

# The wave function

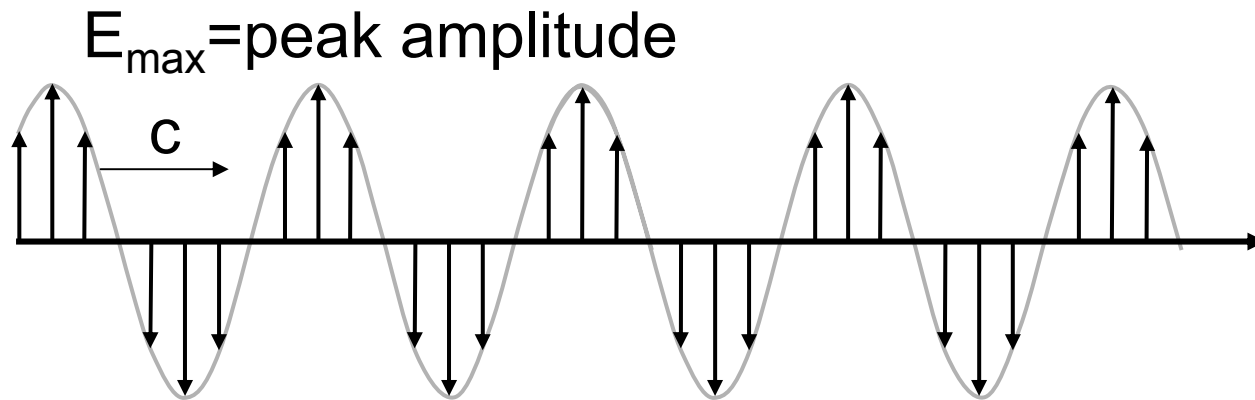
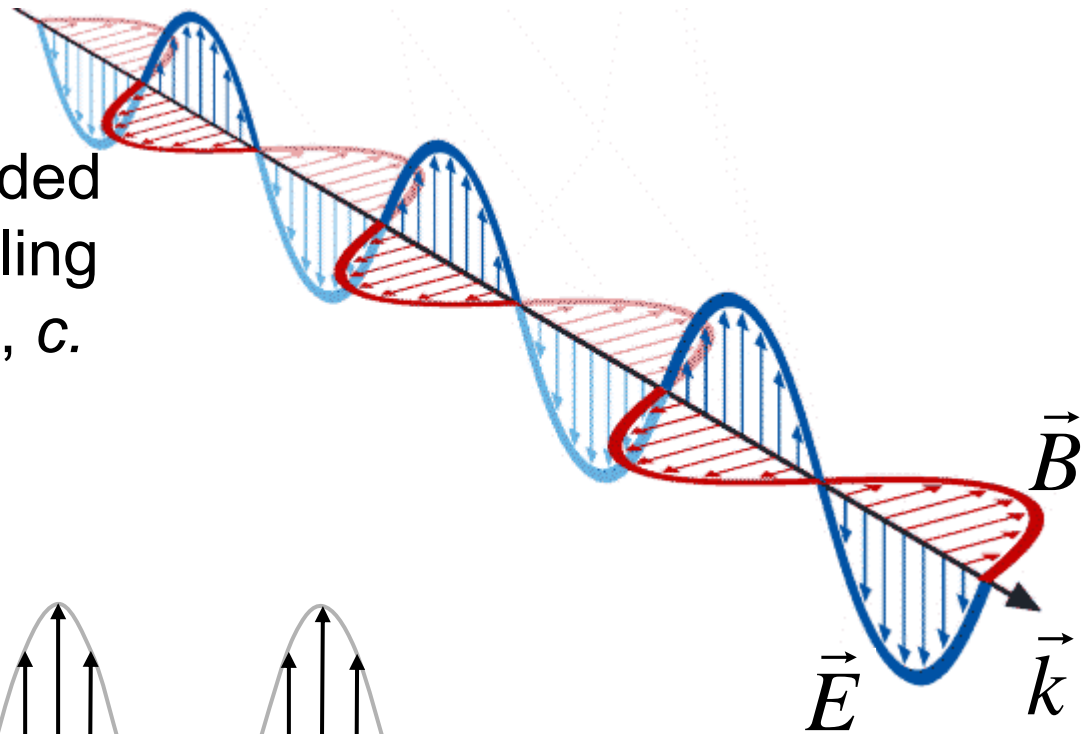
## Announcements:

- Learning Assistant program informational session will be held a week from today 3/11/09 at 6pm in UMC 235.
- Some bullet points about the LA program
  - Interact with students in innovative learning environments
  - We recruit good physics students (A or B students) with good communication skills and are interested in teaching
  - It is a competitive program and I think it pays \$10/hour.
  - Find out all about it at the informational meeting
  - You can pick one up a reminder card now or after class
- The 2<sup>nd</sup> midterm will be April 7 in MUEN E0046 at 7:30pm.

