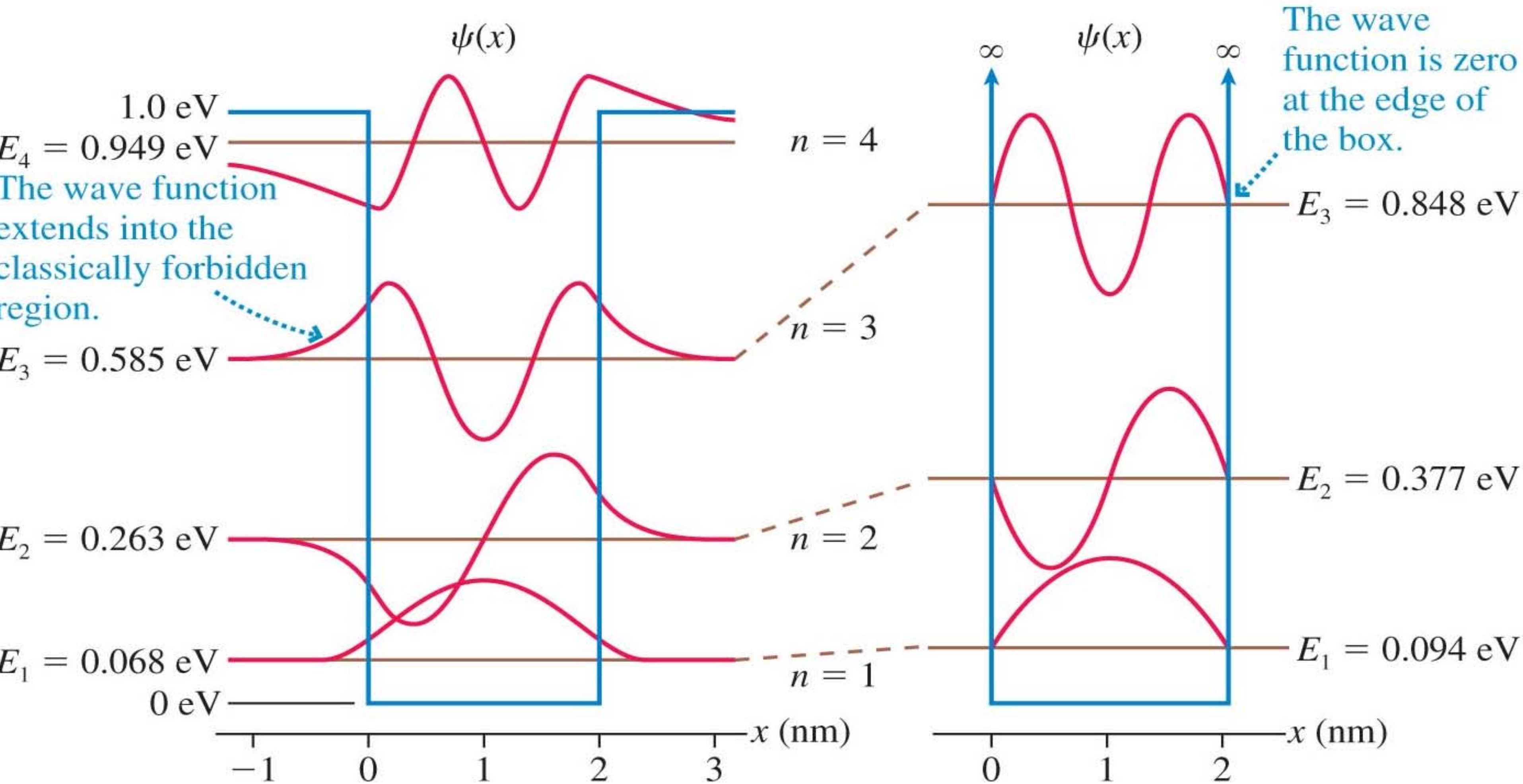
Wave number *k* goes up to the right indicating kinetic energy is going up and potential energy is going down.

Note that left side smoothly goes to 0 indicating a finite well while right side abruptly goes to zero indicating infinite well.

