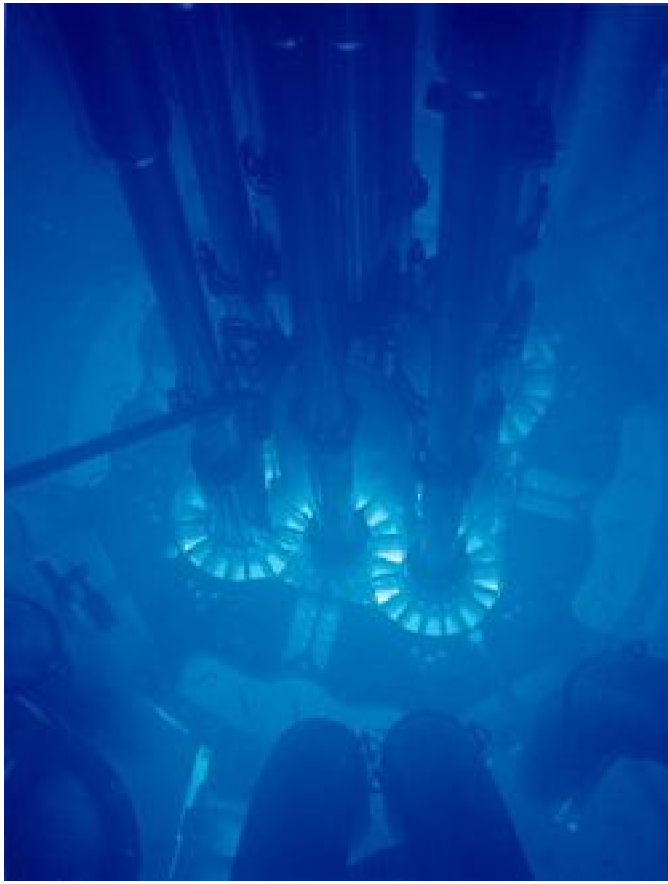
Special relativity

Announcements:

- Homework solutions will be available around 4pm today.
- Next weeks homework should be available by 7pm today.
- The first midterm exam is on February 17 (two weeks from yesterday)



Today we will pretty much finish up special relativity.

Relativistic momentum and energy

Relativistic momentum: $\vec{p} = \gamma_u m\vec{u}$

Total energy: $E = \gamma_u mc^2$

With these definitions for momentum and energy, conservation of momentum and conservation of (total) energy continue to work in isolated relativistic systems

At rest, γ_u=1 so the rest energy is $E_{\text{rest}} = mc^2$

This tells us that mass and energy are equivalent. Mass is a type of energy.

Kinetic energy is *always* the total energy minus the rest energy:

$$KE = E - E_{\text{rest}} = \gamma_u mc^2 - mc^2 = (\gamma_u - 1)mc^2$$

At *low* speeds, this becomes the familiar $KE \approx \frac{1}{2} mu^2$

Clicker question 1 | Set frequency to DA

At what speed is the total energy of a particle equal to twice its rest mass energy? $E = \gamma_u mc^2$ $E_{\text{rest}} = mc^2$ $KE = (\gamma_u - 1)mc^2$

A. 0

B. 0.7c

C. 0.87c

D. 0.94c

E. c

To have total energy equal to twice the rest mass energy, need γ=2

Solve $\gamma = (1-\beta^2)^{-1/2}$ for β.

$$\gamma^2 = \frac{1}{1-\beta^2} \longrightarrow \frac{1}{\gamma^2} = 1-\beta^2 \longrightarrow \beta^2 = 1-\frac{1}{\gamma^2}$$

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{2^2}} = \sqrt{1-0.25} = \sqrt{0.75} = 0.87$$

so you need to be moving pretty fast to get your kinetic energy close to your rest mass energy!

