# Lab 9: Diffraction and Interference

## INTRODUCTION & BACKGROUND:

Light is an electromagnetic wave, and under the proper circumstances, it exhibits wave phenomena, such as constructive and destructive interference. The wavelength of visible light ranges from about 400-750 nm = 0.0004-0.00075 mm, and this wavelength $\lambda$ sets the scale for the appearance of wave-like effects. For instance, if a broad beam of light partly passes through a wide slit (i.e. a slit which is very large compared to $\lambda$), then the wave effects are negligible, the light acts like a ray, and the slit casts a geometrical shadow. However, if the slit is small enough (i.e. around the same size as $\lambda$ or smaller), then the wave properties of light become apparent and a *diffraction pattern* is projected.

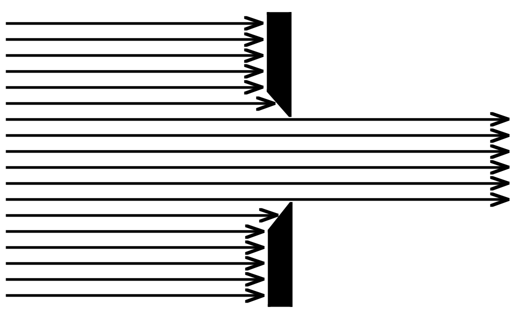

**Figure 1a. Slit large compared to λ.**

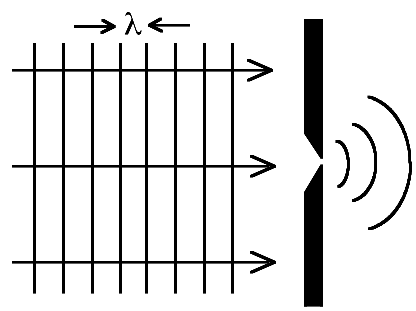


**Figure 1b. Slit small compared to λ.**

Now consider the light from **two coherent light sources** a distance **d** apart. Coherent sources emit light waves that are *in phase*, or in sync. If we think of light like a water wave, we can imagine that coherent sources emit an identical succession of wave crests and troughs, with both emitting crests at the same time. One way to create such coherent sources is to illuminate a pair of narrow slits with a distant light source.

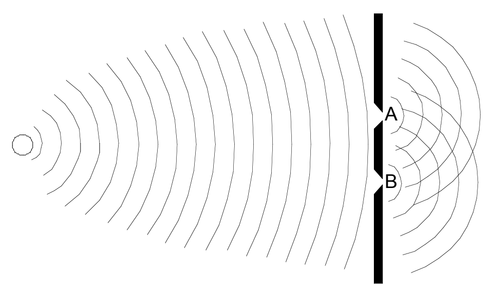


**Figure 2. Points A and B act like coherent sources.**

***Interference from two slits:*** Consider the light rays from the two coherent point sources made from slits a distance d apart (see fig. 3). We assume that the sources are emitting *monochromatic* (single wavelength) light of wavelength $\lambda$. The rays are emitted in all forward directions, but let's concentrate on the rays that are emitted in a direction θ toward a distant screen (θ measured from the normal to the screen, diagram below). One of these rays has further to travel to reach the screen, and the *path difference* is given by $d\sin\theta$.

What happens if this path difference is **exactly** one wavelength λ (or any integer number of wavelengths)? If you look carefully, this is what is represented in fig. 3.

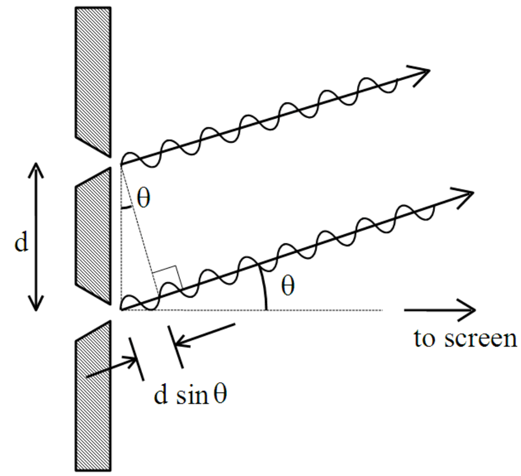


**Figure 3.**

What happens if the path difference is λ/2, or 3λ/2, or 5λ/2, etc.?

