

## Lab 10: The Atom and the Hydrogen Spectrum

### INTRODUCTION & BACKGROUND:

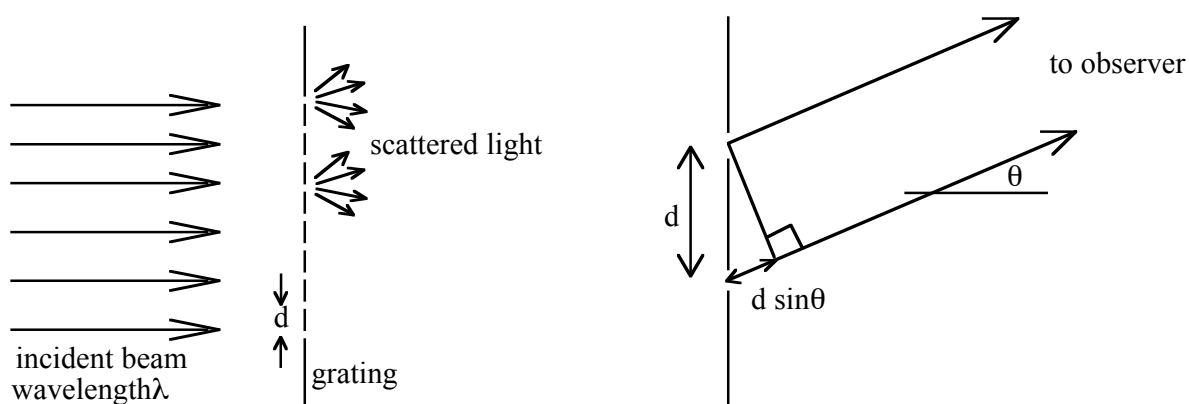
This lab will allow us to look inside an atom. Using tools we have developed over the semester and a simple model of the atom, we will be able to determine which atoms emit which colors of light. Each atom has a unique optical fingerprint and we will use that concept to study the hydrogen atom, and then determine the identity of a mystery atom. This approach is similar to how astronomers determine what stars are made of.

The goals of this lab are to learn how to use a diffraction grating, to understand the Bohr model of the atom and electron orbital structure, energy levels in an atom, and to learn how the orbital structure relates to the spectrum of emitted light.

**Diffraction grating review:** Recall our last lab working with two-slit interference. We observed that the peak separation distance of light projected on a screen depended on the wavelength of light and the separation between the slits. Now instead of using just two slits, we will use thousands... The effect is the same as with two slits, only the peaks get much sharper -- so we observe sharp bands of light corresponding to specific wavelengths.

A diffraction grating is simply a piece of glass or plastic which has a series of very fine scratches or grooves cut in its surface. The grooves are perfectly straight and parallel and are equally spaced so that there are a fixed number of grooves per millimeter, typically around 500 grooves/mm.

A grating behaves essentially like a multi-slit aperture, that is, a mask with many closely spaced slits. If the number of grooves per length is  $n$  (grooves per cm), then the separation between adjacent slits is  $d = 1/n$  (cm per line or simply, cm). Consider what happens when a beam of monochromatic (single wavelength) light strikes a grating at normal incidence, as shown below. Each groove or slit scatters the light in all forward directions. However, in only certain directions will the light scattered from different grooves interfere constructively, producing a strong beam.



The diagram on the right shows two light rays emerging from adjacent slits in the grating and heading toward an observer (or a point on a screen) at an angle  $\theta$  from the normal (perpendicular) direction. In traveling to the observer, the ray from the lower slit has to travel an extra path distance; this path difference is  $\Delta\text{path}=d\sin(\theta)$ . The two rays will interfere constructively only if the path difference is an integer number of wavelengths:

$$(1) \quad d \sin \theta = m\lambda ,$$

where  $\lambda$  is the wavelength of the light and  $m$  is any integer. At only those special angles corresponding to integer  $m$ 's ( $m=0, 1, 2, \dots$ ) will the rays from all the slits interfere constructively, producing a bright beam in that direction. In any other direction, the rays from the various slits interfere destructively and produce no light intensity. The integer  $m$  is called the order of the diffraction.

