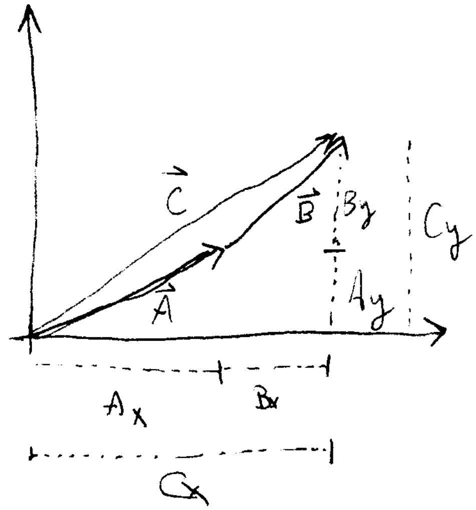
P2010 LECTURE 6 VECTORS cont'd.

Components make addition very easy:

$\vec{C} = \vec{A} + \vec{B}$:

$C_x = A_x + B_x$

$C_y = A_y + B_y$



Vector subtractions: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Graphically: $\vec{A}$ $\vec{B}$ $-\vec{B}$

$\vec{A} - \vec{B} = \vec{D}$:

$\vec{B} + \vec{D} = \vec{A}$

Subtraction by components: $D_x = A_x - B_x$

$D_y = A_y - B_y$

Vectors in physics:

POSITION: $\vec{r} = (x, y)$ by components

VELOCITY: $\vec{v} = (v_x, v_y)$

ACCELERATION: $\vec{a} = (a_x, a_y)$

where $v_x = \frac{\Delta x}{\Delta t}$, $v_y = \frac{\Delta y}{\Delta t}$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \underbrace{\frac{1}{\Delta t}}_{\text{scalar}} \underbrace{\Delta \vec{v}}_{\text{vector}}$$

(Note - time is a scalar)

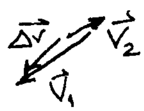
Therefore, $a_x = \frac{\Delta v_x}{\Delta t}$

$a_y = \frac{\Delta v_y}{\Delta t}$

and $\vec{a}$ has the same direction as $\Delta \vec{v}$.

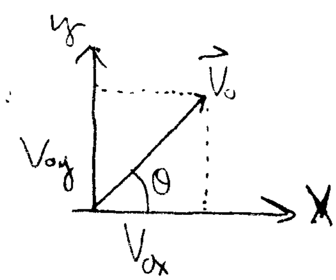
Say $\vec{V}_1 = \nearrow$ $\quad \vec{V}_2 = \rightarrow$

$\vec{V}_1 + \Delta \vec{v} = \vec{V}_2$:



so $\vec{\Delta} v$ DOESN'T have to be in same direction as $\vec{V}_1, \vec{V}_2$.

Motion in 2D: velocity, acceleration vectors can be in any direction. Example: cannonball fired at 45°:



$V_{0x} = V_0 \cos\theta = \frac{1}{\sqrt{2}} V_0 \quad (\cos 45° = \frac{1}{\sqrt{2}})$

$V_{0y} = V_0 \sin\theta = \frac{1}{\sqrt{2}} V_0 \quad (\sin 45° = \frac{1}{\sqrt{2}})$

Acceleration can have any direction, magnitude. But in special case of <u>freefall</u>, it's always straight down:

$\downarrow \vec{a}, \; |\vec{a}| = g \qquad a_x = 0$

$a_y = -g$

Recall in 1D: special case where $a =$ constant:

1) $v = v_0 + at$
2) $x = x_0 + v_0 t + \frac{1}{2} a t^2$
3) $v^2 = v_0^2 - 2a(x - x_0)$

even more special case: $a = 0$, $\frac{\Delta x}{\Delta t}$ constant $= v_0$

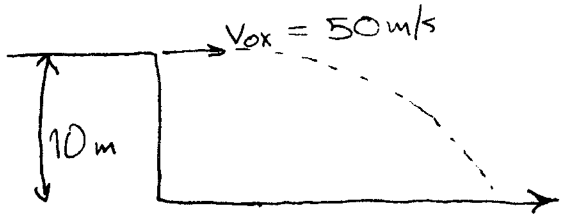
1) $v = v_0$
2) $x = x_0 + v_0 t$

In 2D, can apply these formulae to each component:

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$
$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \qquad y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

So we can treat separately the motion in x and y.
This is very useful in these free-fall problems.