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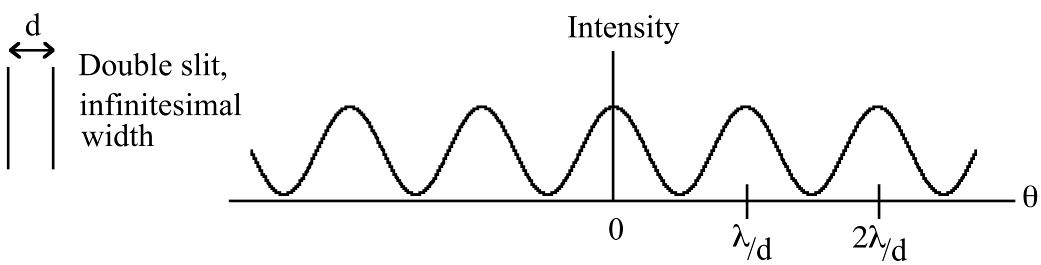
A complete analysis yields a pattern of **intensity vs. angle** that looks like:

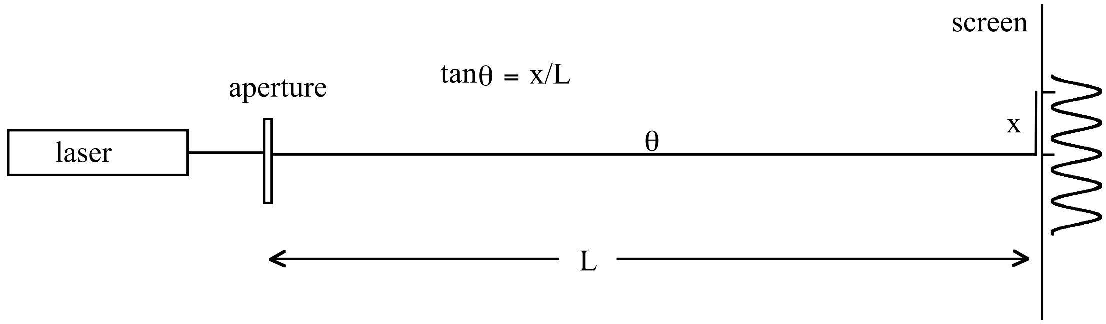



$$\left.\begin{aligned} &\textit{Bright}: \; d\sin\theta = m\lambda \\ &\textit{Dark}: \; d\sin\theta = (m+\tfrac{1}{2})\lambda. \end{aligned}\right\} \mathrm{m} = 0, \pm 1, \pm 2..$$

What happens to the above interference pattern if d is increased? What if d is decreased?

***Geometric simplification:*** If θ is small, then $\sin\theta \cong \theta$ (in radians), and maxima occur on the screen at $\theta = m\dfrac{\lambda}{d}$; minima occur at $\theta = \left(m+\tfrac{1}{2}\right)\dfrac{\lambda}{d}$.

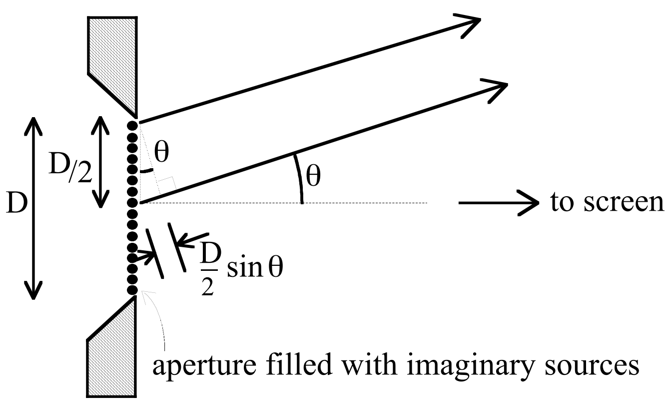
As shown below, the angle θ (measured from the center of the screen) is related to the distance x measured on the screen by tan(θ)=x/L, where L is the distance from the screen to the source of light (the aperture).




