Some Useful Equations:

1D Schrodinger Equation:

$$-\frac{\hbar^2 \partial^2 \Psi(x,t)}{2m\partial x^2} + V(x,t)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

Solution to Schrodinger Equation with
potential energy $V(x,t)$ = constant < total energy E:

$$\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$$

Solution to Schrodinger Equation with
potential energy $V(x,t)$ = constant > total energy E:

$$\Psi(x,t) = (Ae^{\alpha x} + Be^{-\alpha x})e^{-iEt/\hbar}$$

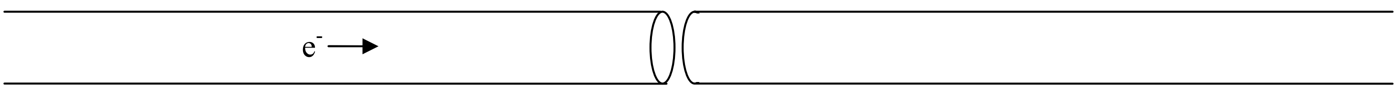
$k = 2\pi/\lambda$
$\lambda = h/p$
$\hbar = h/2\pi$
$KE = \frac{1}{2}mv^2 = p^2/2m$

Name____________________Student ID____________________

Tutorial: Quantum Tunneling

In this tutorial you will explore the physics of an electron traveling through an air gap in a wire – first in the case in which the electron has enough energy to get through the gap classically, and then in the case in which it does not. If you're paying attention, you should be surprised by some of the results.

Consider an electron initially moving to the right through a very long smooth copper wire with a small air gap in the middle. (See figure below.) The work function of copper is V_0.



PART I: $E > V_0$: Suppose the electron shown above has an initial energy $E > V_0$.

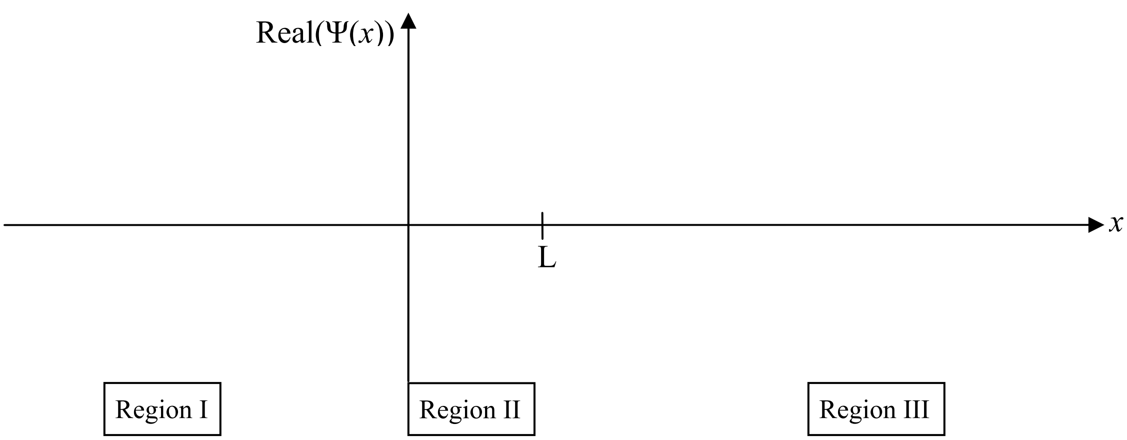
1. In the space below, sketch a graph of the potential energy V of the electron as a function of horizontal position x. Define $V = 0$ inside the wire. Once you have $V(x)$ sketched, use a dashed line to show the energy of an electron that satisfies the $E > V_0$ condition.

2. For the region in the copper wire to the left of the air gap, write down the general solution for $\Psi(x,t)$. Plug it into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

For the region in the air gap, write down the general solution for $\Psi(x,t)$. Is the value of k here the same as the value of k in the previous region? Why or why not? If not, call it k' to distinguish it from the k above. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

For the region in the copper wire to the right of the air gap, write down the general solution for $\Psi(x,t)$. Is the value of k here the same as either of the values of k above? Why or why not? If not, call it k'' to distinguish it from the k's above. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

3. In the plot below, sketch the shape of the real part of the wave function at $t = 0$ in each region for the case where $E > V_0$. The air gap starts at $x = 0$ and ends at $x = L$. Don't worry about the relative magnitudes of the waves in the different regions, but think carefully about the general shape of the graph in each region.