Today we will define the wave function and see how it works.

# Maxwell's Equations describe EM radiation in vacuum:

$E$ ,  $B$ , and  $k$  form a 'right-handed system', with the wave traveling in the direction of  $k$  at speed,  $c$ .



$E_{\max}$  = peak amplitude

$$E(x, t) = E_{\max} \sin(ax - bt) \quad \text{standard wave equation}$$

$$\text{Intensity} \propto E_{\text{avg}}^2 \propto E_{\text{max}}^2$$

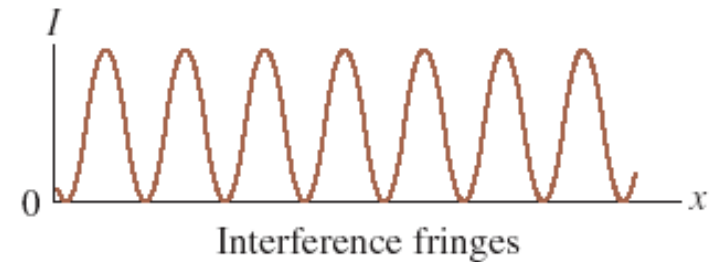
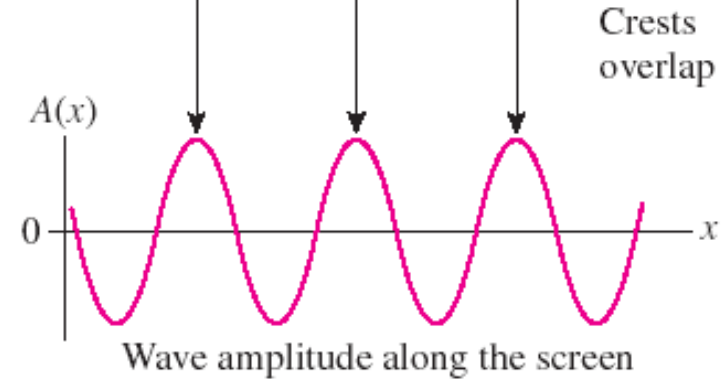
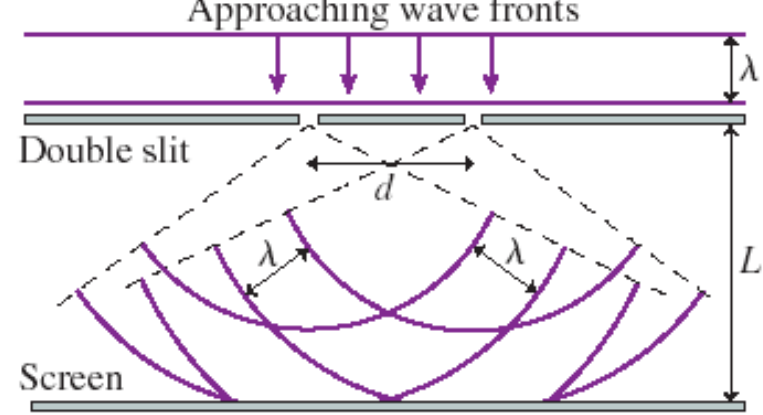
# Two slit interference

Standard two slit interference with light causes waves to interfere

The **amplitude**  $A(x)$  gives the electric and/or magnetic field of the wave as a function of position. It can be positive or negative.

The **intensity** is proportional to the *square* of the amplitude  $I(x) \propto A(x)^2$  and is  $\geq 0$ . This is what we see!

The intensity **also** tells us the probability that a photon will land in a particular  $x$  region.



Please answer this question on your own.  
No discussion until after.

What is the quantity  $\psi(x)$  called?