Take a cannonball fired horizontally off a cliff:



How long in the air?
How far will it go before hitting ground?

Look at y component: $v_{0y} = 0$, $a_y = -g$

x component: $v_{0x} = 50 \frac{m}{s}$, $a_x = 0$

Separate the components:

Y	X
$Y_0 = 10\,m$	$X_0 = 0$
$V_{oy} = 0$	$V_{ox} = 50\,\frac{m}{s}$ (kind of slow for a bullet)
$a_y = -g \cong -10\,\frac{m}{s^2}$	$a_x = 0$

→ Why is a_x zero? Force of gravity is only vertical. Ignoring air resistance, there's no change in V_x.

How long is bullet in the air? We must solve for time to hit the ground. Use const. accel. equ. 2; call impact time t_f.

$$\underbrace{y}_{=0} = \underbrace{y_0}_{=10m} + \underbrace{V_{oy}}_{=0} t - \frac{1}{2} g t^2 \quad t^2 \to t_f^2$$

$$0 = y_0 - \frac{1}{2} g t_f^2$$

$$\frac{1}{2} g t_f^2 = y_0$$

$$t_f^2 = \frac{2y_0}{g}$$

$$t_f = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2 \cdot 10m}{10\frac{m}{s^2}}} = \sqrt{2 s^2} = \underline{\underline{1.41 \text{ seconds}}}$$

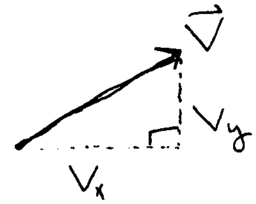
Where in X is it when hits ground?

$$X_f = \underbrace{X_0}_{0} + V_{ox} t_f + \frac{1}{2} \underbrace{a_x}_{0} t_f^2$$

$$X_f = V_{ox} t_f = \left(50\tfrac{m}{s}\right)(1.41\,s) = 71 \text{ meters.}$$

What is the <u>speed</u> of bullet as it falls? Recall speed is magnitude of velocity vector:

$$s = |\vec{v}|$$



Given v_x, v_y, what is $|\vec{v}|$? Components form a right triangle with the vector, so use Pythagorean Theorem:

$$|\vec{v}|^2 = v_x^2 + v_y^2$$

or $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$ works for any vector, not just velocity.

$v_x = v_{0x}$ constant: $v_x^2 = v_{0x}^2$ constant

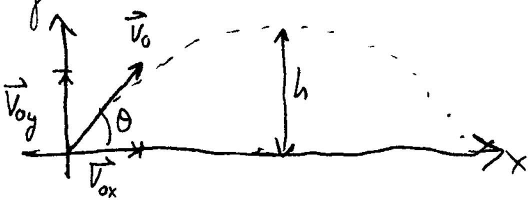
$v_y = \underbrace{v_{0y}}_{0} - gt = -gt$: $v_y^2 = g^2 t^2$ grows.

So speed $s = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + g^2 t^2}$

is minimum at $t = 0$ (i.e. when $v_y = 0$)

is v_{0x} at $t = 0$

Now, take a projectile fired upwards with speed v_0 at angle θ:



Initial velocity $\vec{v}_0$ has magnitude $|\vec{v}_0| = v_0$

v_0 = initial speed.

$\vec{v}_0 = \vec{v}_{0x} + \vec{v}_{0y}$: Components can be vectors too!

But remember that components $\neq$ their magnitude:

$$|\vec{V}_{oy}| \neq V_{oy} \quad \text{since } V_{oy} \text{ can be } + \text{ or } -,$$

$$\text{but } |\vec{V}_{oy}| \geq 0 \text{ always.}$$

What is the maximum height h? Only depends on V_{oy}, since x motion is separate.