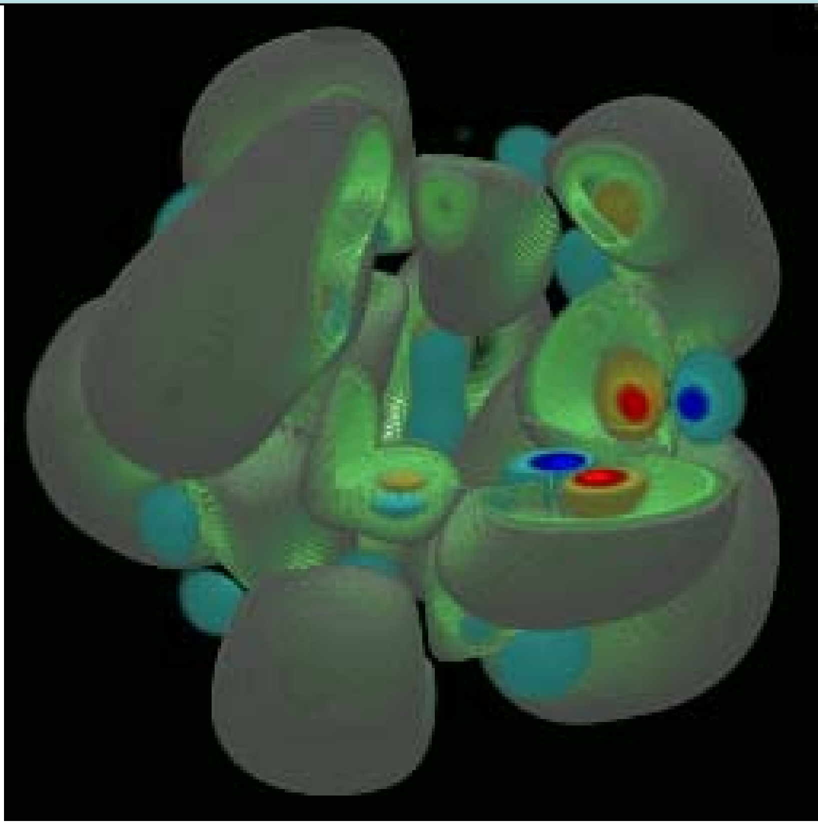
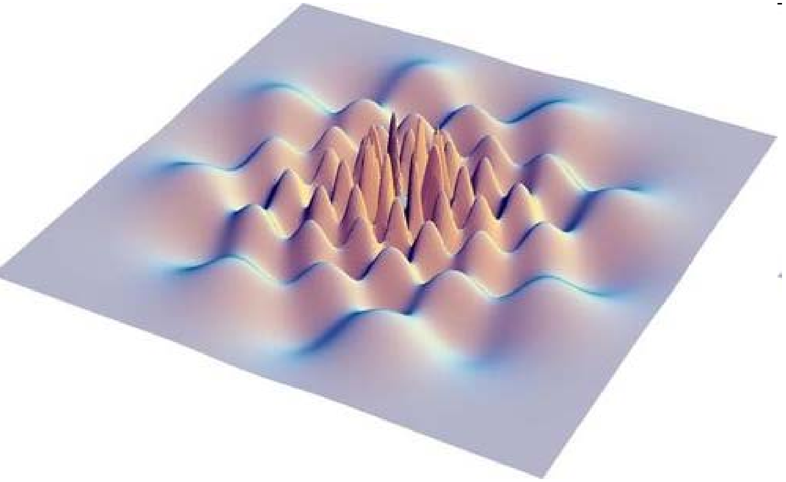
More on waves

Announcements:

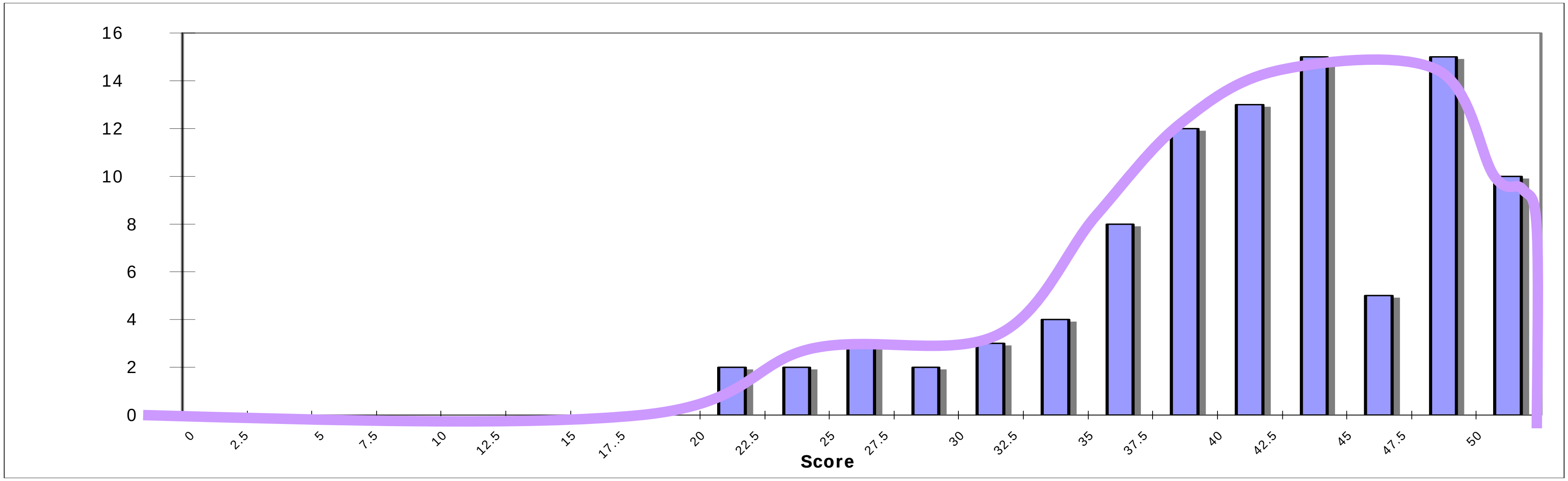
- Learning Assistant program informational session will be held 3/11/09 at 6pm in UMC 235.
- The 2nd midterm will be April 7 in MUEN E0046 at 7:30pm.



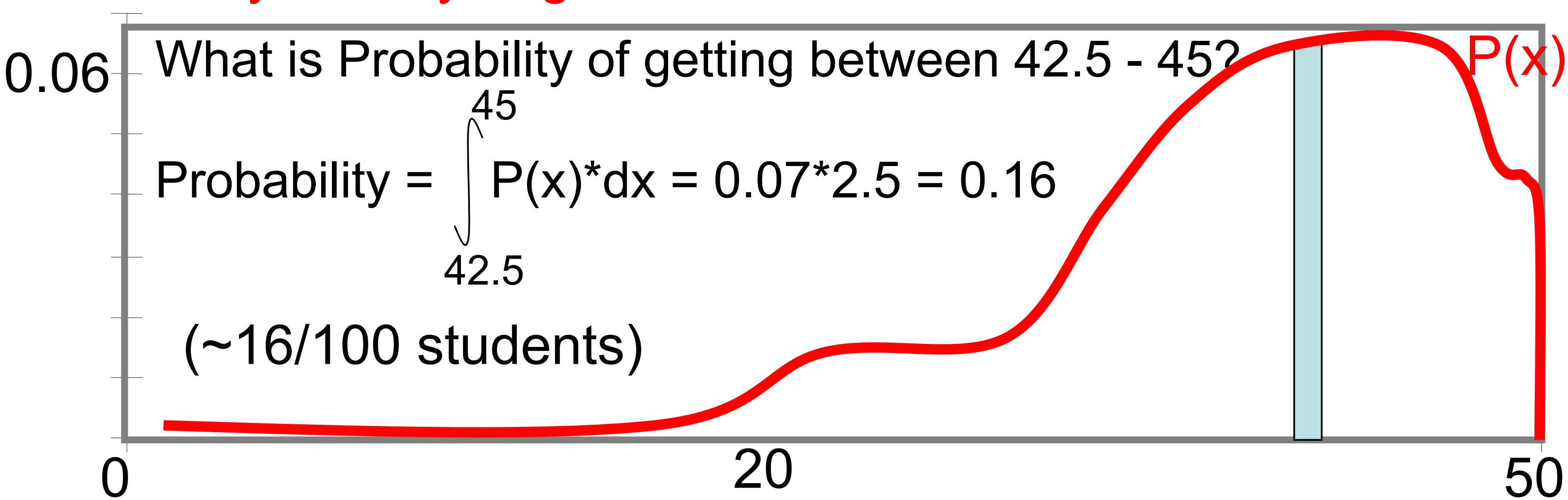
Electron wave function of C_{60}



Working with probabilities



Probability density of grades:



What is Probability of getting between 42.5 - 45?