- A. Probability density
- B. Angular field
- C. Wave function
- D. Potential energy function
- E. Schrödinger function

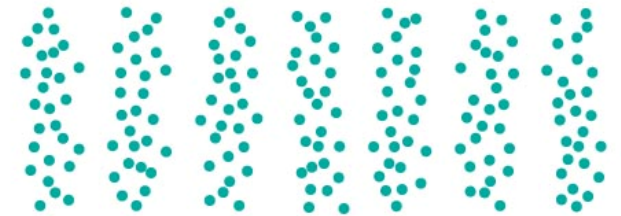
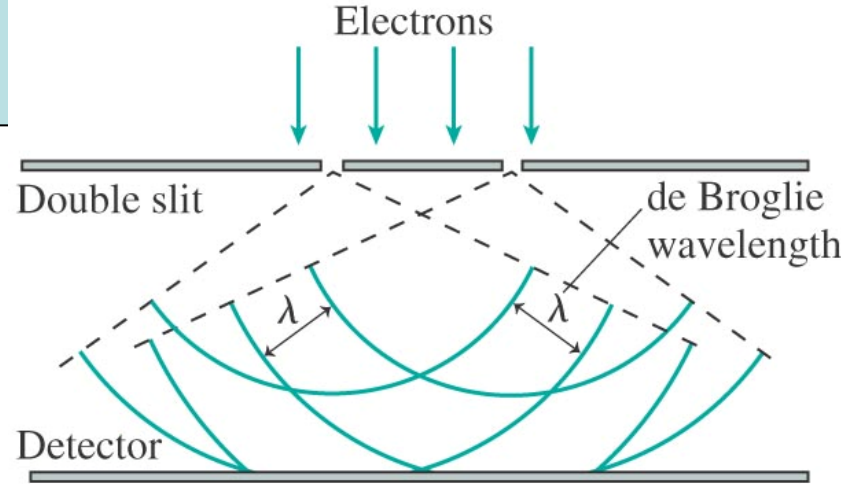
# Same thing with electrons

Electrons have a wavelength and so two slit interference has the same effect as for light

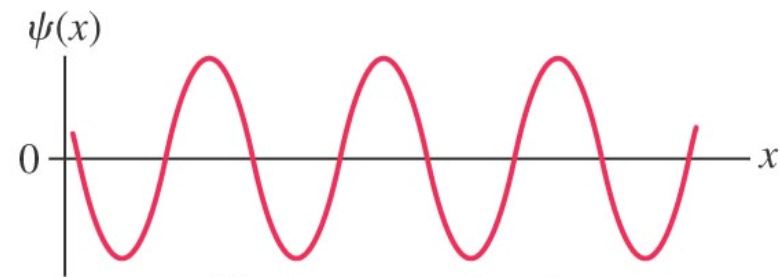
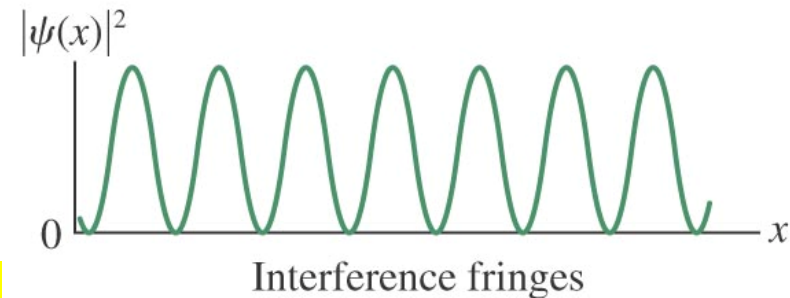
The probability of any given electron hitting near  $x$  is determined by  $|\psi(x)|^2$

$\psi(x)$  is called the **wave function**.

