More on waves

Announcements:

- Learning Assistant program informational session will be held 3/11/09 at 6pm in UMC 235.

Historical quote:

Once at the end of a colloquium I heard Debye saying something like: “Schrödinger, you are not working right now on very important problems…why don’t you tell us some time about that thesis of deBroglie, which seems to have attracted some attention?” So, in one of the next colloquia, Schrödinger gave a beautifully clear account of how deBroglie associated a wave with a particle, and how he could obtain the quantization rules…by demanding that an integer number of waves should be fitted along a stationary orbit. When he had finished, Debye casually remarked that he thought this way of talking was rather childish…To deal properly with waves, one had to have a wave equation.

- Felix Bloch

Clarification of de Broglie relations

I stated that the de Broglie relation is $p = \frac{h}{\lambda} = \hbar k$

Originally came from an analysis of massless photons but also works for massive particles like electrons, neutrons, and atoms.

In fact, there is another relation which is derived from the photon results: $E = hf = \hbar\omega$

Note the momentum relation deals with the *space* part of a wave (wavelength and wave number) while the energy relation deals with the *time* part of the wave (frequency).

For light, the space and time quantities are related by $c = \lambda f$

For massive particles $v_{\text{wave}} = \lambda f$ but $v_{\text{wave}} \neq v_{\text{particle}}$ so it is not very useful in practice.

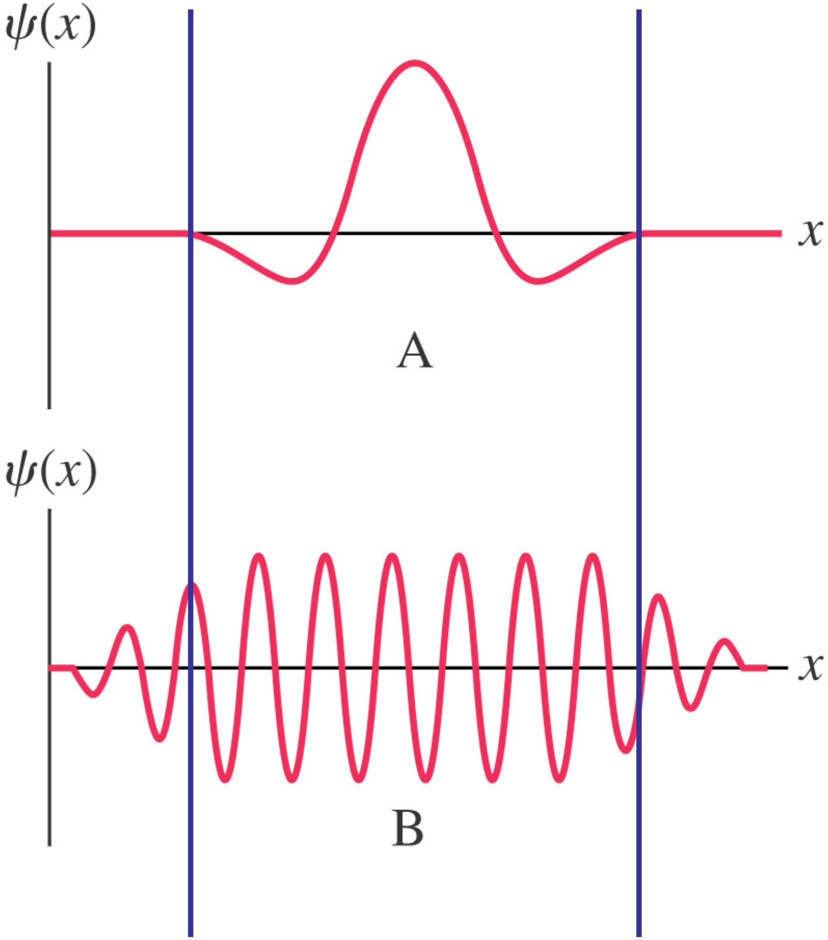
My advice: avoid using velocity. Stick with *E, p, k, T, f,* λ, ω.

Clicker question 1

Set frequency to DA

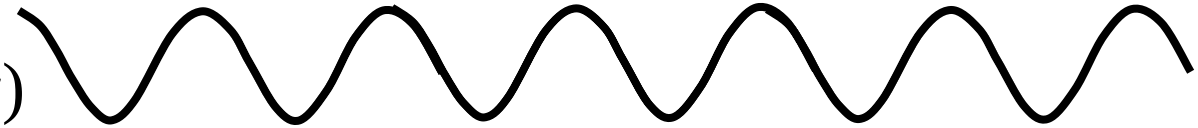
Which of the two particles, A or B, can you locate more precisely?