4. What is the basic shape of the real part of the wave function in each of the three regions? For example, is it linear, constant, quadratic, exponential, sinusoidal, or something else?

5. Is the total energy of the electron to the right of the air gap *greater than*, *less than*, or *equal to* the energy of the electron to the left of the air gap? Explain how you arrived at your answer.

6. Fill in the values for the potential, kinetic, and total energy of the particle in each of the three regions in the table below. Your answers should be in terms of E and V_0.

	Left wire	Air gap	Right wire
Potential Energy			
Kinetic Energy			
Total Energy			

7. On top of the graph you drew in question 1, now sketch the kinetic energy *KE* of the electron as a function of position. Be sure to label each of the energies clearly.

8. Write an equation that relates the kinetic energy of a particle to its deBroglie wavelength.

9. Are the wave functions you sketched in question 3 consistent with your equation in question 8 and your kinetic energies in question 6? Resolve any discrepancies.

PART II: *E* < *V*$_0$: Now suppose the electron has an initial kinetic energy $E < V_0$.

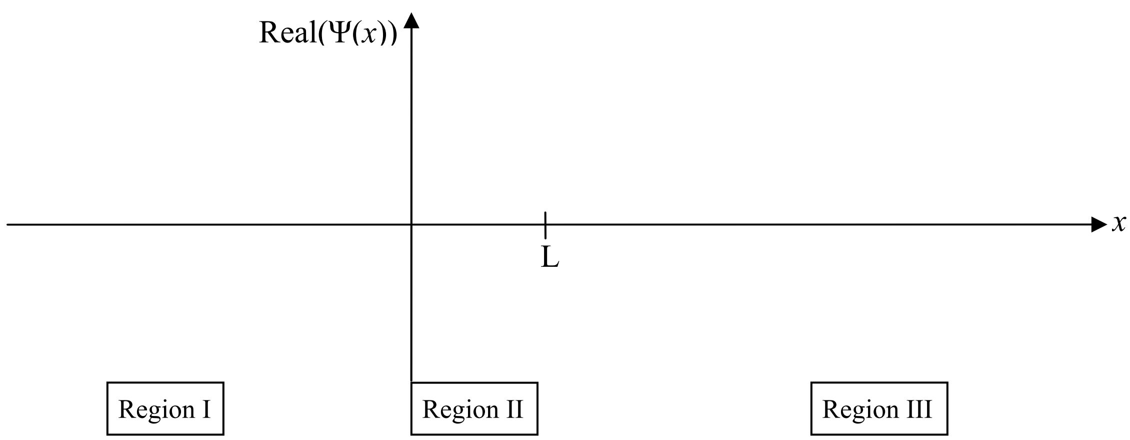
10. In the space below, sketch a graph of the potential energy V of the electron as a function of horizontal position x. Define where $V = 0$ inside the wire. Once you have $V(x)$ sketched, use a dashed line to show the energy of an electron that satisfies the $E < V_0$ condition.

11. For the region in the copper wire to the left of the air gap, write down the general solution for $\Psi(x,t)$. Plug it into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

For the region in the air gap, write down the general solution for $\Psi(x,t)$. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

For the region in the copper wire to the right of the air gap, write down the general solution for $\Psi(x,t)$. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

12. In the plot below, sketch the shape of the real part of the wave function at $t = 0$ in each region for the case where $E < V_0$. The air gap starts at $x = 0$ and ends at $x = \mathrm{L}$. Don't worry about the relative magnitudes of the waves in the different regions, but think carefully about the general shape of the graph in each region.



13. What is the basic shape of the real part of the wave function in each of the three regions? For example, is it linear, constant, quadratic, exponential, sinusoidal, or something else?

14. It is often stated that a particle can quantum mechanically tunnel through a barrier. Explain what is meant by this.

15. Consider an electron that has tunneled through the barrier. Is the energy of the electron to the right of the air gap *greater than*, *less than*, or *equal to* the energy of the electron to the left of the air gap? Explain how you arrived at your answer.

16. Fill in the values for the potential, kinetic, and total energy of the particle in each of the three regions in the table below. Your answers should be in terms of E and V_0.

	Left wire	Air gap	Right wire
Potential Energy			
Kinetic Energy			
Total Energy			

17. On top of the graph you drew in question 10, now sketch the kinetic energy *KE* of the electron as a function of position. Be sure to label each of the energies clearly.

18. Do you notice anything unusual about the kinetic energy?