It contains everything we know about the electron but only gives probabilities about where the electron is located



Electron arrival positions on detector

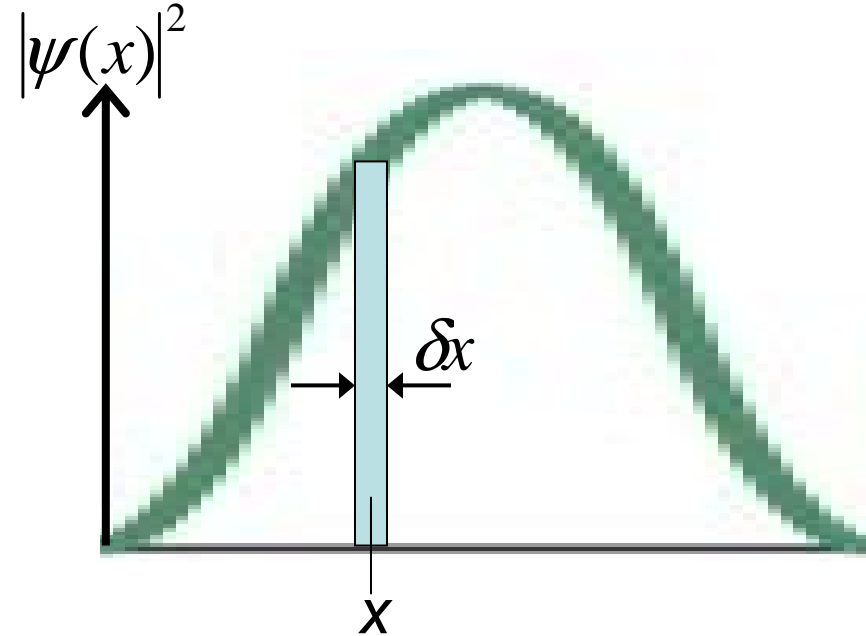


# Probability versus probability density

$|\psi(x)|^2$  is the **probability density** for finding the electron at point  $x$ .

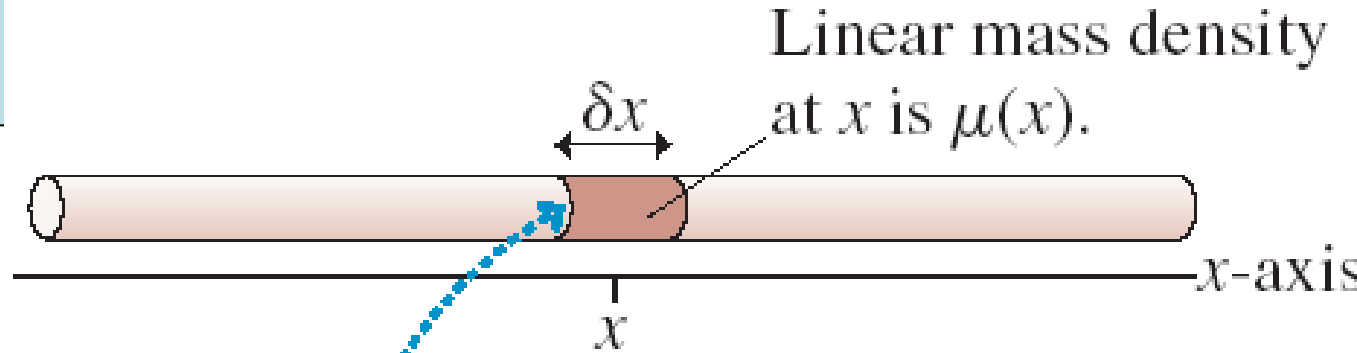
To get probability, multiply the probability density by a region  $\delta x$ .

The probability of finding the electron in the region  $\delta x$  shown is  $|\psi(x)|^2 \delta x$



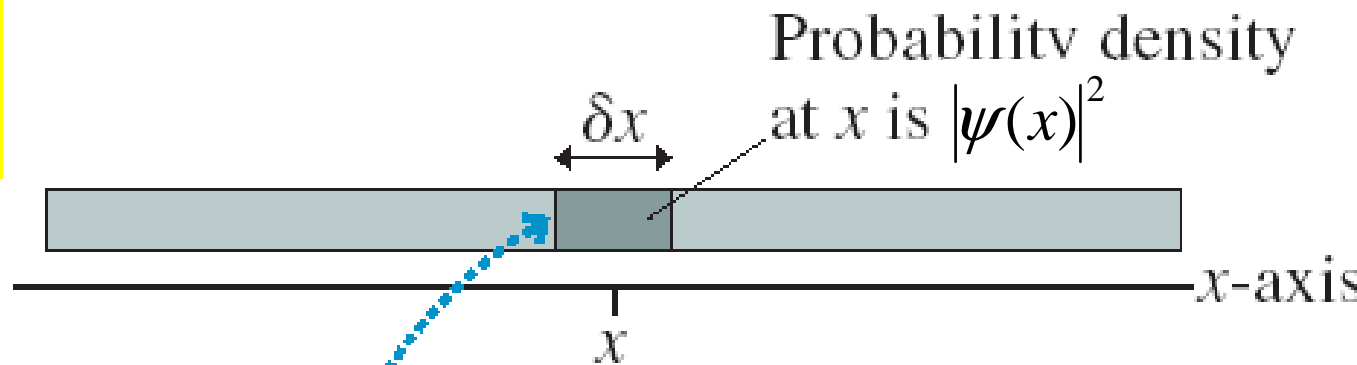
Similar to the relationship between linear mass density and mass.