A. A
B. B
C. Same precision for A and B



Plane Waves vs. Wave Packets

Plane wave:



$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

This wave represents a single *k* and ω. Therefore energy, momentum, and wavelength are well defined.

The amplitude is the same over all space and time so *position* and *time* are <u>undefined.</u>

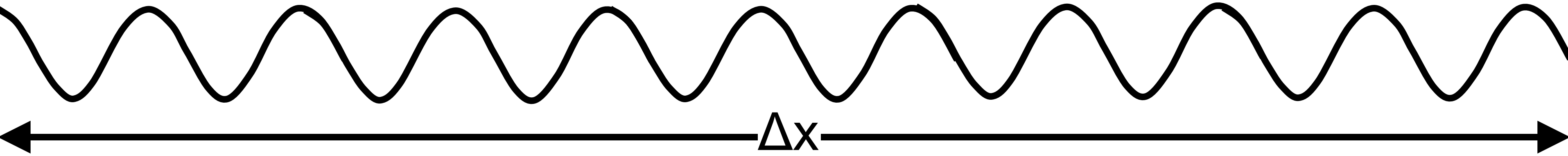
Wave packet:

$$\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)}$$

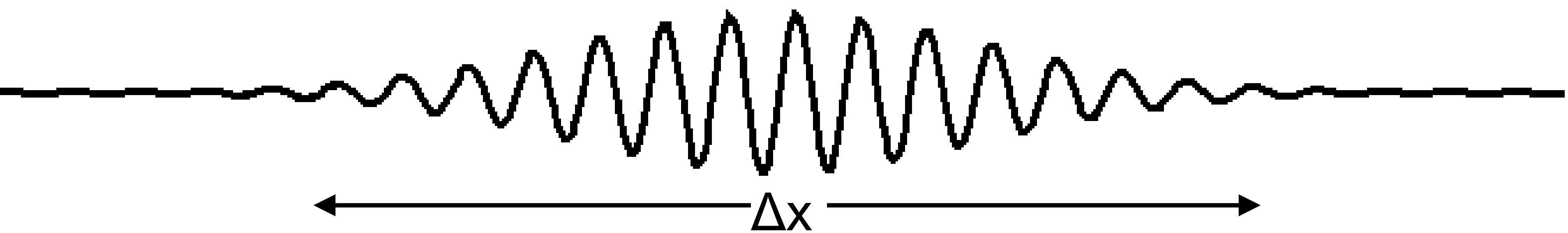
This wave is composed of many different *k* and ω waves. Thus, it is composed of many different energies, momenta, and wavelengths and so these quantities are not well defined.

The amplitude is non-zero in a small region of space and time so the position and time is constrained to be in that region.

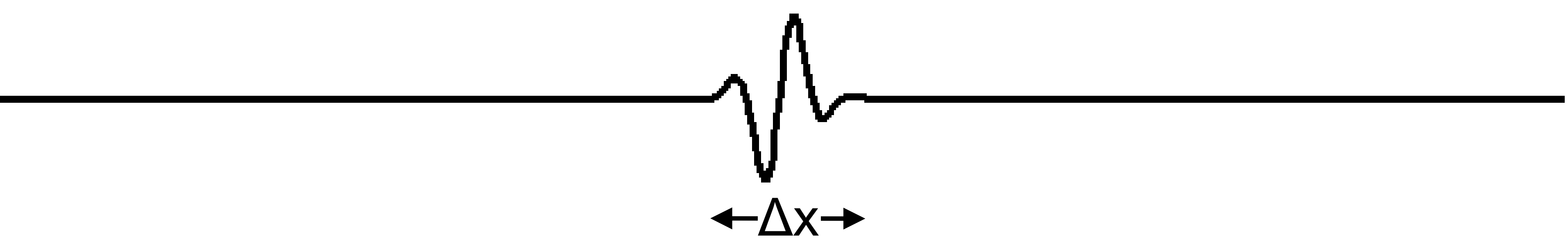
Heisenberg Uncertainty Principle: $\Delta x \, \Delta p \geq \hbar / 2$



