Wave Nature of Light

Light is a wave, an electromagnetic wave. The wavelength $\lambda$ of visible light is very small.

Visible light: $\lambda = 400 \text{ nm (violet)} \rightarrow 700 \text{ nm (red)}$

$$c = \lambda f, \quad f = \frac{c}{\lambda}, \quad \lambda = \frac{c}{f}$$

Wave-like effects are difficult to detect because of the small wavelength. In many situations, light behaves like a ray, exhibiting no obvious wave-like behavior.

Light passing through hole in wall:

Newton (late 1600's) did not believe that light was a wave since he always observed ray-behavior. Wave-like behavior was not clearly observed until around 1800.

Review of Constructive/Destructive interference of Waves:

Consider 2 waves, with the same speed $v$, the same wavelength $\lambda$, (and therefore same frequency $f = \frac{c}{\lambda}$), traveling in the same (or nearly the same) direction, overlapping in the same region of space:

If the waves are in phase, they add $\Rightarrow$ constructive interference

If the waves are out of phase, they subtract $\Rightarrow$ destructive interference
If wave in nearly the same direction:

**Huygen's Principle:** Each point on a wavefront (of given \( f, \lambda \)) can be considered to be the source of a spherical wave.

To see interference of light waves, you need a monochromatic (single \( \lambda \)) light source, which is coherent (nice, clean plane wave). This is not easy to make. Most light sources are incoherent (jumble of waves with random phase relations) and polychromatic (many different wavelengths).

**Young's Double slit experiment (1801):**

What do you expect to see on the screen? If you believe light is a ray, then you expect to see 2 bright patches on the screen, one patch of light from each slit.
But here is what you actually see:
A series of bright and dark fringes:
wave interference

How do we explain this?  Consider the 2 slits as 2 coherent point sources of monochromatic light.  Two sources are coherent is they have the same wavelength $\lambda$ (and therefore the same frequency $f$ ) and they emit peaks and troughs in sync, in phase.

Each slit (source) emits light in all forward directions, but let us consider only the parts of the waves heading toward a particular point on the screen.

If the screen is far away ($L >> d$), then the rays from the two slits to the same point on the screen are nearly parallel, both heading in the same direction, at the same angle $\theta$.

The ray from the lower slit has to travel further by an extra distance ($d \sin\theta$) to reach the screen. This extra distance is called the path difference. When the path difference (p.d.) is one full wavelength, or 2 full wavelengths, or an integer number of wavelengths, then the waves will
arrive in phase at the screen. There will be constructive interference and a bright spot on the screen.

\[ \text{p.d.} = d \sin \theta = m \lambda , \quad m = 0, 1, 2, \ldots \] (constructive interference)

But if the path difference is \( \frac{1}{2} \) wavelengths or \( \frac{3}{2} \) wavelengths, etc, then there will be destructive interference at the screen and the screen will be dark there.

\[ \text{p.d.} = d \sin \theta = (m + \frac{1}{2}) \lambda , \quad m = 0, 1, 2, \ldots \] (destructive interference)

Notice that the formula \( \text{p.d.} = d \sin \theta \) is NOT a definition of path difference. It is a formula for path difference in a specific situation, namely when the screen is "at infinity". The definition of path difference is this: \( \text{p.d.} = (\text{distance to one source}) - (\text{distance to the other source}) \)

A plot of brightness (intensity) vs. angle position on the screen:

Maxima at angles where \( \sin \theta = m \frac{\lambda}{d} \cong \theta \) (rads) [Recall \( \sin \theta \cong \theta \) (rads) if \( \theta \ll 1 \)]

Young's experiment was the first real proof that light is a wave. If you believe that light is a ray, there is no way to explain the destructive interference seen on the screen. In the ray-view, when you hit a screen with two rays, the brightness of the 2 rays always adds and you see a bright spot there. It is impossible to explain destructive interference of two light sources, unless you admit that light is a wave.

**Single Slit Diffraction**

"Diffraction" = interference due to infinitely-many sources packed infinitely close via Huygen's Principle. Huygen's Principle says that a slit that is illuminated by a plane wave can be consider to be filled with an array of coherent point sources.
Consider the light from just two of the infinitely-many sources: one at the top of the slit, and one exactly in the middle of the slit. When the path difference between these two sources and the screen is \( \frac{1}{2} \) wavelength, that is, when \( \frac{D}{2} \sin \theta = \frac{\lambda}{2} \), then the light from these two sources interfere destructively and no light from those two sources illuminates the screen at that particularly angle \( \theta \).