If the angle θ is small (less than a few degrees), then to an excellent approximation, sin(θ) ≈ tan(θ) ≈ θ (in radians) so the locations of the interference maxima are given by

$$\frac{x}{L} = m\frac{\lambda}{d}.$$

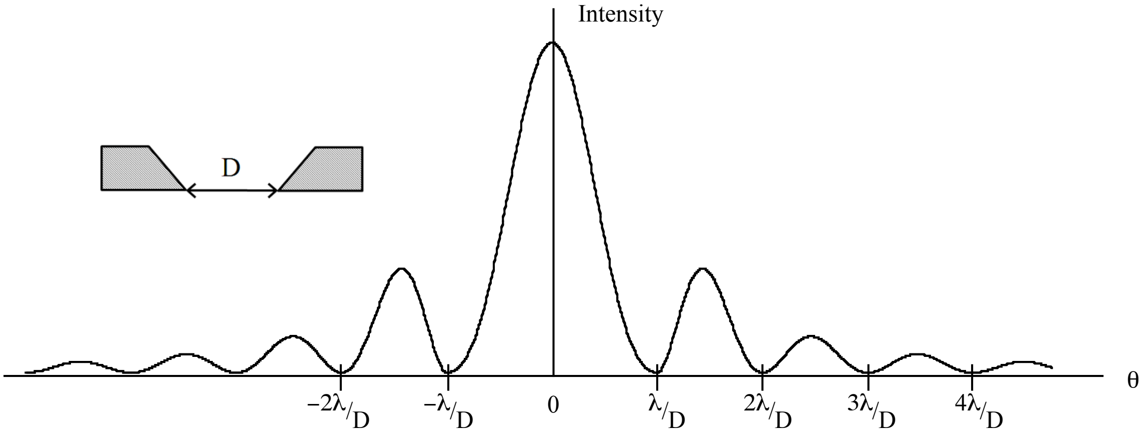
***Single slit diffraction:*** The uniform 2-slit interference pattern shown above is seldom observed in practice, because real slits always have finite width (not an infinitesimal width). We now ask: what is the intensity pattern from a **single slit** of **finite width** D? *Huygens' Principle* states that the light coming from an aperture is the same as the light that would come from a collection of coherent point sources filling the space of the aperture. Its like we constructed the large slit out of a whole set of small slits, all adjacent to each other. To see what pattern the entire array produces, consider first just two of these imaginary sources: one at the edge of the slit and one in the center. These two sources are separated by a distance D/2.




The path difference for the rays from these two sources, going to the screen at an angle θ, is $\frac{D}{2}\sin\theta$, and these rays will interfere destructively if $\frac{D}{2}\sin\theta = \frac{\lambda}{2}$. But the same can be said for every pair of sources separated by D/2. Consequently, the rays from all the sources filling the aperture cancel in pairs, producing zero intensity on the screen when $\frac{D}{2}\sin\theta = \frac{\lambda}{2}$ or, if θ is small,

$$\theta = \frac{\lambda}{D}. \qquad \text{(First minimum in single slit pattern.)}$$

The complete intensity pattern, called a *diffraction* pattern, looks like...