Probability and probability density are similar to mass and linear mass density.



The mass of this small segment of string is

$$\text{mass(in } \delta x \text{ at } x) = \mu(x) \delta x$$



The probability that a photon lands in this small segment of the screen is

$$\text{Prob(in } \delta x \text{ at } x) = |\psi(x)|^2 \delta x$$

# Clicker question 1

## Set frequency to DA

This is the wave function for a neutron. Around which point is the neutron most likely to be found?

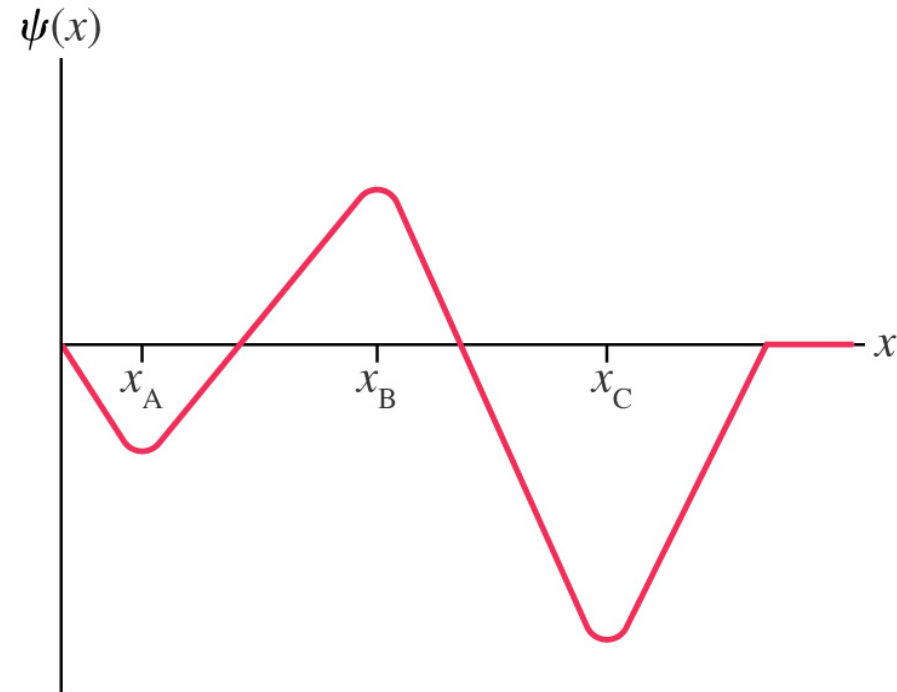
A.  $x = 0$

B.  $x = x_A$

C.  $x = x_B$

D.  $x = x_C$

E. There is no most likely place



Wave functions exist for any object including electrons and neutrons.