But notice that all the sources can grouped in pairs, with each pair's members \( D/2 \) apart. The light from all the sources (the entire slit) cancel in pairs, and there is no light at the position on the screen at the angle \( \theta \) such that

\[
\frac{D}{2} \sin \theta = \frac{\lambda}{2}, \quad \text{or} \quad D \sin \theta = \lambda.
\]

The angle \( \sin \theta = \lambda / D \) is the first intensity minimum on the screen. The intensity pattern on the screen looks like this:

The angular width of the "central maximum" is \( \Delta \theta \text{(rads)} = \frac{2\lambda}{D} \). Notice that in the limit, \( D \rightarrow \lambda \) (slit width becomes as small as the wavelength of light), the central max becomes so broad, that we get spherical wave behavior.
Diffraction Grating

A diffraction grating is an array of many narrow slits with a uniform inter-slit spacing \( d \). A grating with "500 lines per cm" has a slit separation of

\[
d = \frac{1 \text{ cm}}{500} = 0.002 \text{ cm} = 0.02 \text{ mm} = 20 \mu\text{m}
\]

A typical diffraction grating has thousands of slits. With exactly the same argument we used in the double-slit case, we see that maximum brightness occurs when

\[
p.d. = d \sin \theta = m \lambda , \quad m = 0, 1, 2, ...
\]

The maxima occur at the same angles as with a double slit of the same \( d \), but the peaks are much sharper and much brighter.

As \( N \) (the number of slits) increases, the width of each peak decreases. Why? With just 2 slits, when we are near the maximum at the angle \( \theta = \lambda / d \), then the waves from the two slits are nearly in phase and we have nearly complete constructive interference and nearly maximum brightness. But with \( N \)-slits, when we are near the angle \( \theta = \lambda / d \), any two adjacent slits are nearly in phase, but the next slit over is a little more out of phase, and the next one over is even more out of phase. With many slits, if you are just a bit off the special angle for maximum brightness, the phase differences among the slits quickly add up and gives destructive interference.
Another nice feature of the grating is that, with many slits for the light to get through, the pattern on the screen is brighter than in the double-slit case.

The diffraction grating is very useful because the angle of the peaks depends on the wavelength $\lambda$. Measuring the angle of the peak give a direct measurement of the wavelength of the light.

The $m = 1$ peak is at $\sin \theta = \frac{\lambda}{d}$. Distance $x$ on screen is related to angle $\theta$ by $\tan \theta = \frac{x}{L}$.

If $\theta$ is small ($<< 1$ rad), then $\sin \theta \approx \tan \theta \approx \theta \Rightarrow \frac{\lambda}{d} \approx \frac{x}{L} \Rightarrow x \approx L \frac{\lambda}{d}$ (for $m=1$).

Notice that (if $\theta$ small) the position $x$ on the screen is proportional to the wavelength $\lambda$: $x \propto \lambda$ [If $\theta$ is not small, we must use exact relation: $x = L \tan \theta = L \tan \left( \sin^{-1} \left( \frac{\lambda}{d} \right) \right)$].

With a grating spectrometer, one can determine the spectral composition of a multi-wavelength light source.

In a spectrometer, we want a large spread ("dispersion") of the colors so that we can easily distinguish close wavelengths. This means we want a big $x$, small $d$. Small $d$ means large number of "lines per mm" in the grating.
Review of angle measure in radians

Definition of angle in radians: \[ \theta \text{(rads)} = \frac{s}{R} \text{ (dimensionless)} \]

A circle of radius R, arc length s along the rim of the circle. Notice that for a given angle \( \theta \), the ratio \( s/R \) is independent of the size of the circle.

Example: How many radians in 180°?

Answer: Circumference \( C = 2\pi R \), \( \theta = \frac{s}{R} = \frac{\pi R}{R} = \pi \text{ rads} \)
\( \pi \text{ rads} = 180^\circ \), 1 rad = 57.3°

The small angle approximation

For small \( \theta \) (\( \theta \ll 1 \text{ rad} \)), \( \theta \text{(in radians)} \cong \sin \theta \cong \tan \theta \)

\[ \theta = \frac{s}{R} \cong \frac{x}{R} = \tan \theta \]
\[ \sin \theta = \frac{x}{R + \delta} \cong \frac{x}{R} \cong \frac{s}{R} = \theta \]

<table>
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<th>angle</th>
<th>10°</th>
<th>5°</th>
<th>1°</th>
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<td>( \theta \text{(rads)} )</td>
<td>0.174533</td>
<td>0.087266</td>
<td>0.017453</td>
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<tr>
<td>\sin \theta</td>
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<td>\tan \theta</td>
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</tbody>
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When describing the size of a feature (such as the width of the central max in single slit diffraction), sometimes we give its *angular width* (in radians or degrees), and the sometimes we give its *linear size* on the screen (in mm or cm). The relation between angular width $\theta$ and linear size $x$ on the screen is seen here:

If $\theta$ is small, then $\theta \approx \frac{x}{L}$. When dealing with single- and double-slit patterns, the angles are usually much less than $1^\circ$ and the small angle approximation is valid. However, when dealing with diffraction gratings, the angles can be large ($40^\circ$ or more), and we must then use exact relations. Diffraction gratings generally produce larger angles because they usually have a very small slit spacing $d$.

$$\tan \theta = \frac{x}{L} \approx \theta \Rightarrow$$

Approximate: $x = L \theta$

Exact: $x = L \tan \theta$
Some loose ends:

Prisms can also be used as spectrometers (device used to measure the spectrum of light). A prism bends different colors of light through different angles, because the index of refraction $n$ of glass depends on the wavelength. The index $n$ is larger for shorter wavelengths, and therefore shorter-wavelength light is bent more.

The dependence of $n$ on $\lambda$ is called dispersion, since a prism disperses the colors.