Comparison of infinite and finite potential wells

Electron in finite square well
(a=2 nm and V=1.0 eV)

Infinite potential well
(a = 2 nm and V = ∞)



Note that energy level *n* has *n* antinodes

Clicker question 2

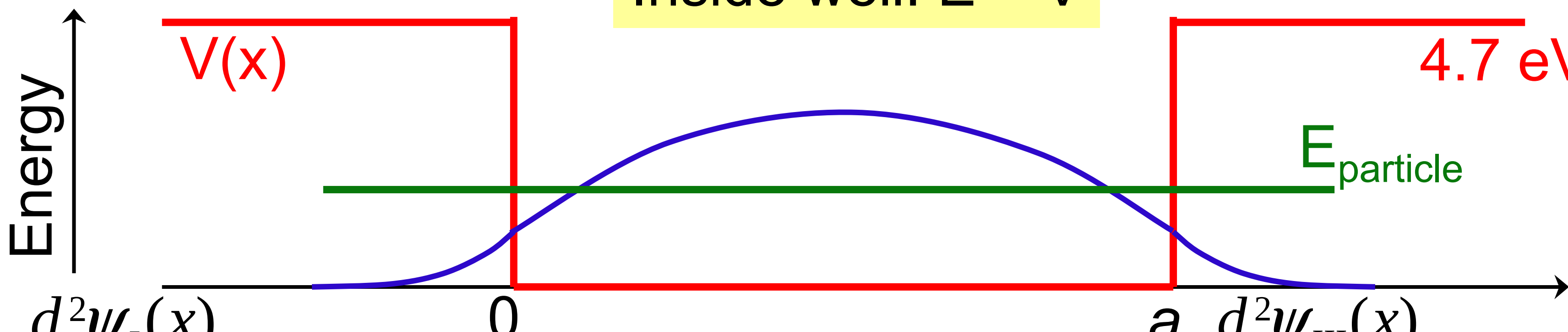
ssically forbidden regions

How far does the particle extend into forbidden region?

Outside well: E < V

Inside well: E > V

Outside well: E < V



$$\frac{d^2\psi_{\mathrm{I}}(x)}{dx^2} = \alpha^2\psi_{\mathrm{I}}(x)$$

$$\frac{d^2\psi_{\mathrm{III}}(x)}{dx^2} = \alpha^2\psi_{\mathrm{III}}(x)$$

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x) = \alpha^2\psi(x)$$

so

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

What are the units of α?

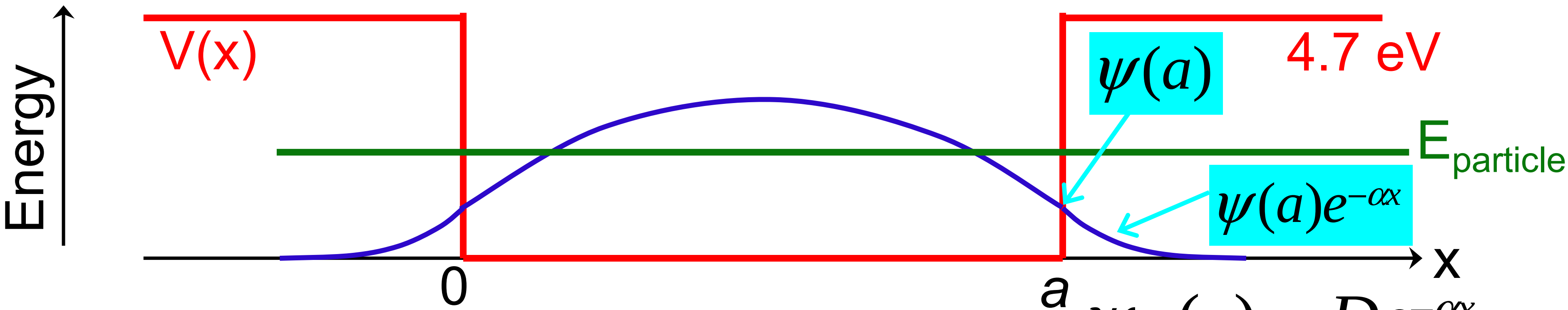
A. J   B. J^{-1} C. m^2 **D. m^{-1}**   E. $J^{-1/2}$

$$\alpha = \frac{\sqrt{\mathrm{eV}/c^2 \cdot \mathrm{eV}}}{\mathrm{eV}\cdot\mathrm{s}} = \frac{\mathrm{eV}}{\mathrm{eV}\cdot\mathrm{s}\cdot\mathrm{m/s}} = \frac{1}{\mathrm{m}}$$

or note $\psi_{\mathrm{III}}(x) = De^{-\alpha x}$ and recall exponent must be dimensionless.

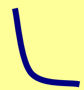
Particles in classically forbidden regions

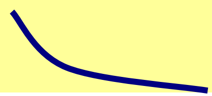
<u>How far does the particle extend into the forbidden region?</u>



$$\psi_{III}(x) = De^{-\alpha x}$$

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Large α means fast drop off 

Small α means slow drop off 

A measure of the *penetration depth* is $1/\alpha = \lambda$

Distance at which ψ(x) is reduced by a factor of 1/*e*.