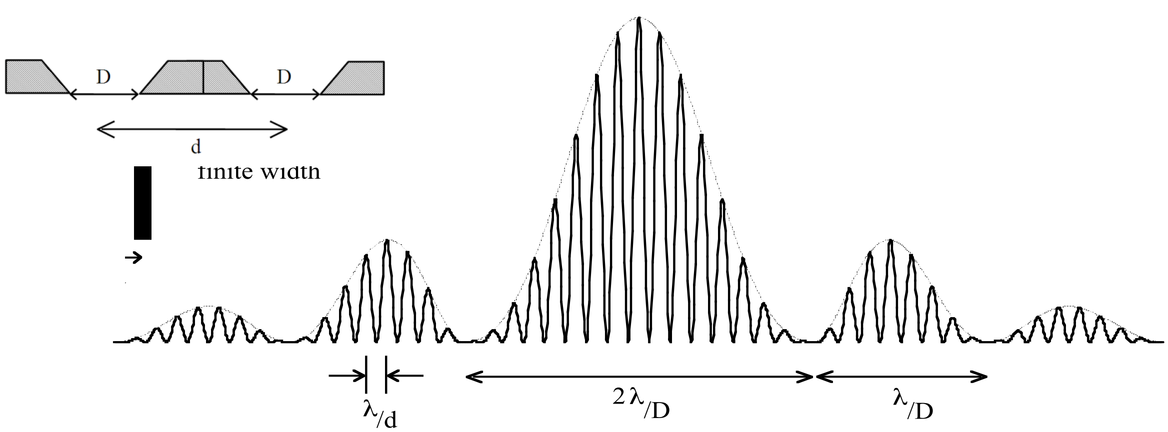



[The central maximum is actually much higher than shown here. It was reduced by a factor of 6, for clarity.] The single slit diffraction pattern has minima at

$$\theta = \pm\frac{\lambda}{D},\ \pm\frac{2\lambda}{D},\ \pm\frac{3\lambda}{D},\ \ldots \qquad \text{(Minima of single slit pattern.)}$$

So the separation of minima is λ/D, except for the first minima on either side of the central maximum, which are separated by 2λ/D. If x is the distance on the screen between minima, then $\theta = \frac{x}{L} = \frac{\lambda}{D}$.

***Combine interference (2-slit) with diffraction (finite-width slit):*** When the aperture consists of **two finite** slits, each of width D, separated by a distance d, then the intensity pattern is a combination of both the single-slit pattern and the double slit pattern: the amplitude of the two slit interference pattern is modulated by a single slit diffraction pattern:



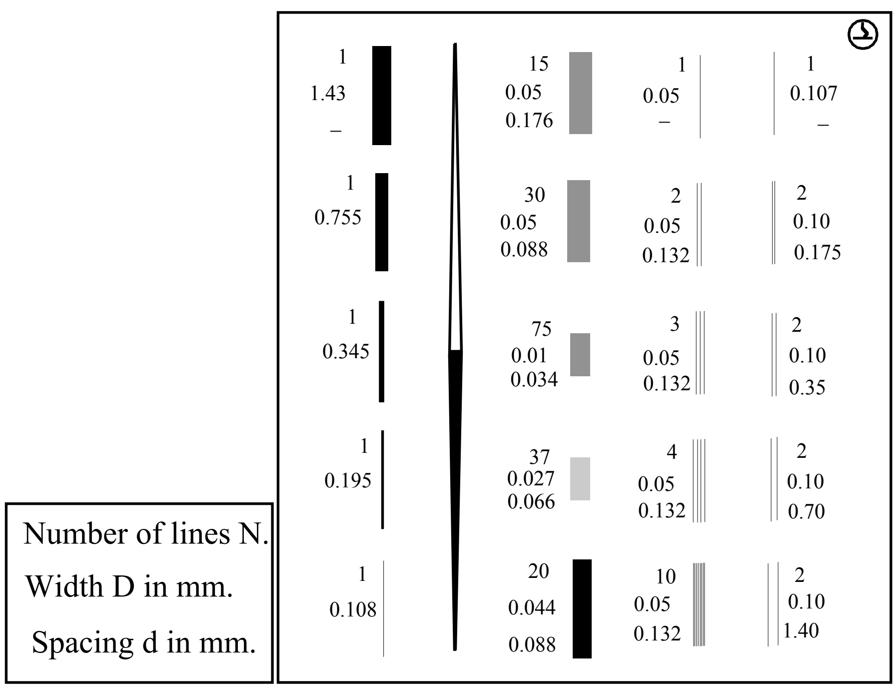
In this full pattern, the **finely spaced interference maxima** are spaced $\Delta\theta = \frac{\lambda}{d}$ apart, while the more **widely spaced minima of the single-slit diffraction pattern** are separated by $\Delta\theta = \frac{\lambda}{D}$ or $\frac{2\lambda}{D}$. Note that an interference maximum can be wiped out if it coincides with a diffraction minimum.

