Voltage ( = Electric Potential )

An electric charge alters the space around it. Throughout the space around every charge is a vector thing called the electric field. Also filling the space around every charge is a scalar thing, called the voltage or the electric potential. Electric fields and voltages are two different ways to describe the same thing.

(Note on terminology: The text book uses the term "electric potential", but it is easy to confuse this with "potential energy", which is something different. So I will use the term "voltage" instead.)

Qualitative description of voltage

The voltage at a point in empty space is a number (not a vector) measured in units called volts (V is the abbreviation for volts). Near a positive charge, the voltage is high. Far from a positive charge, the voltage is low. Voltage is a kind of "electrical height". Voltage is to charge like height is to mass. It takes a lot of energy to place a mass at a great height. Likewise, it takes a lot of energy to place a positive charge at a place where there voltage is high.

The electric field is related to the voltage in this way: Electric field is the rate of change of voltage with distance. E-field is measured in units of N/C, which turn out to be the same as volts per meter (V/m). E-fields points from high voltage to low voltage. Where there is a big E-field, the voltage is varying rapidly with distance.

Mathematically, we write this as $E = \frac{\Delta V}{\Delta x}$ or $|\Delta V| = E |\Delta x|$.

(This equation assumes that the E-field is along the x-axis and that $E = \text{constant}$)

The technical definition of voltage involves work and potential energy, so we review these first.

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**Review of Work and PE**

**Definition of work done by a force:** consider an object pulled or pushed by a force \( \vec{F} \). While the force is applied, the object moves along some axis (x-axis, say) through a displacement of magnitude \( \Delta x \).

![Diagram showing work](image)

Notice that the direction of displacement is not the same as the direction of the force, in general.

Work done by a force \( F = W \equiv F_x \cdot \Delta x = F \cos \theta \Delta x = F_{||} \Delta x \)

\( F_{||} = \) component of force along the direction of displacement

Work is not a vector, but it does have a sign (+) or (-). Work is positive, negative, or zero, depending on the angle between the force and the displacement.

\[
\begin{align*}
\theta < 90, & \quad W \text{ positive} \\
\theta = 90, & \quad W = 0 \\
\theta > 90, & \quad W \text{ negative}
\end{align*}
\]

**Definition of Potential Energy PE:** The change in potential energy \( \Delta PE \) of a system is equal to the work done by an external agent (assuming no friction and no change in kinetic energy)

\[
\Delta PE = W_{\text{ext}}
\]

This is best understood with an example: A book of mass \( m \) is lifted upward a height \( h \) by an "external agent" (a hand which exerts a force external to the field). In this case, the work done by the hand is \( W_{\text{ext}} = \text{force} \times \text{distance} = +mgh \). The change in the potential energy of the book is \( \Delta PE = W_{\text{ext}} = +mgh \). The work done by the external agent went into the increased

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gravitational potential energy of the book. (The initial and final velocities are zero, so there was no increase in kinetic energy.)

**Quantitative description of voltage**

We define *electrostatic potential energy* in the same way as we defined gravitational potential energy, with the relation \( \Delta \text{PE} = W_{\text{ext}} \). Consider two parallel metal plates (a capacitor) with equal and opposite charges on the plates which create a uniform electric field between the plates. The field will push a charge \( +q \) down toward the negative plate with a constant force of magnitude \( F = q E \). (The situation is much like a mass in a gravitational field, but there is no gravity in this example.) Now imagine grabbing the charge with tweezers (an external agent) and *lifting* the charge \( +q \) a distance \( \Delta y \) against the electric field toward the positive plate. By definition, the change in electrostatic potential energy of the charge is \( \Delta \text{PE} = W_{\text{ext}} = \text{force} \times \text{distance} \Rightarrow \)

\[
\Delta \text{PE} = +q E \Delta y
\]

This formula assumes that the \( E \)-field is in the \( -y \) direction. But don't try to get the signs from the equations – it's too easy to get confused. Get the sign of \( \Delta \text{PE} \) by asking whether the work done by the external agent is positive or negative and apply \( \Delta \text{PE} = W_{\text{ext}} \).

Now we are ready for the definition of voltage difference between two points in space. Notice that the increase in PE of the charge \( q \) is proportional to \( q \), so the ratio \( \Delta \text{PE}/q = E \Delta y \) is independent of \( q \). Recall that electric field is defined as the force *per* charge: \( \vec{E} \equiv \frac{\vec{F}_{\text{on q}}}{q} \).

Similarly, we define the voltage difference \( \Delta V \) as the change in PE *per* charge:

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We showed above that $\Delta PE = + q E \Delta y$, so $\Delta V = \frac{\Delta PE}{q} = \frac{q E \Delta y}{q} = E \Delta y$. Again, this formula assumes that the E-field is along the $-y$ direction, but don't try to get the signs from the equations – it's too easy to get confused. Instead, just remember that the E-field always points from high voltage to low voltage:

$$|\Delta V| = E |\Delta x|$$

(if E-field = constant and is along the $x$-axis)

To say that "the voltage at a point in space is $V$" means this: if a test charge $q$ is placed at that point, the potential energy of the charge $q$ (which is the same as the work required to place the charge there) is $PE = q V$. If the charge is moved from one place to another, the change in $PE$ is $\Delta PE = q \Delta V$. Only changes in $PE$ and changes in $V$ are physically meaningful. We are free to set the zero of $PE$ and $V$ anywhere we like.