small Δp – only one wavelength

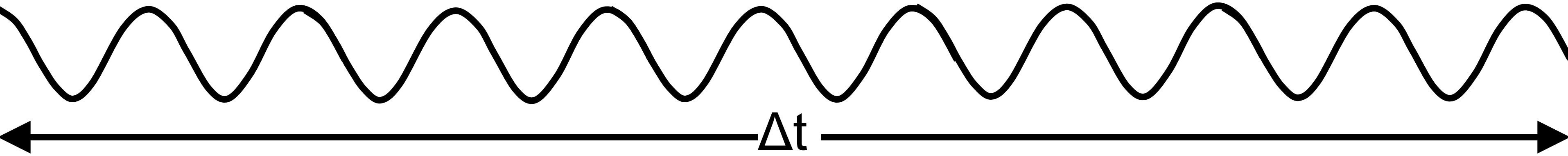


medium Δp – wave packet made of several waves

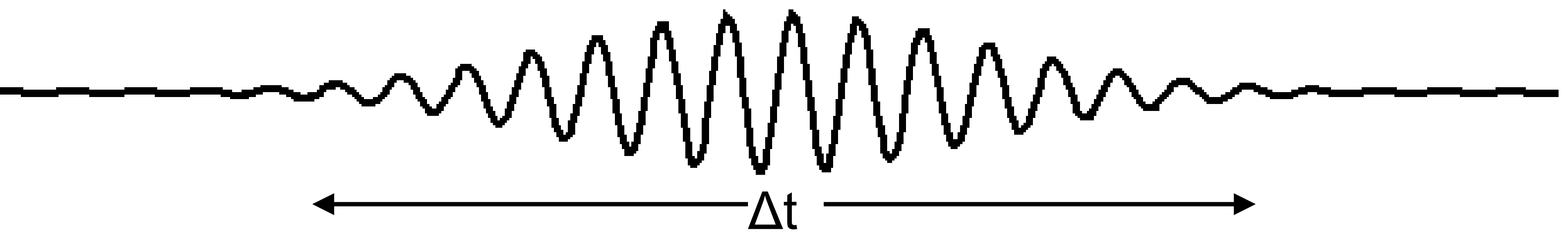


large Δp – wave packet made of lots of waves

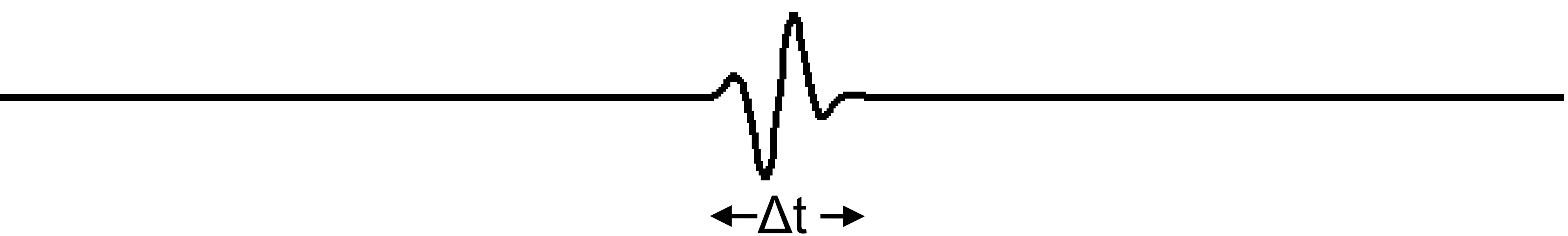
Heisenberg Uncertainty Principle: $\Delta t \, \Delta E \geq \hbar / 2$



small ΔE – only one period



medium ΔE – wave packet made of several waves



large ΔE – wave packet made of lots of waves

Heisenberg Uncertainty Principle

There are two Heisenberg uncertainty relations:

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

$$\Delta t \, \Delta E \geq \frac{\hbar}{2}$$

What does this uncertainty principle *mean*?

The wave nature of things prevents a precise determination of both momentum and position or of both energy and time.

This is a fundamental limitation which has nothing to do with the actual equipment used to measure things.

Another way of seeing why this makes sense is Heisenberg's microscope.

Microscopes are limited to resolutions ~ wavelength of light

Smaller wavelengths allow a better measurement of *x* but the photons have larger momentum giving larger kicks to the particle, making the momentum more uncertain.