$$\text{Probability} = \int_{42.5}^{45} P(x)*dx = 0.07*2.5 = 0.16$$

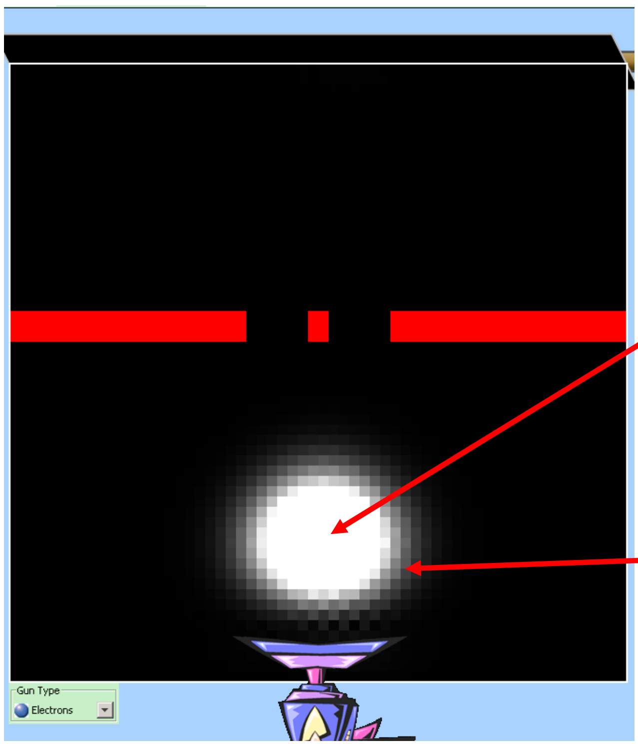
(~16/100 students)

Summary of wave function

The wave function ψ(x) is just the ***spatial*** part of the wave function. The general wave function is also a function of time and is Ψ(x,t).

$\Psi(x,t)$ is the full wave function

$|\Psi(x,t)|^2$ gives the space and time dependent probability of detecting the particle



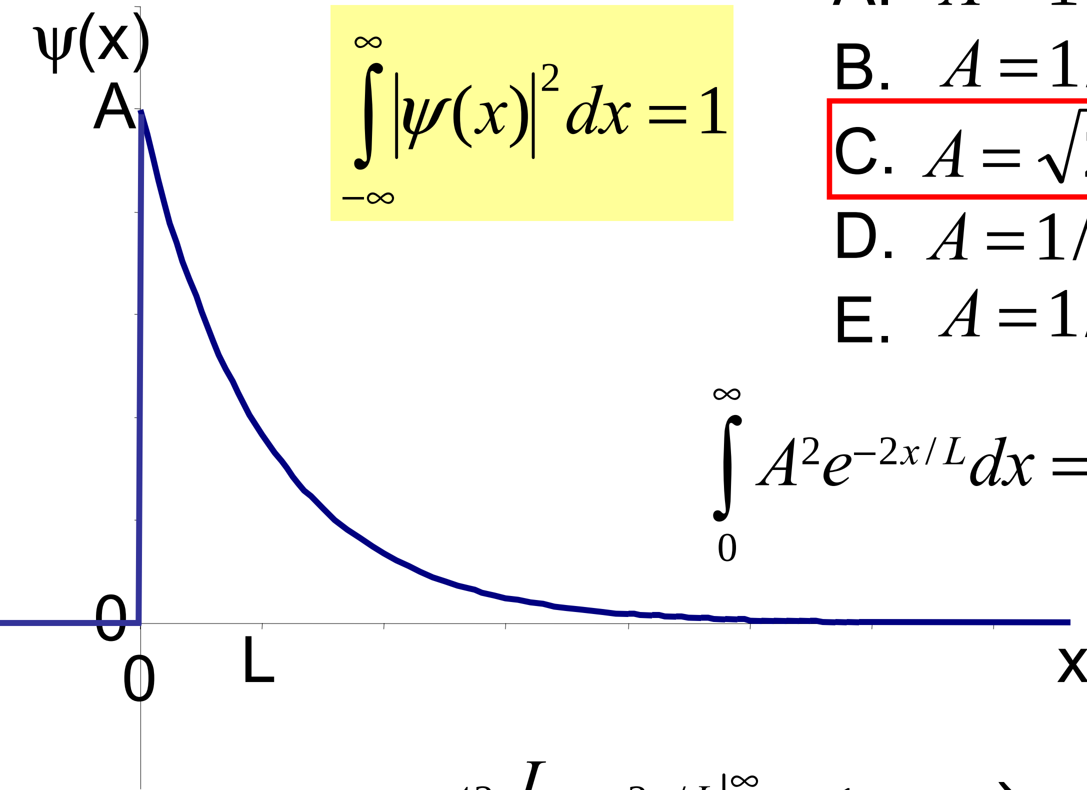
<u>Electron double slit experiment:</u>

Intensity is magnitude of wave function

Large Magnitude (|Ψ|)=
probability of detecting electron here is high

Small Magnitude (|Ψ|)=
probability of detecting electron here is low

$$\psi(x) = 0 \text{ for } x<0$$
$$= Ae^{-x/L} \text{ for } x \geq 0$$



$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

What must be the value of *A* to have a properly normalized wave function?