Relativistic momentum and energy

Relativistic momentum: $\vec{p} = \gamma_u m\vec{u}$

Total energy: $E = \gamma_u mc^2$

With some algebra we can eliminate the velocity variable from these two relations and obtain the triangle relation:

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2$$

A new unit of energy is the electron-volt (eV). It's the energy obtained by an electron moving through 1 V. It is not an SI unit but is very common.

$\Delta E = q\Delta V = 1\text{ eV} = 1.6 \cdot 10^{-19}\text{ C} \cdot 1\text{ V} = 1.6 \cdot 10^{-19}\text{ J}$

Since mc^2 is a unit of energy, dividing energy by c^2 gives a unit of mass. Also, dividing energy by c gives a unit of momentum.

A proton has a mass of 938 MeV/c^2. What is this in kg?

$$938\text{ MeV}/c^2 = \frac{938\times10^6\text{ eV}\cdot1.60\times10^{-19}\text{ J/eV}}{\left(3.00\times10^8\text{ m/s}\right)^2} = 1.67\times10^{-27}\text{ kg}$$

Also use eV/c or MeV/c units for momentum

Please answer this question on your own.
No discussion until after.

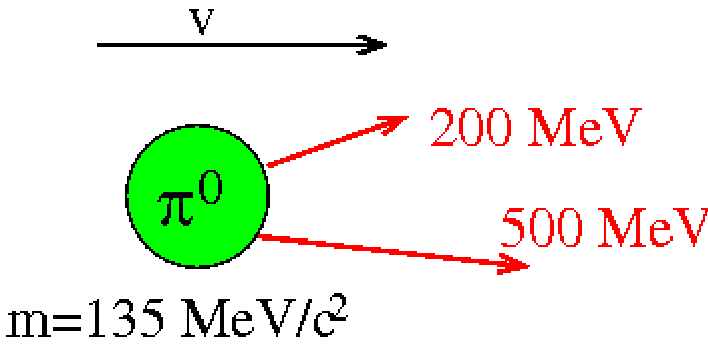
True or false: A massless particle has a finite momentum.

A. $\boxed{\text{True}}$
B. False
C. More than one of the above
D. None of the above

We will see how in the next few minutes

Conservation of energy for subatomic particles

A π^0 particle decays into two photons. The photon energies are measured to be 200 MeV and 500 MeV. The π^0 rest mass is 135 MeV/c^2. What is the π^0 velocity?



Conservation of energy requires that the sum of the photon energies equal the total π^0 energy.

$$E(\pi^0) = E(\gamma_1) + E(\gamma_2) = 200\text{ MeV} + 500\text{ MeV} = 700\text{ MeV}$$

Although we don't need it, we can see that the π^0 kinetic energy is KE = E – mc^2 = 700 MeV – 135 MeV = 565 MeV

Can get velocity from

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

From $E = \gamma mc^2$

we note that $\gamma = \dfrac{E}{mc^2}$

Conservation of energy for subatomic particles

What is the velocity of a π^0 with rest mass 135 MeV/c^2 and total energy 700 MeV?

$$\gamma = \frac{E}{mc^2} = \frac{700\,\text{MeV}}{135\,\text{MeV}/c^2 \cdot c^2} = \frac{700\,\text{MeV}}{135\,\text{MeV}} = 5.2$$

We found before that we could solve $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

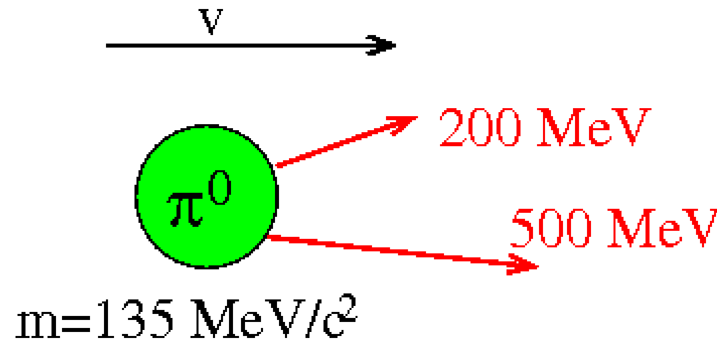
to get $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$

So this gives us: $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{5.2^2}} = \sqrt{0.96} = 0.98$

So the π^0 velocity was 0.98c

Clicker question 2

Set frequency to DA



What is the π^0 momentum?