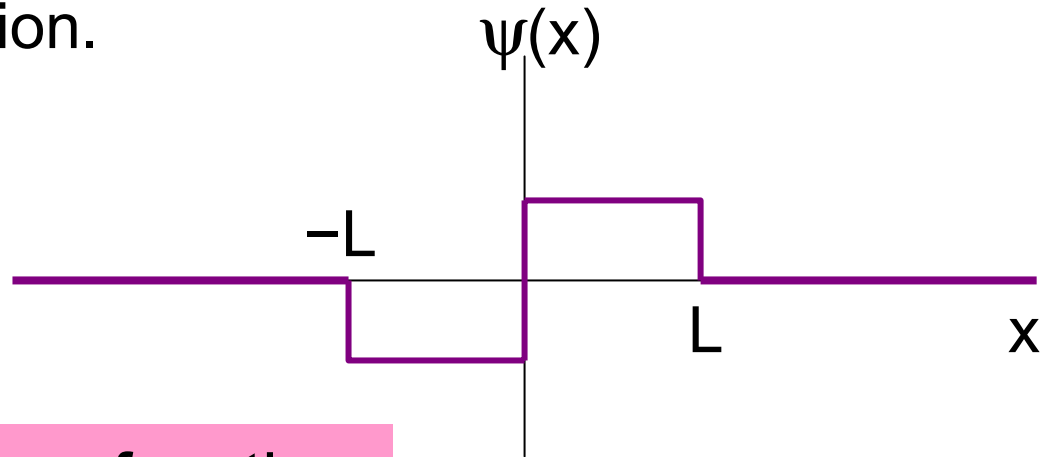
The wave functions we have been looking at came from two-slit interference and so were simple sine waves but different functions are also possible.

Remember the probability is related to  $|\psi(x)|^2$  so need to square the wave function.

# The wave function

The wave function does not tell you the ***path of an object*** like a normal traveling wave function.

The wave function (squared) gives the ***probability of finding the object*** at a given  $x$ .



Some details about the wave function:

Wave function is really a function of the 3D position,  $\psi(x,y,z)$  but many of the problems are 1D so we will start with that:  $\psi(x)$ .

Wave functions are complex valued (have real and imaginary parts) and are ***never*** directly observable.

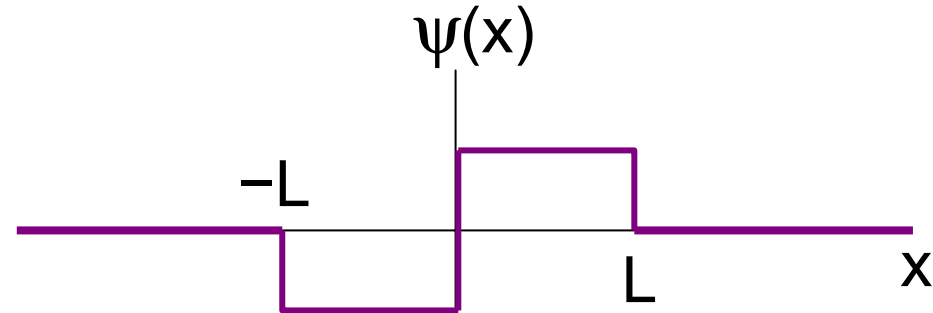
The probability density ***is*** observable and is

$$|\psi(x)|^2 = \psi^*(x) \psi(x) = \psi_{\text{real}}^2(x) + \psi_{\text{imag}}^2(x)$$

$\psi^*(x)$  is complex conjugate: replace  $i$  by  $-i$  in  $\psi(x)$ .

# Getting probabilities from wave functions

Wave function  $\psi(x)$ :



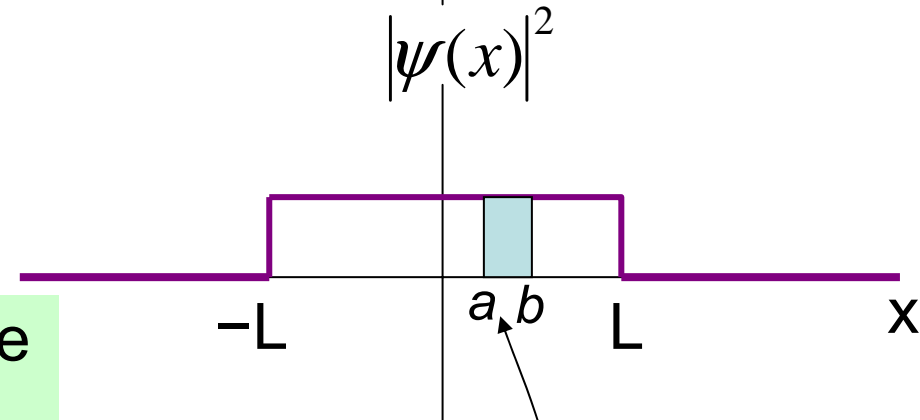
Probability density  $|\psi(x)|^2$

For an infinitesimal distance  $\delta x$  the probability is  $|\psi(x)|^2 \delta x$

To find the probability over a finite distance we need to integrate.