**PART I: Diffraction Pattern from Single Slits**

The light source in parts 1 and 2 of this experiment is a He-Ne laser which produces a monochromatic beam with a wavelength of $\lambda$ = 632.8 nm and a beam diameter of about 1 mm. The power output of our lasers is about 1 mW, a small amount, but still enough to damage your retina if you look directly into the beam. What color is light at $\lambda$ = 632.8 nm?

NEVER LOOK INTO A LASER BEAM.

The aperture consists of an opaque photographic negative, containing several single, double, and multiple slits. The arrangement of slits on the plate is shown here. The numbers are those given by the manufacturer and are *not* always accurate. (This particular plate is often called a Cornell Plate, since apertures of this kind where first used at Cornell University.)

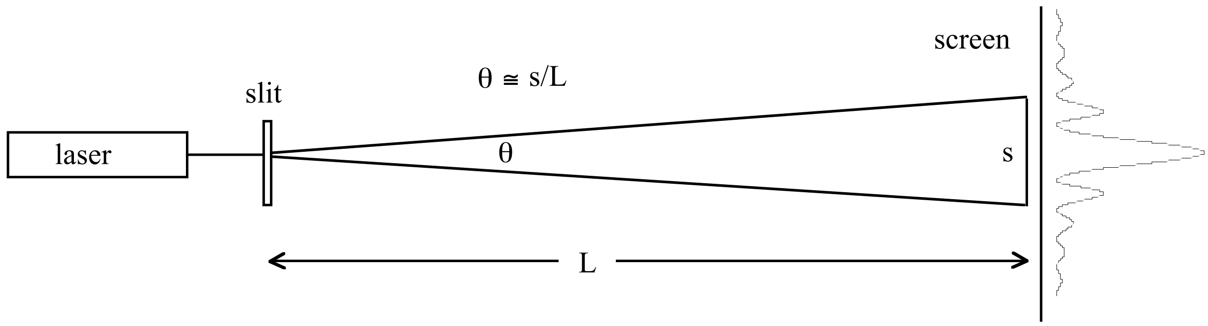


Place the Cornell plate in its holder and mount it on the optical bench a few centimeters in front of the laser. Place a piece of white paper in the clipboard and place it at the far end of the bench. Then shine the laser through the various apertures and observe the diffraction patterns. What do you notice? Is it what you expect? Spend a few minutes exploring, and draw a diagram of your setup here.

Based on the formulas listed above, how do you **expect** the appearance of the single-slit pattern to vary with slit width?

In general, how does the **observed** single slit pattern change as you vary the width of the slit?

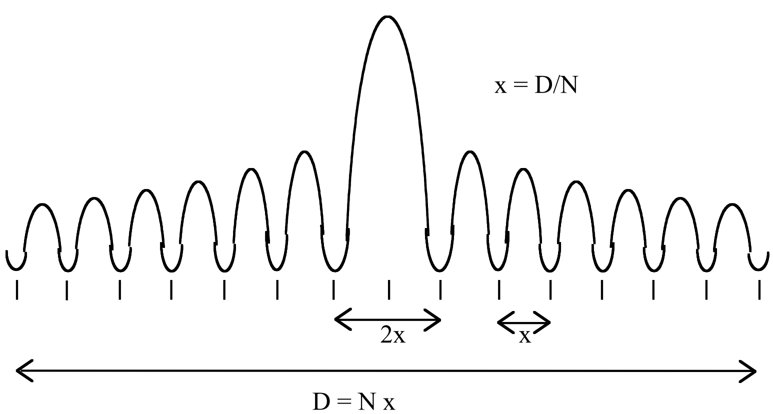
***More geometry:***




This diagram shows the relation between the width ***s*** of some feature (any feature) on the screen, the angular width θ of that feature, and the distance L from the aperture to the screen.