From the relativistic equation for momentum we get

$$p = \gamma m u = 5.2 \cdot 135\ \text{MeV}/c^2 \cdot 0.98c = 690\ \text{MeV}/c$$

Q. What is the net momentum of the two photons?

A. 0
B. 690 MeV/c
C. 700 MeV/c
D. 135 MeV/c

Momentum is conserved so the π^0 momentum is carried by the photons

But photons are massless so $p = \gamma m u$ doesn't work. What does?

Energy and momentum of massive & massless particles

Remember our triangle relation: $E^2 = (pc)^2 + (mc^2)^2$

For a massless particle like a photon, this reduces to $E = |pc|$
so the energy and momentum are the same

Keep in mind that momentum is a vector and energy is a scalar

As you will find out soon, Einstein won the Noble Prize for finding the energy of a photon is $\boxed{E = hf}$

f = frequency and *h* is Plank's constant: $6.63\times10^{-34}\ \mathrm{J\cdot s} = 4.14\times10^{-15}\ \mathrm{eV\cdot s}$

This can then be used to obtain the photon momentum

Set frequency to DA

Please answer this question on your own. No discussion until after.

The correct formula for Newton's 2nd law in special relativity is:

A. $\vec{F} = \frac{Gm_1m_2}{r^2}\hat{r}$

B. $\vec{F} = m\vec{a}$

C. $\vec{F} = \gamma m\vec{a}$

D. $\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$

Forces still work in relativity, pretty much the same as in classical physics.

Since we need to allow for massless particles, we need to use $\vec{F} = d\vec{p}/dt$ for Newton's 2nd law.

The other "derivation" of E=γmc^2

Back when we were discussing spacetime diagrams I mentioned that just like distance between two points is invariant in 3D, there is a 4D invariant called the *spacetime interval*

$$\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (c\Delta t)^2 - (\Delta \vec{r})^2$$

Or, back to just one space dimension: $\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2$

What happens if we multiple everything by $\dfrac{m^2c^2}{(\Delta\tau)^2}$ where $\Delta\tau$ is the *proper time*

$$m^2c^2\left(\frac{\Delta s}{\Delta \tau}\right)^2 = m^2c^4\left(\frac{\Delta t}{\Delta \tau}\right)^2 - c^2\left(m\frac{\Delta x}{\Delta \tau}\right)^2$$

This is *c*

This is γ

This is γmv = p

We end up with a different writing of the triangle equation:

$$(mc^2)^2 = (\gamma mc^2)^2 - (pc)^2$$

or

$$(mc^2)^2 = E^2 - (pc)^2$$

Four vectors

Position and momentum are 3-vectors since in a 3D space, they measure three things $\vec{r} = (x, y, z)$ $\quad \vec{r} = (r, \theta, \phi)$ $\quad \vec{p} = (p_x, p_y, p_z)$

In the 4 dimensions of spacetime we have 4-vectors which are a 3-vector plus a 4th scalar. Whether we put the 4th scalar at the end or beginning of the order list depends on the convention.

The 4th scalar associated with the position 3-vector is, of course, time or *c*t: (x,y,z,ct) or (ct,x,y,z) and the invariant spacetime interval is $\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (c\Delta t)^2 - (\Delta\vec{r})^2$

The 4th scalar associated with the momentum 3-vector is energy or E/*c* to keep the units straight: (p_x,p_y,p_z,E/*c*) or (E/*c*,p_x,p_y,p_z) and the invariant is $m^2c^2 = \frac{E^2}{c^2} - p^2$ (our triangle relation again)

End of special relativity

We will be starting something new on Friday. I haven't decided what yet. So no reading assignment for Friday.