For an electron with V-E = 4.7 eV this is only 10^{-10} m (size of an atom). Not very far!

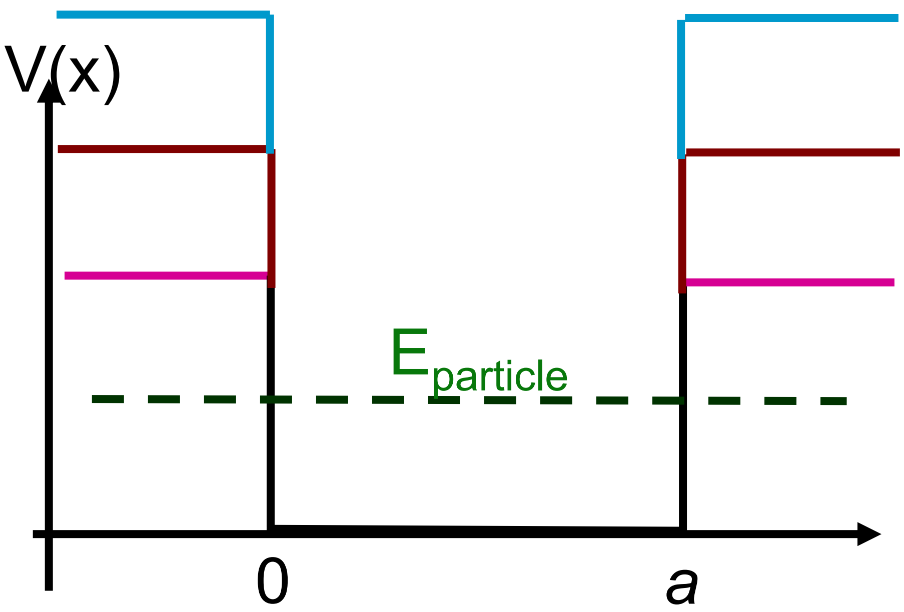
What changes would increase the penetration depth?

Consider changing the potential energy curve or the particle energy. What changes increase or decrease the penetration depth: $\lambda = 1/\alpha$

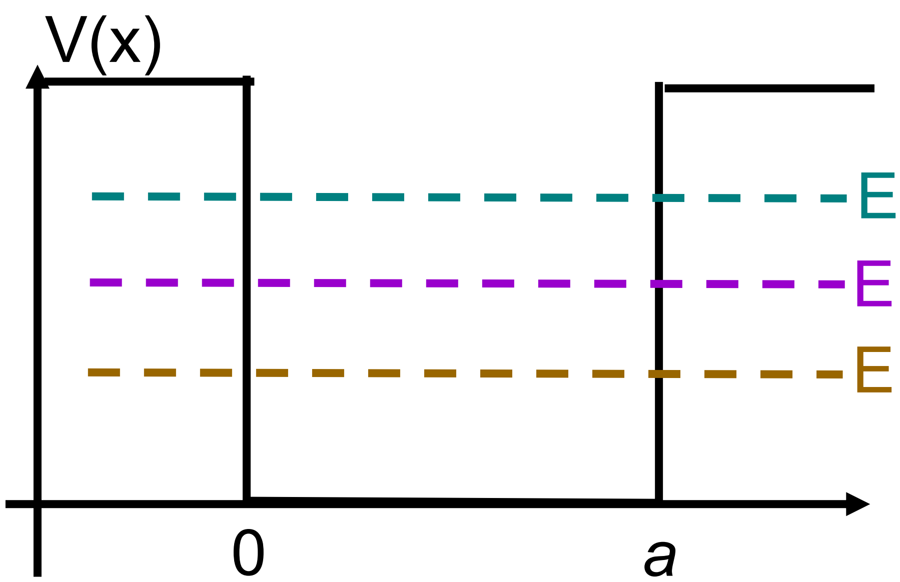
$$\psi_{\text{III}}(x) = De^{-\alpha x}$$

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Changing potential curve.



