Magnetism: a new force!

So far, we've learned about two forces: gravity and the electric field force.

\[ \vec{E} = \frac{\vec{F}_E}{q}, \quad \vec{F}_E = q \vec{E} \quad \leftarrow \text{Definition of E-field} \]

- E-fields are created by charges: \( |\vec{E}| = \frac{kQ}{r^2} \)
- E-field exerts a force on other charges: \( \vec{F}_E = q \vec{E} \).

The gravitational force is similar:

- Gravitational fields are created by mass: \( |\vec{g}| = \frac{GM}{r^2} \).
- The gravitational field exerts a force on other masses: \( \vec{F}_{\text{grav}} = m \vec{g} \).

There is a different kind of field, called a magnetic field or B-field.

- B-fields are created by moving charges (currents).
- B-fields exert forces on moving charges.

The force \( \vec{F}_B \) from a B-field on a moving charge depends on the velocity of the charge in a peculiar way: a charge \( q \), moving with velocity \( \vec{v} \) in a magnetic field \( \vec{B} \), feels a force of magnitude

\[ |\vec{F}_B| = F_B = |q| v B \sin \theta = |q| v_{\perp} B \]

If \( \vec{v} \perp \vec{B} \), then \( \sin \theta = 1 \Rightarrow F_B = |q| v B \)
If \( \vec{v} \parallel \vec{B} \), then \( \sin \theta = 0 \Rightarrow F_B = 0 \)
If \( v = 0 \), then \( F_B = 0 \) (unlike gravity or E-field force)

The direction of the force \( \vec{F}_B \) is perpendicular to the plane formed by \( \vec{v} \) and \( \vec{B} \). Direction of \( \vec{F}_B \) is determined by the "Right-hand rule". Use your right hand. Point your fingers in the direction...
of \( \vec{v} \), curl your fingers toward \( \vec{B} \). Thumb then points in the direction of \( \vec{F}_B \) if the charge \( q \) is positive. (Orient hand so that fingers curl thru angle < 180°). If \( q \) is (-), \( \vec{F}_B \) is other way

![Diagram of magnetic force](image)

Units of \( B \): \([B] = [\mathbf{F}] = \frac{[\mathbf{F}]}{[q][v]} = \frac{N}{\text{C} \cdot \text{m/s}} = 1 \text{ tesla (T)}

Older, non-SI, unit of \( B \): 1 gauss = \( 10^{-4} \) T, 1 T = \( 10^4 \) gauss

- Earth's magnetic field \( \approx 0.5 \text{ gauss} = 5 \times 10^{-5} \) T
- kitchen magnet: 50 – 500 gauss = 0.005 – 0.05 T
- iron core electromagnet: 2 T (max) (Strong enough to yank tools out of your hand.)
- superconducting magnet: 20 T (max)

**Currents make B-fields**

You make a B-field with a current \( I \). (Charges make E-fields, currents make B-fields).

The **B-field produced by a long straight wire**, carrying current \( I \), has magnitude

\[
B = \frac{\mu_0 I}{2\pi r}
\]

\( r \) = distance from wire, \( \mu_0 \) = constant = \( 4\pi \times 10^{-7} \) (SI units)

(This formula can be derived from a fundamental law called Ampere's Law, which relates currents and B-fields.)
The direction of the B-field is perpendicular to the direction of the current.

The B-field lines form circles around the current, according to "Right-hand Rule II": Point thumb of RH along current I, fingers curl along direction of B-field.

A loop of wire carrying a current I forms a rather complicated B-field:

B-field lines are fundamentally different from E-field lines in this way: E-field lines begin and end on charges (or go to $\infty$). But B-field lines always form closed loops with no beginning or end.

It is possible to make a uniform, constant B-field with a solenoid = cylindrical coil of wire:
Motion of a charged particle in magnetic field

Consider a charge q moving in a uniform magnetic field $B$. Since the force $F_B$ is always perpendicular to the velocity $v$, the force $F_B$ does no work: $F_B \cdot \vec{v} \Rightarrow W_{v_B} = 0$. The magnetic force cannot change the KE or PE of the particle (since Work done = $\Delta KE + \Delta PE$). The B-field changes the direction of the velocity $v$, but does not change the speed. If the velocity $v$ is perpendicular to the field $B$, the magnetic force bends the path of the particle in a circle.

We can relate the radius $R$ of the circular path to the magnitude of the field $B$ and the speed $v$ with Newton's Second Law:

$$|\vec{F}_{net}| = m|\vec{a}| \Rightarrow qvB = \frac{mv^2}{R} \quad (\text{recall that for circular motion } |\vec{a}| = \frac{v^2}{R})$$

Solving for $R$, we get $R = \frac{mv}{qB}$. Notice that the radius is proportional to the mass of the particle. In a mass-spectrometer, the mass of an unknown particle is determined from measurement of the radius (charge, speed and B-field are all known).

The Velocity Selector

The velocity selector is a device which measures the speed $v$ of an ion. (ion = charged atom with one or more electrons missing). A magnet produces a uniform B-field and a capacitor produces a uniform E-field, with $E \perp B$. 

Last update: 3/1/2006
The B and E fields are adjusted until the particle goes straight through. If the path is straight, then $F_B = F_E \Rightarrow qvB = qE \Rightarrow v = \frac{E}{B}$.

**Magnetic force on a current-carrying wire**

A B-field exerts a force on a moving charge. A current-carrying wire is full of moving charges, so a B-field exerts a force on the current-carrying wire.

Recall that $F_{on\_q} = qv_B = qvB\sin\theta$.

We will show below that this leads to $F_{on\_wire} = I L B \sin\theta$ where $I$ is the current, $L$ is the length of the wire, and $\theta$ is the angle between the direction of $B$ and the direction of the current.

Proof of $F_{on\_wire} = I L B$: Consider a section of wire, length $L$, carrying $N$ moving charges.

Assume $B \perp$ wire.

speed of moving charges $= v = \frac{L}{t}$

current $I = \frac{Nq}{t}$

$$F_{on\_wire} = N \cdot F_{on\_q} = NqvB = \frac{Nq}{t} \cdot L B = I L B \quad \text{(done)}$$
Current-carrying wires exert magnetic forces on each other. Wire 1 creates a B-field 
\[ B = \frac{\mu_0 I}{2\pi r} \]. Wire 2 feels a force due to the B-field from wire A \( F_{\text{on wire}} = I_1 L B \).

Parallel currents attract,  
Anti-parallel currents repel.  
(Can you see why? Use Right hand rule.)

**Permanent Magnets**  
Currents make B-fields. So where's the current in a permanent magnet (like a compass needle)?

An atom consists of an electron orbiting the nucleus. The electron is a moving charge, forming a tiny current loop — an "atomic current".

In most metals, the atomic currents of different atoms have **random** orientations, so there is no net current, no B-field.

In **ferromagnetic** materials (Fe, Ni, Cr, some alloys containing these), the atomic currents can all line up to produce a large net current.

In a magnetized iron bar, all the atomic currents are aligned, resulting in a large net current around the rim of the bar. The current iron bar then acts like a solenoid, producing a uniform B-field inside:
Why do permanent magnets sometimes attract and sometimes repel? Because parallel currents attract and anti-parallel current repel.

**opposite poles attract:**

\[
\begin{array}{c}
S \\ N \\
\end{array}
\quad |
\begin{array}{c}
S \\ N \\
\end{array}
\]

parallel currents on ends attract

**like poles repel:**

\[
\begin{array}{c}
S \\ N \\
\end{array}
\quad |
\begin{array}{c}
N \\ S \\
\end{array}
\]

anti-parallel currents on ends repel