An incident light beam made of a several distinct wavelengths will be split by the grating into its component wavelengths, with each separate wavelength heading in different directions, determined by the condition  $d \sin \theta = m\lambda$ . In this way, the various wavelengths can be determined by measuring the angles.

**Bohr model of the hydrogen atom:** In the 19<sup>th</sup> century, it was known that hydrogen gas, when made to glow in an electrical discharge tube, emitted light at four particular visible wavelengths. In 1885, a Swiss high school teacher named Balmer discovered that the four wavelengths, here labeled  $\lambda_i$  (where  $i = 1, 2, 3, 4$ ) precisely obeyed a curious mathematical relation:

$$(2) \quad \frac{1}{\lambda_i} = R \left( \frac{1}{2^2} - \frac{1}{n_i^2} \right)$$

where  $R$  is a constant, and  $n_i = 3, 4, 5, 6$ . The four wavelengths (or "lines") were henceforth called the Balmer lines of hydrogen. **Why** hydrogen emitted only those visible wavelengths and why the wavelengths obeyed the Balmer formula was a complete mystery.

The mystery was solved in 1913 by the Danish physicist Niels Bohr. According to the Bohr model, the electron orbiting the proton in a hydrogen atom can only exist in certain orbital states labeled with a quantum number  $n$  ( $n=1, 2, 3, 4, \dots$ ). When the electron is in orbit  $n$ , the total energy of the hydrogen atom is given by the formula:

$$(3) \quad E_n = -R hc \cdot \frac{1}{n^2} = -\frac{13.6\text{eV}}{n^2},$$

where  $c$  is the speed of light,  $h$  is a constant (Planck's constant), and  $R$  is a number predicted by the Bohr model to be  $R = 1.09737 \times 10^7 \text{ m}^{-1}$ . The different energies  $E_n$  correspond to different orbital states of the electron. Smaller-radius orbits correspond to lower values of  $n$  and lower, more negative, energies. The  $n=1$  state is the lowest possible energy state and is called the ground state.

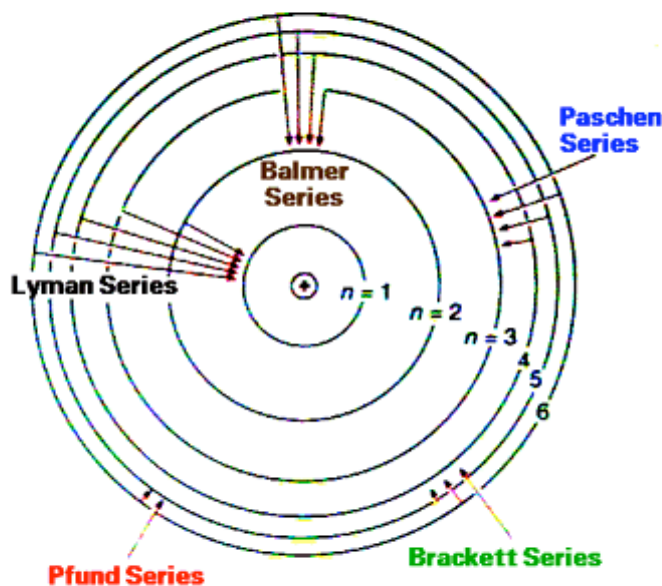
When an electron makes a transition from an initial state of higher energy  $E_i$  to a final state of lower energy  $E_f$ , the atom emits a photon of energy

$$(4) \quad E_\gamma = hf = h \frac{c}{\lambda} = E_i - E_f.$$

Here we have used the expression for the energy of a single photon:  $E = hf$ , where  $h$  is Planck's constant and  $f$  is the frequency of the light. From equations (3) and (4), the wavelength of the emitted photon is related to the initial and final quantum numbers like so:

$$(5) \quad \frac{hc}{\lambda} = E_i - E_f = -R hc \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right), \quad \frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

This is none other than Balmer's formula! Transitions between any pair of states such that  $n_i > n_f$  produces a photon; however, only those transitions with  $n_f = 2$  and  $n_i = 3, 4, 5,$  or  $6$ , happen to produce photons in the visible range of wavelengths.