Changing particle energy



Clicker question 3

Set frequency to DA

Which of the four possible scenarios (A,B,C,D) would give the *shortest* penetration depth $\lambda = 1/\alpha$

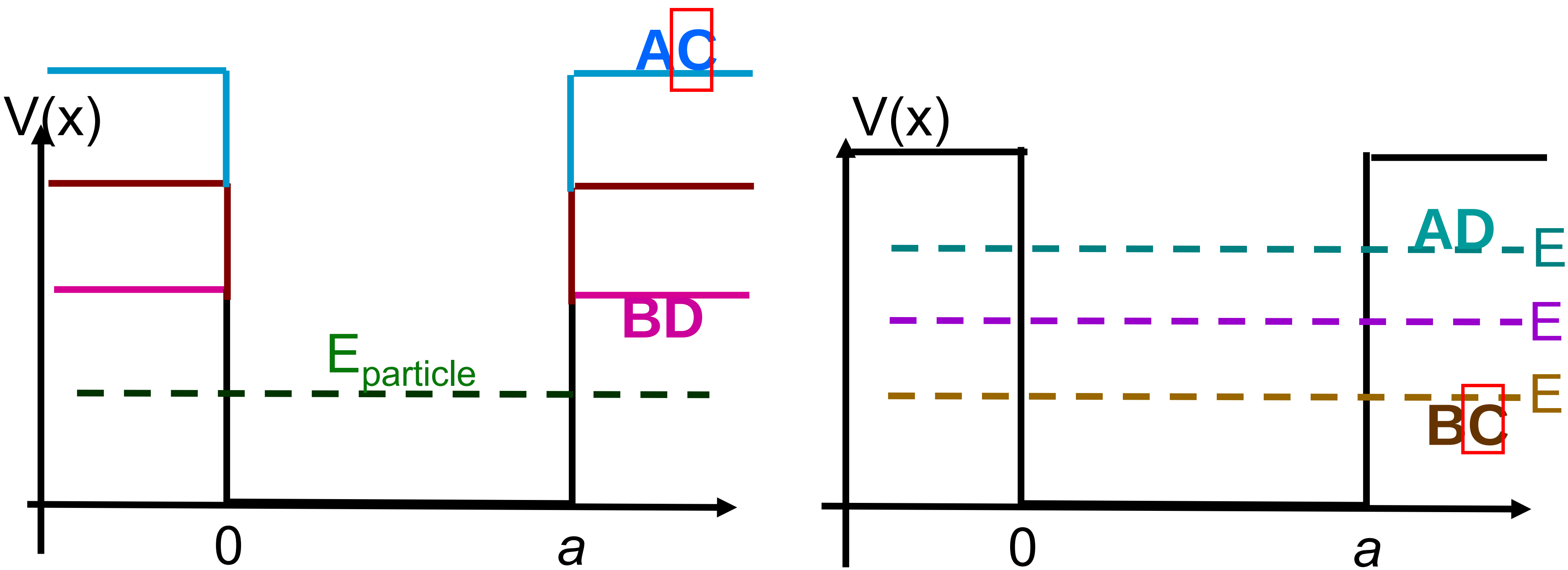
$$\psi_{\text{III}}(x) = De^{-\alpha x}$$

Small λ implies large α.

Large α comes from large V and/or small E

Answer **C** gives largest value for (V-E).

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar}$$



Please answer this question on your own.
No discussion until after.