Units of voltage = [V] = \frac{\text{energy}}{\text{charge}} = \frac{\text{joule}}{\text{coulomb}} = \text{volt (V)}. \quad 1 \text{ V} = 1 \text{ J/C}

**Voltage near a point charge**

(This is hard to compute, since $E$ = constant. Need calculus. See appendix for math details.)

$$V = \frac{kQ}{r}$$

Answer: $V(r) = \frac{kQ}{r}$

Notice that this formula gives $V = 0$ at $r = \infty$. When dealing with point charges, we always set the zero of voltage at $r = \infty$.
What does a graph of voltage vs. position look like?

![Graph of voltage vs. position](image)

- **V near (+) charge is large and positive.**
- **V near (–) charge is large and negative.**

If we have several charges \( Q_1, Q_2, Q_3, \ldots \), the voltage at a point near the charges is

\[
V_{\text{tot}} = V_1 + V_2 + V_3 + \ldots
\]

(from \( \Delta P E_{\text{tot}} = W_{\text{tot}} = W_1 + W_2 + \ldots \))

Voltages add like numbers, not like vectors.

What good is voltage?

- Much easier to work with V's (scalars) than with \( \vec{E} \)'s (vectors).
- Easy way to compute PE.

**Voltage example:** Two identical positive charges are some distance \( d \) apart. What is the voltage at point \( x \) midway between the charges? What is the E-field midway between the charges? How much work is required to place a charge \( +q \) at \( x \)?

\[
V_{\text{tot}} = V_1 + V_2 = \frac{kQ}{(d/2)} + \frac{kQ}{(d/2)} = \frac{2kQ}{d}
\]

The E-field is zero between the charges (Since \( \vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 = 0 \). Draw a picture to see this!)
The work required to bring a test charge +q from far away to the point x is positive, since it is hard to put a (+) charge near two other (+) charges. You have to push to get the +q in place. The work done is

\[ W_{ext} = \Delta PE = +q \Delta V, \quad \text{where} \quad \Delta V = V_{\text{final}} - V_{\text{initial}} = V(x) - V(\infty) = \frac{4kQ}{d} \]

**Units of electron-volts (eV)**

The SI units of energy is the joule (J). 1 joule = 1 newton-meter. Another, non-SI unit of energy is the electron-volt (eV), often used by chemists. The eV is a very convenient unit of energy to use when working with the energies of electrons or protons.

From the relation \( \Delta PE = q \Delta V \), we see that energy has the units of charge \( \times \) voltage. If the charge \( q = 1 \text{ e} = |\text{charge of the electron}| \) and \( \Delta V = 1 \text{ volt} \), then \( \Delta PE = q \Delta V = 1 \text{ e} \times 1\text{ V} = \) a unit of energy called an "eV". Notice that the name "eV" reminds you what the unit is: it's an "e" times a "V" = 1 e \( \times \) 1 volt.

How many joules in an eV? 1 eV = 1 e \( \times \) 1V = \((1.6 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.6 \times 10^{-19} \text{ J}\)

If \( q = e \) (or a multiple of e), it is easier to use units of eV instead of joules when computing \( \text{(work done)} = \text{(change in PE)} \).

**Example of use of eV.** A proton, starting at rest, "falls" from the positive plate to the negative plate on a capacitor. The voltage difference between the plates is \( \Delta V = 1000 \text{ V} \). What is the final KE of the proton (just before it hits the negative plate)?

As the proton falls, it loses PE and gains KE.

\[ |\Delta KE| = |\Delta PE| = |q \Delta V| = 1\text{e} \times 1000 \text{ V} = 1000 \text{ eV} \]
"Equipotential Lines" = constant voltage lines

For a constant E-field, we showed before that $|\Delta V| = E |\Delta x|$, but this is only true if the E-field is parallel (or anti-parallel) to the displacement $\Delta \mathbf{x}$. If we move in a direction perpendicular to the E-field, the voltage does not change: if $\Delta \mathbf{x} \perp \mathbf{E}$, then $\Delta V = 0$. Why? Recall the definition

$$\Delta V \equiv \frac{\Delta \text{PE}_{\text{of } q}}{q} = \frac{W_{\text{ext}}}{q} = \frac{F_x \Delta x}{q} = -\frac{q E_x \Delta x}{q} = -E_x \Delta x,$$

where $E_x = E_{\parallel}$ is the component of the E-field along the movement, which we call the x-direction. If the signs confuse you, remember "the hi-voltage people look down their electric field noses at the low-voltage people". Anyway $|\Delta V| = |E_x| |\Delta x|$. Only the component of the E-field along the displacement involves non-zero work and produces a change in voltage.

Equipotential (constant voltage) lines are always at right angles to the electric field.

![Diagram of equipotential lines and electric field](image-url)