Spring 2009 Physics 2170 Homework Assignment 9

Show all work; the answer alone is not sufficient. This assignment is due Wednesday, March 18, 2009 at 12:50pm in the wood cabinet at the entrance to the physics help room (Duane G2B90). The assignment is worth 50 points.

1. (5 points): TZ&D 7.4.

2. (5 points): TZ&D 7.10.

3. (4 points): TZ&D 7.14. We are already using the Euler formula for complex exponentials. This problem allows you to see where it comes from. Prior to Euler, most people worked with functions of real numbers. An interesting question arose when trying to see what such functions do when you replace real numbers with complex numbers. Since multiplying complex numbers is no big deal it is easy to generalize polynomial functions of real numbers to polynomial functions of complex numbers. However, what do you do when you want to generalize more complicated functions like sine or cosine to be functions of complex variables? Euler's answer was to use the power series. First, notice that any function can be written in terms of its Taylor series. Then you have a polynomial in which we can put complex variables. Then we just sum the series and see what it looks like.

4. (4 points): TZ&D 7.16.

5. (6 points): TZ&D 7.18. Nuclei have quantum energy levels that the protons and neutrons occupy just like the electrons occupy quantum energy levels of atoms. These levels can theoretically be derived from a combination of quantum chromodynamics (QCD) and quantum electrodynamics (QED). While these calculations are very difficult, it turns out the simple particle in a box idea gets the basic energy scale correct.

6. (6 points): TZ&D 7.30. You may find it helpful to look at the answers for question 7.31. For the probability over a small region, as requested here, you can assume that the probability density is not changing. So you are assuming the probability is a rectangle.

7. (6 points): TZ&D 7.32. This is similar to problem 4 from last week but for a different function.

8. (14 points total) We will use the time independent Schrödinger equation to solve most of the quantum mechanics problems we come across. This name is a little misleading because wave functions always have time dependent part as well. What we will find is that as long as the potential term does not change as a function of time, the time dependent part is always the same. In this problem we will start from the *time dependent* Schrödinger equation and see how we can solve the time-dependence and position-dependence of the wave function separately. This method of solving a differential equation is called *separation of variables.*

a. **(3 points): Start from the time dependent Schrödinger equation (TDSE)** $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$**. Assuming that the potential term only depends on position, show that if you write the total wave function** $\Psi(x,t)$ **as the** $\psi(x)\phi(t)$ **that you can rearrange the TDSE so that all terms involving position are on one side and all terms involving time are on the other side. This is why it is called separation of variables (we are separating** x **and** t**).**

b. **(2 points): Explain why the potential term could not depend on both position and time in order for this to work.**

c. **(2 points): Explain why the only possible solution for the equation you found is for both sides to be equal to a constant (which we will call** C**). That is** $-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt} = C$**.**

d. **(4 points): To find the time dependence we need to find** $\phi(t)$ **which means solving the time equation** $i\hbar\frac{1}{\phi(t)}\frac{d\phi(t)}{dt} = C$**.**

   (a) **Show that** $\phi(t) = e^{-iCt/\hbar}$ **is a solution**

   (b) **Express this solution in terms of sines and cosines.**

   (c) **What is the angular frequency** ω **and the frequency** f **for this solution?**

   (d) **What is the energy of the particle in terms of** C**?**

   (e) **What is** $\phi(t)$ **in terms of** E**?**

e. **(3 points): Using your findings from above, derive the time independent Schrödinger equation** $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$**.**