Q. Which of the following is an example of quantum tunneling?

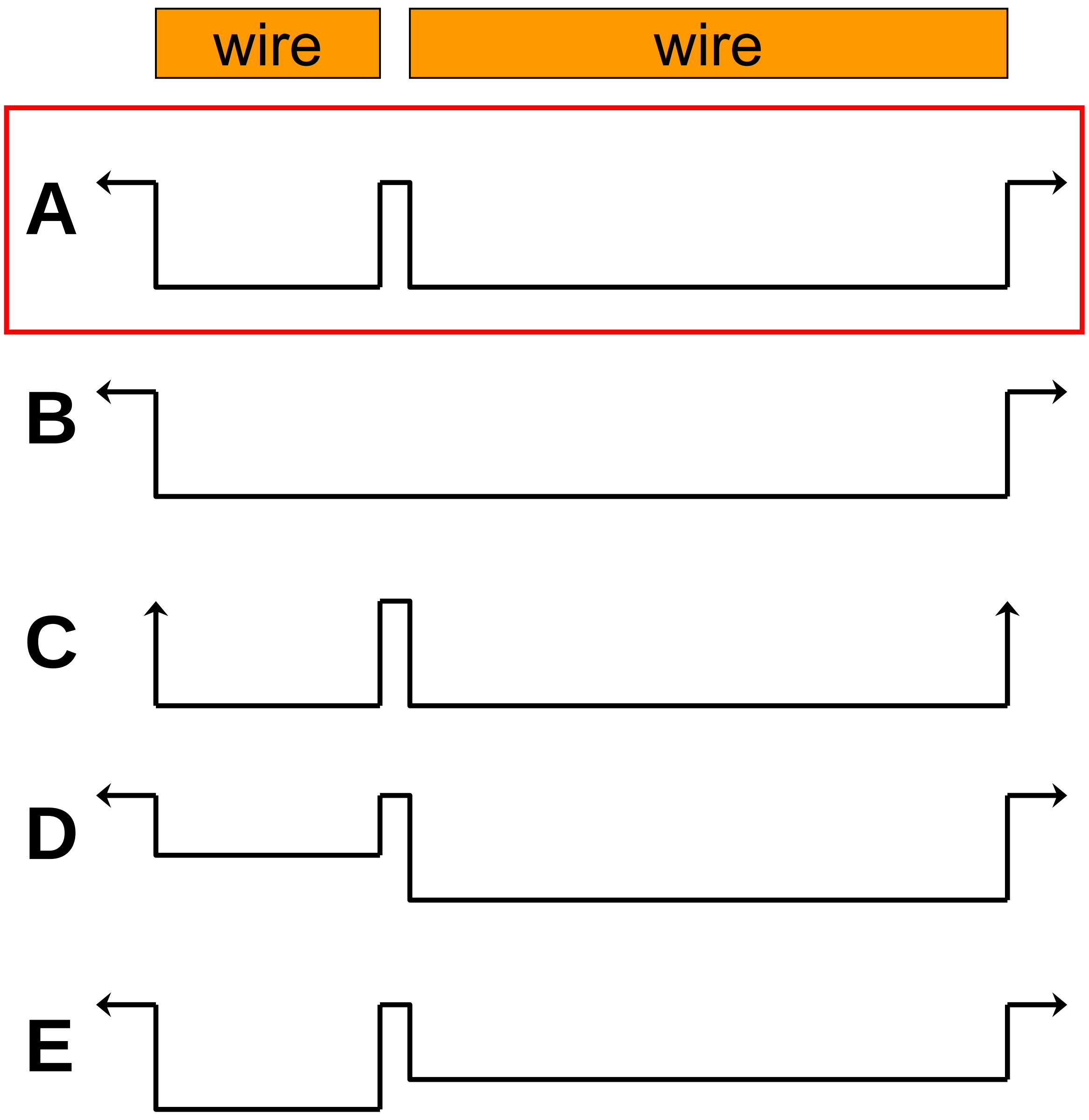
A. Radioactive decay (α decay)
B. Photoelectric effect
C. Scanning tunneling microscope
D. Time dilation
E. More than one of the above

Clicker question 4

Set frequency to DA

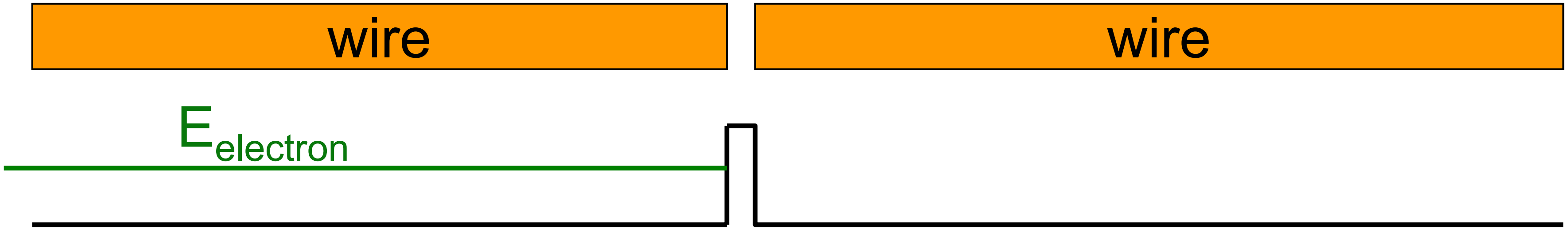
Two copper wires with different lengths are brought close together as shown. What does the potential energy function V(x) look like?

Two potential wells of the same depth (4.7 eV) given by the work function for copper.



Quantum tunneling through potential barrier

Consider the slightly simpler case of two very long wires separated by a small gap:



This is an example of a ***potential barrier.***

Quantum tunneling occurs when a particle which does not have enough energy to go over the potential barrier somehow gets to the other side of the barrier.

This is due to the particle being able to penetrate into the *classically forbidden region*.

If it can penetrate far enough (the barrier is thin enough) it can come out the other side.

You will investigate this effect on Friday.