In this part we will test the relation $\theta = \frac{x}{L} = \frac{\lambda}{D}$, where x is the separation of minima on the screen in the single slit diffraction pattern. Measure the distance L from the slit to the screen. Record it here.

Observe the diffraction pattern on your paper screen for each of the three smallest single slits (the three with approximate widths D = 0.10, 0.20, 0.35 mm). With a pencil, mark the positions of as many of the minima that you can see and measure the spacing x between adjacent minima on the screen, for each of the three slits. To do this most accurately, measure the width of the entire pattern and divide by the number of maxima in the pattern (central max counts as two!). From your measurements, using the wavelength λ=632.8 nm, compute the slit width D for each of the three slits and compare with the manufacturer's widths.



## Part II. Double Slit Diffraction Patterns

How far apart would you expect the peaks in the intensity to be for a slit-separation of 0.35 mm and $\lambda$=632.8 nm?

The manufacturing precision of the Cornell plates might not be perfect – would a 5% manufacturing error make any difference? How far apart would the peaks be if the slit separation was 5% wider than the previous value of 0.35 mm? How about 5% closer?

Repeat the procedure from part 1, using the double slit aperture with nominal slit separation of 0.35 mm (the one in the middle of the right-most column on the Cornell plate). What measurements do you need to make to allow you to compute the actual slit separation? Record them here.

Compute d, the actual slit separation. How close is it to the manufacturer's number? (Within 10%? Within 1%?)

**Part III. Resolving power of the human eye**

In this part, you will measure the resolving power of your eyes to see how close your vision is to "perfect", that is, diffraction-limited performance.

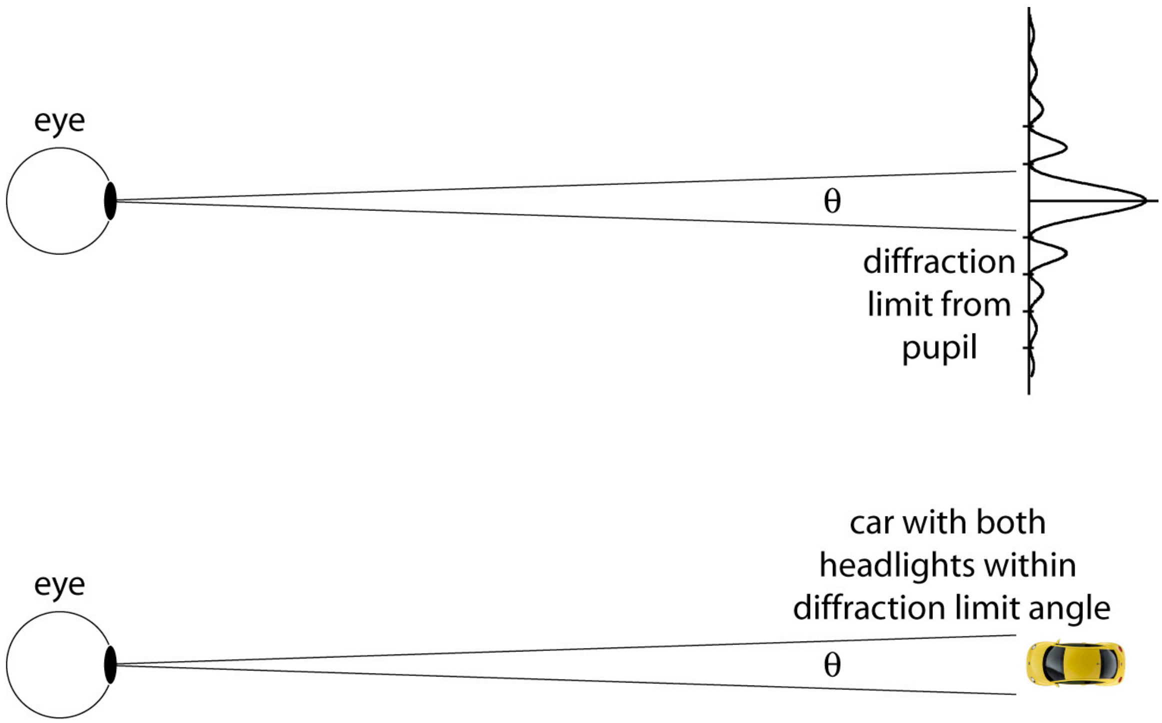



**Figure 4. Diffraction limit from pupil of eye, and car with both headlights within the diffraction limit angle. The two headlights would be unresolvable in this case, and would look like a single light.**