NOTE: this figure is *not* a picture of the electron orbits of the Bohr model atom; rather it shows the **ENERGY** levels and transitions for various electronic transitions.

**PART I: QUALITATIVE OBSERVATIONS**

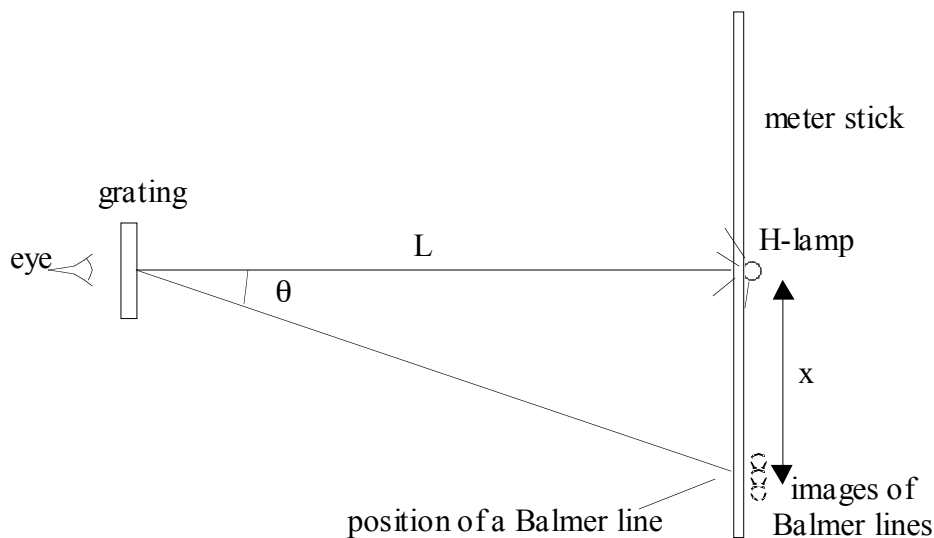
Begin by using the "Project Star" cardboard spectrometers provided to examine **qualitatively** the spectrum of the mercury lamp, the hydrogen lamp, the white light filament, and the overhead fluorescent lights. Record what you see, as descriptively as possible. Note the range of visible wavelengths.

Do these observations make sense with what you saw in class with the diffraction grating? Are there fundamental differences between the different light sources? Which sources appear to be emitting photons from specific transition lines, and which appear to be emitting a blackbody (continuous) spectrum?

## PART II: MEASURING THE $\lambda$ 'S OF THE BALMER LINES IN HYDROGEN

Most people can only see three of the four Balmer lines, because the 4<sup>th</sup> line is faint and very close to the violet edge of the visible spectrum. Look at the hydrogen source through the grating provided. In the first order spectrum ( $m=1$ ), you should clearly see three lines: red, blue-green, and violet (some people can see a second violet line, if the room is dark.) Record what you can see.

Arrange the hydrogen lamp and the diffraction grating on your lab bench, as shown below. Set them on supports to place them at a convenient height for your eye. Position the grating exactly  $L=1.00$  m from the lamp and orient the grating so that it is perpendicular to the line from the lamp (that is, have the grating squarely face the lamp). Place your eye very close to the grating and look through it toward the hydrogen lamp. On **both** sides of the lamp, you should clearly see the images of the first-order Balmer lines. (Be careful to keep the grating facing the lamp. Move your eye position, not the grating, to see the lines to the side of the lamp.)



With a meter stick as close as possible to the lamp, as shown, measure the  $x$ -positions (on **both** sides of the central position) of each of the three lines as accurately as possible. With your measured  $L$  and  $x$ 's, compute the angle  $\theta$  of each of the first-order Balmer lines. Make a table of your results on the next page.