A. $A = 1$

B. $A = 1/\sqrt{L}$

C. $A = \sqrt{2/L}$

D. $A = 1/\sqrt{2L}$

E. $A = 1/L$

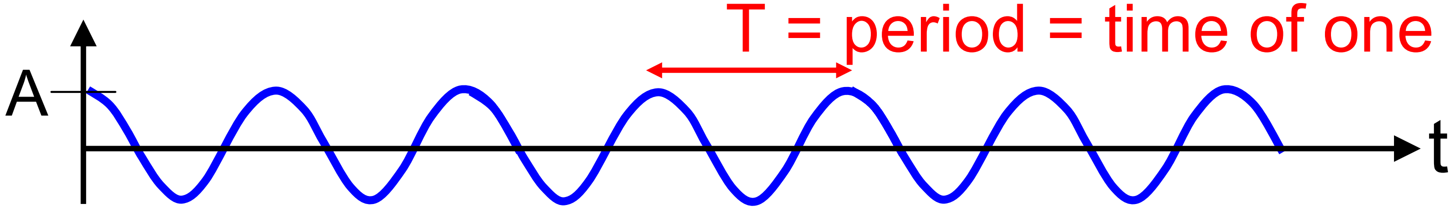
$$\int_0^{\infty} A^2 e^{-2x/L} dx = 1 \quad \Rightarrow \quad A^2 \int_0^{\infty} e^{-2x/L} dx = 1$$

$$\Rightarrow -A^2 \frac{L}{2} e^{-2x/L} \Big|_0^{\infty} = 1 \quad \Rightarrow \quad A^2 \frac{L}{2} = 1 \quad \Rightarrow \quad A = \sqrt{2/L}$$

Review of waves

Waves in time: $y(t) = A\cos(\omega t) = A\cos(2\pi f t) = A\cos(2\pi t/T)$

T = period = time of one cycle

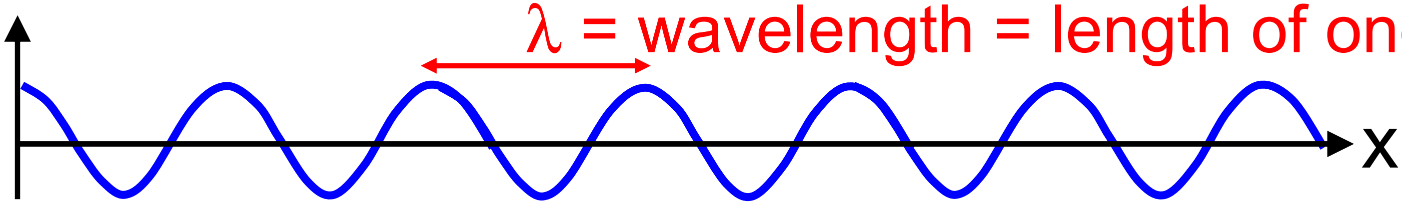


f = 1/T = $\omega/2\pi$ = frequency = number of cycles per second

$\omega = 2\pi f$ = angular frequency = number of radians per second

Waves in space: $y(x) = A\cos(2\pi x/\lambda) = A\cos(kx)$

λ = wavelength = length of one cycle



k = $2\pi/\lambda$ = wave number = number of radians per meter

More on sinusoidal waves

A traveling wave changes in both space and time:

$$y(x,t) = A\cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right) = A\cos(kx - \omega t)$$

Using *k* and ω gives us the simplest equation so we will generally use them instead of λ, *T*, and *f*.

Note we can write the energy of a photon as $E = hf = \hbar\omega$

and the momentum of any particle as $p = \frac{h}{\lambda} = \hbar k$

and for more than 1 dimension: $\vec{p} = \hbar\vec{k}$

For a massive particles can also write kinetic energy as $K = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

Plane waves

Plane waves extend infinitely in space and time.