The resolution of an optical instrument such as a camera, a telescope or the human eye is defined as the smallest angle which the instrument can resolve. Diffraction effects limit the resolution of any optical instrument to $\theta \cong \frac{\lambda}{D}$, where $\lambda$ is the wavelength of the light used, and D is the diameter of the main light-gathering optical element (the objective lens of a telescope, the pupil of the human eye, etc.). Many textbooks use the formula $\theta = 1.22\frac{\lambda}{D}$, called the Rayleigh criterion, but the minimum angle of resolution $\theta$ is difficult to measure precisely, and in practice the factor 1.22 is not experimentally significant.

One way to think about this is as follows: If you had a laser shooting out of the pupil of your eye and recorded the light pattern on a distant screen, you would see an extended diffraction pattern with a wide central peak defined by D, which in this case is the width of your pupil (see figure 4 on the next page). If you reverse the situation, and admit light *into* your eye, that same diffraction pattern describes the size of the smallest thing you can resolve. Any details smaller than that peak just blur together, even if you have excellent vision.

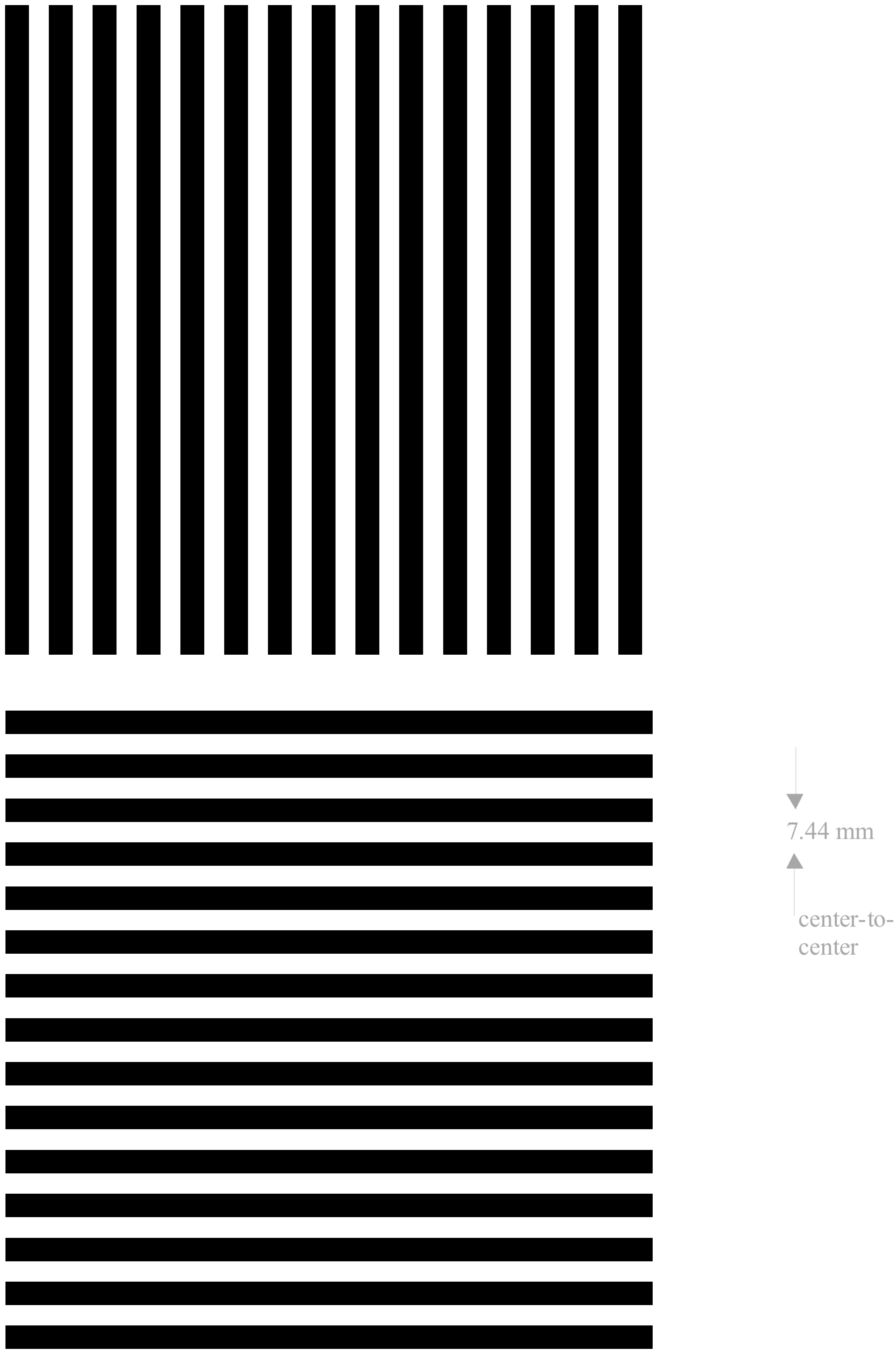
First, measure the diameter D of the pupil of your eye (under normal room-light conditions). With one eye open, look closely at the image of your pupil in a mirror and measure your pupil's diameter with a clear plastic ruler placed on the mirror or over your eye. Record it here.

