## Spring 2009 Physics 2170 Final Exam Formula Sheet

Gamma: $\gamma = 1/\sqrt{1-\beta^2}$
Time dilation: $\Delta t = \gamma \Delta t_0$
Length contraction: $L = L_0/\gamma$
Lorentz transformations for frame $S'$ traveling at speed $v$ with respect to frame $S$.

$$\begin{aligned} x' &= \gamma(x - vt) & x &= \gamma(x' + vt') \\ y' &= y & y &= y \\ z' &= z & z &= z \\ t' &= \gamma(t - vx/c^2) & t &= \gamma(t' + vx'/c^2) \end{aligned}$$

Relativisitc velocity addition:

$$\begin{aligned} u'_x &= \frac{u_x - v}{1 - u_x v/c^2} & u_x &= \frac{u'_x + v}{1 + u'_x v/c^2} \\ u'_y &= \frac{u_y}{\gamma(1 - u_x v/c^2)} & u_y &= \frac{u'_y}{\gamma(1 + u'_x v/c^2)} \\ u'_z &= \frac{u_z}{\gamma(1 - u_x v/c^2)} & u_z &= \frac{u'_z}{\gamma(1 + u'_x v/c^2)} \end{aligned}$$

Doppler effect: $f_{\text{observer}} = f_{\text{source}}\sqrt{\frac{1+\beta}{1-\beta}}$ for source and observer moving toward each other at speed $\beta$.

Relativistic momentum: $\vec{p} = \gamma m \vec{u}$
Relativistic energy: $E = \gamma mc^2$
Rest energy: $E_{\text{rest}} = mc^2$
Relativistic kinetic energy: $KE = E - E_{\text{rest}} = E - mc^2 = (\gamma - 1)mc^2$
Useful relation: $\beta = pc/E$
Energy-momentum relation: $E^2 = (pc)^2 + (mc^2)^2$
Force: $\vec{F} = d\vec{p}/dt$

Non-relativistic kinetic energy: $K = mv^2/2 = p^2/2m$
Angular momentum: $\vec{L} = I\vec{\omega} = \vec{r} \times \vec{r}$
Work function: minimum energy needed to eject an electron from a metal: $\phi$
Photoelectric effect: $K_{\max} = hf - \phi$
de Broglie relations: $p = h/\lambda = \hbar k$ and $E = hf = \hbar\omega$
Heisenberg uncertainty relations: $\Delta x \Delta p > \hbar/2$, $\Delta E \Delta t > \hbar/2$
Sinusoidal wave parameters: $k = 2\pi/\lambda$, $f = \omega/2\pi = 1/T$
1D time dependent Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$
General solution for time independent potential: $\Psi(x,t) = \psi(x)\phi(t)$
Time solution for time independent potential: $\phi(t) = e^{-iEt/\hbar}$
1D time independent Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$
Spatial solution to 1D infinite square well of length $a$: $\psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$
Allowed energies for 1D infinite square well of width $a$: $E_n = n^2\pi^2\hbar^2/(2ma^2)$
Generic solution for free particle traveling right: $\Psi(x,t) = Ae^{i(kx-\omega t)}$
Generic solution for free particle traveling left: $\Psi(x,t) = Ae^{-i(kx+\omega t)}$
Generic spatial solution in classically forbidden regions ($E < V$) in 1D: $\psi(x) = Ae^{\alpha x} + Be^{-\alpha x}$
Normalization condition: $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$
Euler formula: $e^{i\theta} = \cos\theta + i\sin\theta$

Penetration depth: $\lambda = 1/\alpha = \hbar/\sqrt{2m(V-E)}$
Quantum tunneling probability: $P \approx e^{-2\alpha L}$
Balmer-Rydberg formula: $1/\lambda = R(1/n'^2 - 1/n^2)$
Angular momentum quantization in Bohr model: $L = n\hbar$
Bohr model energy levels for Hydrogen atom: $E_n = -E_R/n^2$
Bohr model classical orbital radii for hydrogen atom: $r = n^2 a_B$
Bohr model energy levels for hydrogen-like ions (one electron bound to a charge of $Ze$): $E_n = -Z^2 E_R/n^2$
Bohr model classical orbital radii for hydrogen-like ions: $r = n^2 a_B/Z$
For orbits: $E = \frac{1}{2}U$ and $K = -E = -\frac{1}{2}U$
Hydrogen wave function: $\psi(r,\theta,\phi) = R_{n\ell}(r)\Theta_{\ell m}(\theta)e^{im\phi} = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$
Probability of being around $r$ is $P(r) = r^2|R_{n\ell}(r)|^2$

Electron quantum numbers in atom:

- $n$ = principal quantum number. $E_n = -Z_{\text{eff}}^2 E_R/n^2$
- $\ell$ = orbital angular momentum quantum number. $\ell < n$ and $L = \sqrt{\ell(\ell+1)}\hbar$
- $m$ = z-component of orbital angular momentum quantum number. $|m| \le \ell$ and $L_z = m\hbar$
- $s$ = electron spin quantum number = 1/2. $S = \sqrt{s(s+1)}\hbar$
- $m_s$ = z-component of spin quantum number. $m_s = \pm 1/2$. $S_z = m_s\hbar$

Total angular momentum: $\vec{J} = \vec{L} + \vec{S}$ and $J_z = L_z + S_z$
Atomic energy level order: 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 6s
Energy levels for atoms: $E_n = -Z_{\text{eff}}^2 E_R/n^2$
Most probable radius for electron: $r \approx n^2 a_B/Z_{\text{eff}}$

Constants and conversions

$c = 3.00 \times 10^8$ m/s
$h = 6.63 \times 10^{-34}$ J · s $= 4.14 \times 10^{-15}$ eV · s
$hc = 1.99 \times 10^{-25}$ J · m = 1240 eV · nm = 1240 MeV · fm
$\hbar = h/2\pi = 1.05 \times 10^{-34}$ J · s $= 6.58 \times 10^{-16}$ eV · s
$\hbar c = 3.15 \times 10^{-26}$ J · m = 197 eV · nm = 197 MeV · fm
Elementary charge: $e = 1.60 \times 10^{-19}$ C
Bohr radius: $a_B = \hbar^2/(ke^2 m_e) = 5.39 \times 10^{-11}$ m
Rydberg constant: $R = m_e k^2 e^4/(4\pi c\hbar^3) = 0.0110$ nm$^{-1}$
Rydberg energy: $E_R = hcR = ke^2/2a_B = m_e k^2 e^4/(2\hbar^2) = 13.6$ eV
energy conversion: 1 eV $= 1.60 \times 10^{-19}$ J
mass conversion: 1 eV/$c^2$ $= 1.783 \times 10^{-36}$ kg
electron mass: $m_e = 0.511$ MeV/$c^2$ $= 9.11 \times 10^{-31}$ kg
proton mass: $m_p = 938.3$ MeV/$c^2$ $= 1.6726 \times 10^{-27}$ kg
neutron mass: $m_n = 939.6$ MeV/$c^2$ $= 1.6749 \times 10^{-27}$ kg
unified atomic mass unit: $u = 931.5$ MeV/$c^2$ $= 1.6605 \times 10^{-27}$ kg
Avogadro's constant: $N_A = 6.022 \times 10^{23}$
Boltzmann's constant: $k_B = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Coulomb force constant: $k = 8.99 \times 10^9$ N m$^2$/C$^2$