Can be made up of sine and cosine waves or complex exponentials

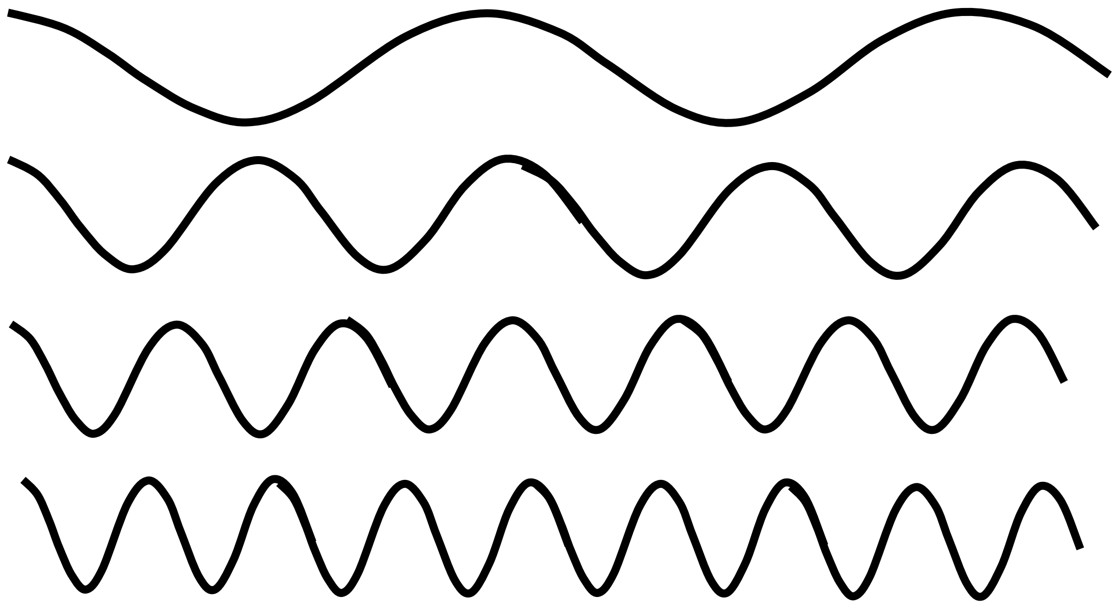
Same thing by Euler's theorem: $e^{i\theta} = \cos\theta + i\sin\theta$

$$\Psi_1(x,t) = e^{i(k_1 x - \omega_1 t)}$$

$$\Psi_2(x,t) = e^{i(k_2 x - \omega_2 t)}$$

$$\Psi_3(x,t) = e^{i(k_3 x - \omega_3 t)}$$

$$\Psi_4(x,t) = e^{i(k_4 x - \omega_4 t)}$$



Different *k*'s correspond to different energies since $K = \frac{\hbar^2 k^2}{2m}$

Superposition

If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are both solutions to the wave equation then the sum $\Psi_1(x,t) + \Psi_2(x,t)$ is also a solution.

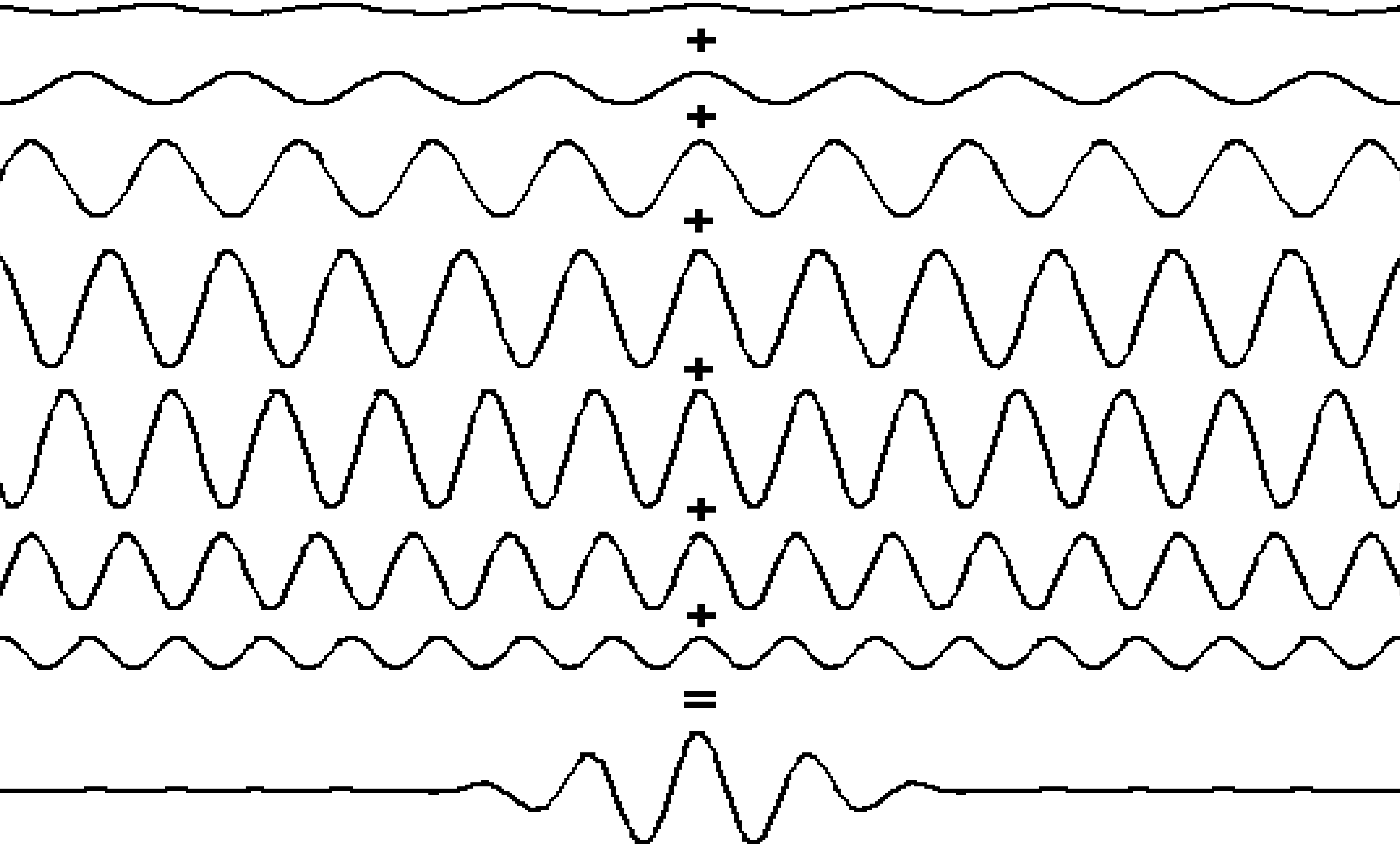
Actually, any linear combination $A\Psi_1(x,t) + B\Psi_2(x,t)$ is a solution.