Probability of finding electron between  $a$  and  $b$  is the area under the curve.

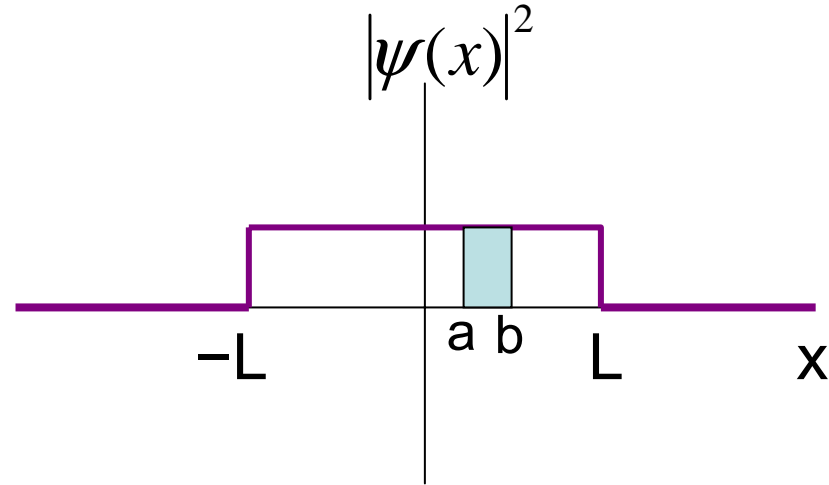
Probability of electron being between  $a$  and  $b$  is  $\int_a^b |\psi(x)|^2 dx$



# Normalization

Probability density =  $|\psi(x)|^2$

Probability of electron being between  $a$  and  $b$  is  $\int_a^b |\psi(x)|^2 dx$



What if this calculation gives a number larger than 1?

Doesn't make any sense!

A properly **normalized** wave function obeys the normalization constraint:  $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

This is simply a statement that the electron is located *somewhere*!

That is, the probability of finding the electron somewhere is 100%.

## Clicker question 2

## Set frequency to DA

If  $\psi(x)$  is a properly normalized wave function, what is the value of  $A$  as shown in the figure?

A.  $A = 1$

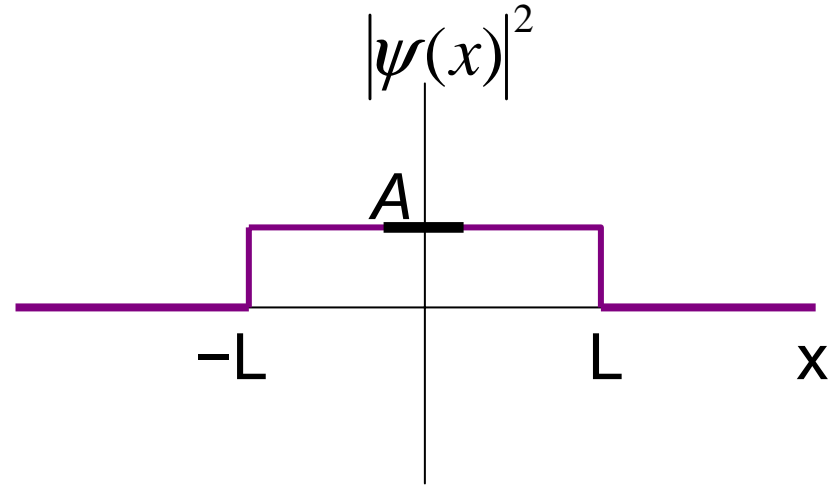
B.  $A = 1 / L$

C.  $A = 1 / 2L$

D.  $A = 1 / \sqrt{2L}$

E.  $A = 1 / 4L^2$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$



The probability is 0 for  $|x| > L$  so we only need to consider the region between  $-L$  and  $L$ . The electron must be somewhere in that region. In that region we know  $|\psi(x)|^2 = A$  so applying the normalization condition we get:

$$\int_{-L}^L A dx = 1 \quad \Rightarrow \quad Ax \Big|_{-L}^L = 1 \quad \Rightarrow \quad AL - A(-L) = 1 \quad \Rightarrow \quad 2AL = 1$$

so  $A = 1 / 2L$

Notice it has units of 1/Length