<b>Color</b>	<b><math>x_{hi}</math></b>	<b><math>x_{lo}</math></b>	<b><math>x_{avg}</math></b>	<b><math>\theta_{hi}</math></b>	<b><math>\theta_{lo}</math></b>	<b><math>\theta_{avg}</math></b>
Red (left)						
Red (right)						
Blue/Green (left)						
Blue/Green (right)						
Violet (left)						
Violet (right)						

From the spread in your  $x$  measurements, estimate an error-bar for  $\theta$ . (This can be done simply by calculating  $\theta$  based the largest and smallest possible  $x$  value for each line – including the uncertainty in each measurement and the dual measurements for each line.) Fill these in in the  $\theta_{hi}$ ,  $\theta_{lo}$ , and  $\theta_{avg}$  columns in the table

The number of lines per mm is marked on the grating. From this, you can compute the spacing  $d$  of the grating. Using your measured  $\theta$ 's and computed  $d$ , compute the wavelengths of each of the three Balmer lines.

Using the wavelengths of the three measured Balmer lines, calculate the energy of the photons from each of these lines. Go back to the diagram on page 3, and indicate which transitions correspond to the Balmer lines that you observed, and mark down the corresponding photon energy on the diagram.

Using the same technique as you did to “propagate” the error in  $\theta$ , estimate the error in each of the computed wavelengths. State your three measured wavelengths here, including the error bar (i.e.  $\lambda = 555 \pm 4$  nm). Don't include significant figures that are irrelevant based on your error-bar.

**PART III: MYSTERY ATOM**

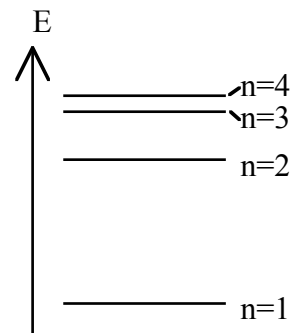
Now that you have worked with spectrometers, identified spectral lines, and correlated them with the emission fingerprint of the Hydrogen atom, you can use a similar technique to determine the identity of an unknown element. Get from your TA a lamp filled with an unknown gas (labeled A, B, C, or some other cryptic label), and use the spectrometer to record the wavelengths of the brightest few lines. Also write down the (encoded) label from the gas tube.

Using your identified emission lines and the chart of common “spectral fingerprints”, identify the gas (or gases) in your lamp tube.

Based on the estimated error in your spectral line identification, how confident are you that you identified your element correctly? Namely, are there any other spectral fingerprints that are similar to the element that you identified? If so, how might you narrow down your choice?

**POTENTIAL EXAM QUESTIONS:**

1. A partial hydrogen atom energy level diagram is shown. How many different lines in the spectrum of hydrogen correspond to transitions among the bottom four energy levels ( $n=1,2,3$ , and  $4$ )?



- a) 3  
 b) 4  
 c) 6  
 d) 12  
 e) None of the above
2. Referring to the diagram above, consider the following four transitions in the hydrogen atom:  $n=2 \rightarrow n=1$ ,  $3 \rightarrow 2$ ,  $4 \rightarrow 3$ ,  $4 \rightarrow 1$ . How do the wavelengths of the photons emitted by each of these transitions compare? Order the emitted photons from shortest wavelength to longest wavelength.

- a) (shortest  $\lambda$ )     $2 \rightarrow 1$              $3 \rightarrow 2$              $4 \rightarrow 3$              $4 \rightarrow 1$  (longest  $\lambda$ )  
 b)                     $4 \rightarrow 1$              $4 \rightarrow 3$              $3 \rightarrow 2$              $2 \rightarrow 1$   
 c)                     $4 \rightarrow 3$              $3 \rightarrow 2$              $2 \rightarrow 1$              $4 \rightarrow 1$   
 d)                     $4 \rightarrow 1$              $2 \rightarrow 1$              $3 \rightarrow 2$              $4 \rightarrow 3$   
 e)                     $3 \rightarrow 2$              $4 \rightarrow 3$              $4 \rightarrow 1$              $2 \rightarrow 1$