This is the superposition principle.

This seems pretty straightforward but is actually an important result

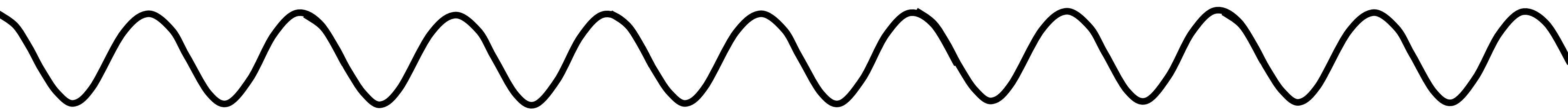
We are going to make a *wave packet* out of a superposition of plane waves

Superposition

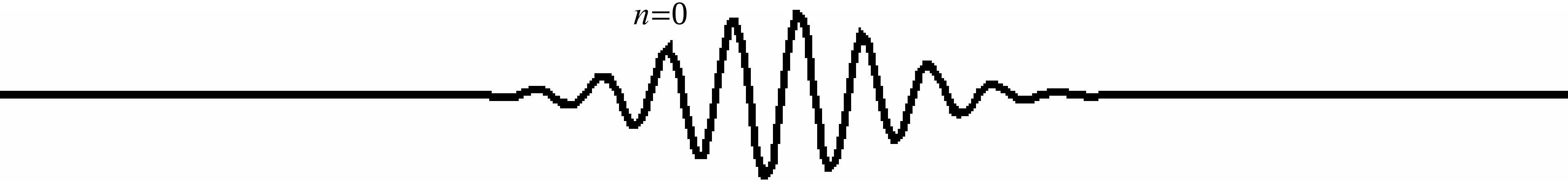


Plane Waves vs. Wave Packets

Plane wave: $\Psi(x,t) = Ae^{i(kx-\omega t)}$



Wave packet: $\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)}$

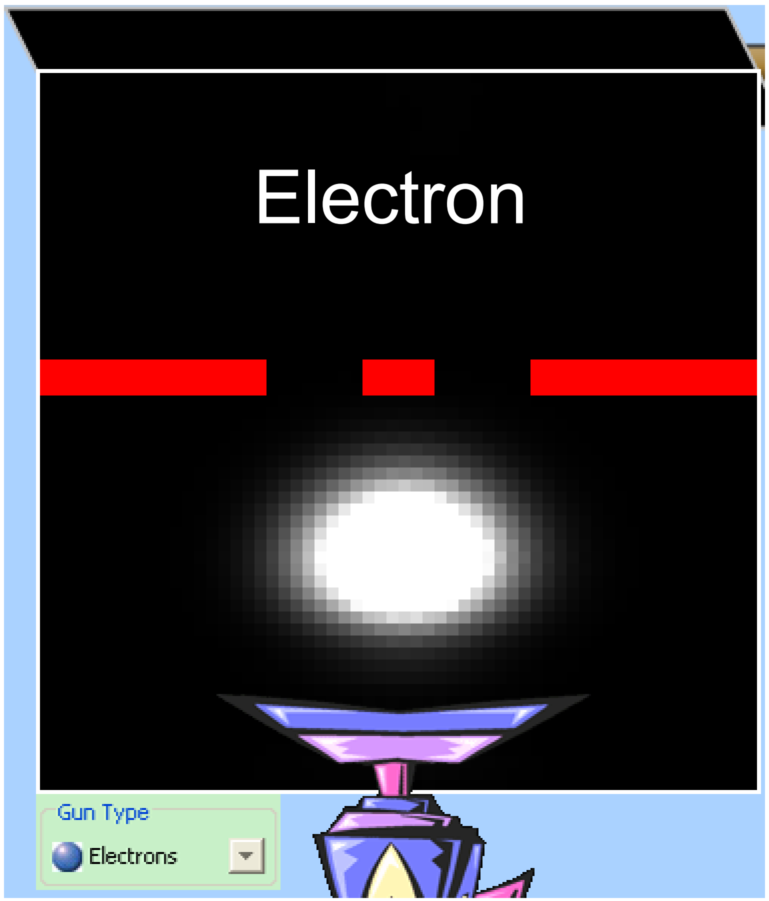


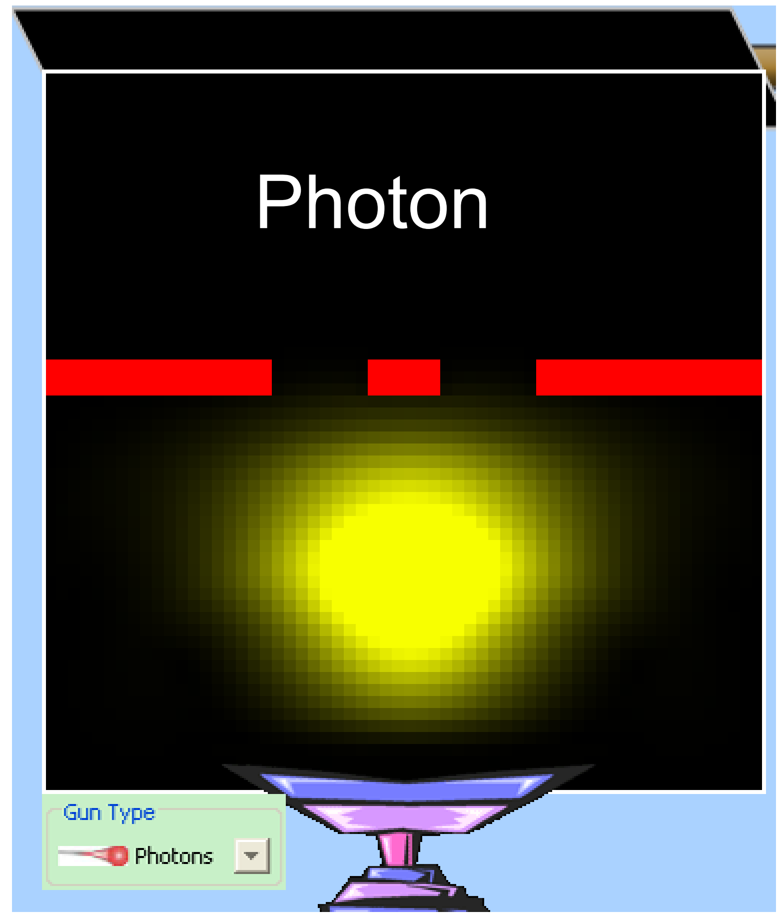
Matter waves are more like wave packets than plane waves.

Mathematically, we obtain wave packets by adding up plane waves.

Method of adding up sine waves to obtain an arbitrary function (like a wave packet) is called Fourier Analysis.

- The “blob” in Quantum Wave Interference is a 2D wave packet.
- In this simulation, intensity is represented by brightness.