Where we go from here

We are going to spend some time thinking about classical waves

Waves on a string (like on a guitar)

Electromagnetic waves

Classical waves obey the wave equation: $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Once we understand these waves we will go back to matter waves which obey a different wave equation called the time dependent Schrödinger equation: $$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

Finally, we will derive the time independent Schrödinger equation: $$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

Reading quiz 1 — Set frequency to DA

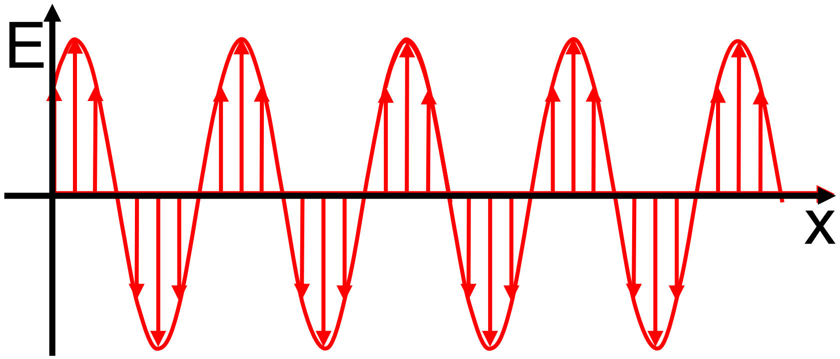
Which of the following is a true statement about *standing waves*?

A. Standing waves have points called antinodes which are motionless
B. Standing waves have points called nodes which are motionless
C. Standing waves can be constructed from two traveling waves moving in opposite directions
D. A and C are both true
E. B and C are both true

Antinodes move the most while nodes do not move at all.

Example Wave Equations You Have Seen

Electromagnetic waves:



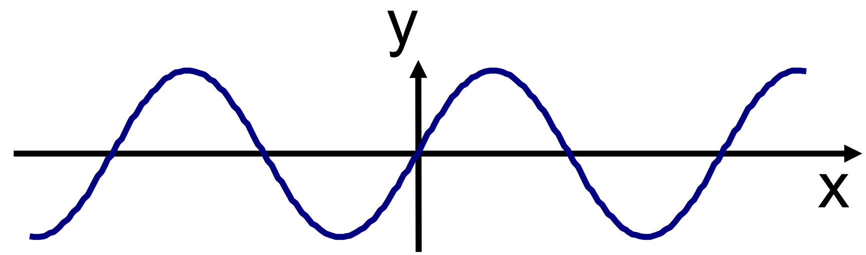
$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

c = speed of light

Solutions: E(x,t)

Magnitude is non-spatial:
= Strength of Electric field

Vibrations on a string:



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

v=speed of wave

Solutions: y(x,t)

Magnitude is spatial:
= Vertical displacement of String

Solving the standard wave equation

Physics laws are often differential equations.

F=ma can be written as $F = m\frac{\partial^2 x}{\partial t^2}$

If the force is a restoring force so $F = -kx$ then $m\frac{\partial^2 x}{\partial t^2} = -kx$

The standard *wave equation* is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Generic prescription for solving differential equations in physics:

1. Guess the function form(s) of the solution
2. Plug into differential equation to check for correctness, find any constraints on constants
3. Need as many independent functions as there are derivatives.
4. Apply all boundary conditions (more constraints on constants)

Clicker question 2

Set frequency to DA

Step 1: Guessing the functional form of the solution

The standard *wave equation* is $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Which of the following function forms is a possible solution to this differential equation?

A. $y = Ax^2t^2$

B. $y = A\cos(Bx)$

C. $y = A\cos(Bt)$

D. $y = A\cos(Bx)\sin(Ct)$

E. More than one of the above

The A. form leads to $2At^2 = \frac{1}{v^2}2Ax^2$ or $t^2 = \frac{x^2}{v^2}$ Different functions on the left side and right side. This is incorrect.

The B. form leads to $-AB^2\cos(Bx) = 0$ which doesn't work.

The C. form leads to $0 = -AB^2\cos(Bt)$ which doesn't work.

Step 2: Check solution and find constraints

Claim that $y = A\cos(Bx)\sin(Ct)$ is a solution to $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

Time to check the solution and see what constraints we have

LHS: $\dfrac{\partial^2 y}{\partial x^2} = -AB^2\cos(Bx)\sin(Ct)$

RHS: $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = -\dfrac{AC^2}{v^2}\cos(Bx)\sin(Ct)$

Setting LHS = RHS: $-AB^2\cos(Bx)\sin(Ct) = -\dfrac{AC^2}{v^2}\cos(Bx)\sin(Ct)$

This works as long as $B^2 = \dfrac{C^2}{v^2}$

We normally write this as $y = A\cos(kx)\sin(\omega t)$

so this constraint just means $k^2 = \dfrac{\omega^2}{v^2}$ or $v = \dfrac{\omega}{k} = \lambda f$