Measure the angular resolution of your eye with the following procedure: One of your partners will stand on a "zero position" mark on the floor some distance away and will hold up an eye-test chart for you to view with both eyes open. The test chart consists of an array of vertical bars and an array of horizontal bars, printed on the next page. Beyond a certain distance, the human eye cannot resolve the bars and the arrays appear to be gray blotches rather than black stripes on a white background.

Begin by standing so far from your partner that the chart cannot possibly be resolved, and then signal your partner to hold up the test chart. The chart is held up with either the vertical bars or the horizontal bars to your right, but you will not know which orientation is used. Slowly, approach your partner and when you believe you can resolve the bars, indicate with hand signals which side of the chart has the horizontal bars. Repeat this test several times, varying your distance somewhat. Each time you "read" the chart, your partner should give the thumbs-up (correct) or thumbs-down (incorrect) signal and then you should turn away briefly while your partner randomly rearranges the chart for the next test. Find the maximum distance L (marked on the floor) at which you can consistently read the chart correctly. The center-to-center separation x of the bars is marked on the chart. Record the maximum distance here.

The angle $\theta$ which you can resolve is then $\theta = x/L$. Record this angle in radians, degrees, and in arcminutes (60 arcminutes = 1 degree).

Compare this angle with the theoretical diffraction-limited resolution of $\theta = \lambda / D$. Use $\lambda$ = 550 nm (the middle of the visible spectrum) in your calculations. Also report the results of your partners' eye tests. How close are your vision and your partners' vision to "perfect"?


7.44 mm
center-to-
center

## POTENTIAL EXAM QUESTIONS:

**1.** Consider a diffraction pattern produced by a laser shining through two slits separated by a distance d. Now suppose the slit-separation d is **decreased** a little, while everything else is kept fixed. In order to maintain the same pattern on the screen (i.e. with the same peak separation), which of the following statements is true?

a) The wavelength of the light should be increased.
b) The wavelength of the light should be decreased.
c) The pattern didn't change when d changed, so nothing needs to be done.
d) Changing the wavelength of the light cannot return the old pattern.
e) None of the above are true.

**2.** A single slit pattern is formed by shining a laser of wavelength $\lambda$ through a single slit onto a screen. The position on the screen of the first intensity minimum (to the side of the central maximum) is a little closer to one edge of the slit than to the other edge of the slit. How much closer is it to the nearer side of the slit?

a) $\lambda/4$
b) $\lambda/2$
c) $\lambda$
d) $2\lambda$
e) None of the above