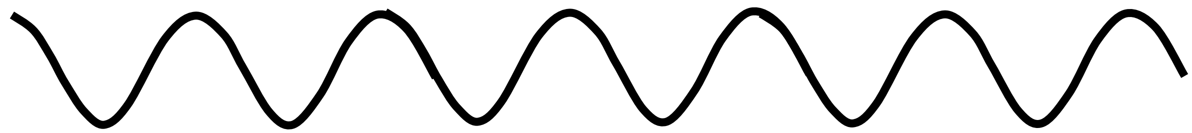




Set frequency to DA

Plane wave:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$



Wave packet:

$$\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)}$$

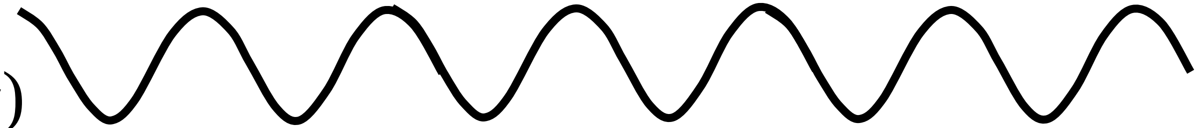
For which type of wave is the momentum and position most well defined?

A. *p* is well defined for plane wave, *x* is well defined for wave packet
B. *p* is well defined for wave packet, *x* is well defined for plane wave
C. *p* is well defined for one but *x* is equally well defined for both
D. *p* is equally well defined for both but *x* is well defined for one
E. Both *p* and *x* are well defined for both

Plane Waves vs. Wave Packets

Plane wave:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$



This wave represents a single *k* and ω. Therefore energy, momentum, and wavelength are well defined.

Amplitude is the same over all space so position is ***undefined***.

Wave packet:

$$\Psi(x,t) = \sum_{n=0}^{\infty} A_n e^{i(k_n x - \omega_n t)}$$

This wave is composed of many different *k* and ω waves. Thus, it is composed of many different energies, momenta, and wavelengths and so these quantities are not well defined.

The amplitude is non-zero in a small region of space so the position is constrained to be in that region. Well defined position.

Heisenberg Uncertainty Principle

One version of the Heisenberg uncertainty principle is written as $\Delta x \, \Delta p \geq \hbar/2$

When we write Δx or Δp we are really referring to the uncertainty on the measurement. Sometimes this is described as the "spread" of values.

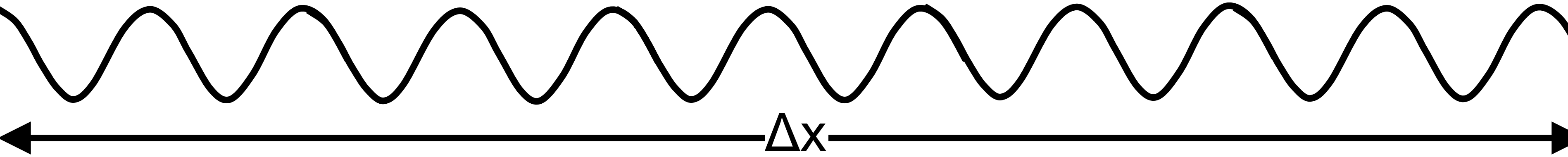
What does this uncertainty principle *mean*?

The position and momentum cannot both be determined precisely. The more precisely one is determined, the less precisely the other is determined.

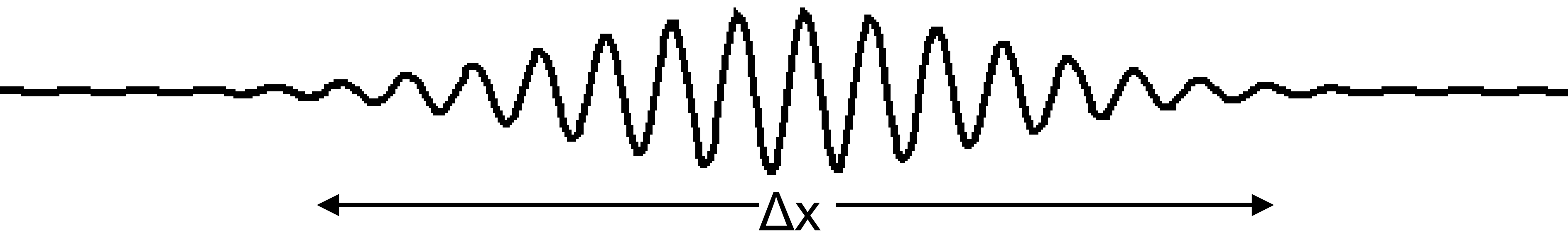
This is a fundamental limitation which has nothing to do with the actual equipment used to measure x or p.

This is a pretty weird concept in the particle view but makes a lot more sense in the wave view.

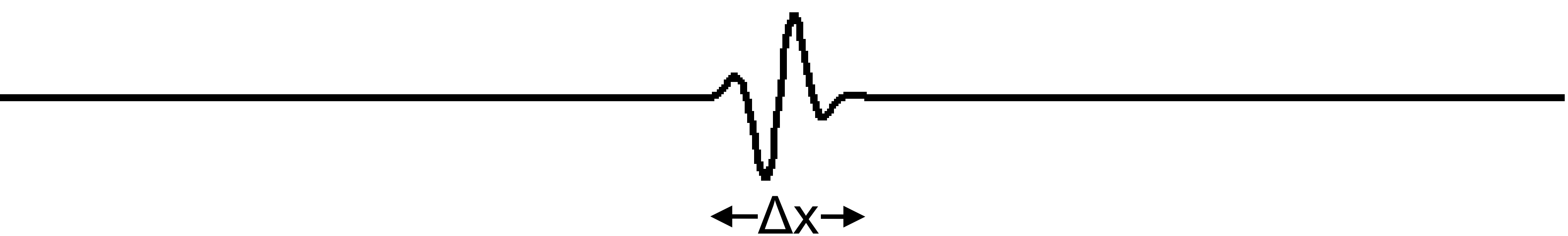
Heisenberg Uncertainty Principle



small Δp – only one wavelength



medium Δp – wave packet made of several waves



large Δp – wave packet made of lots of waves