

Moving on to 2 dimensions. Need to introduce a new mathematical concept: VECTOR: Quantity with magnitude (size) and direction.

Think how velocity (in 1D) differs from speed. Speed tells only how fast you're going, but velocity tells you which direction as well. [In 1D, there are only two directions (left & right, or fwd & backward, or + and -).] Velocity is therefore a VECTOR.

In 2D, represent vector by an arrow:



length of arrow = magnitude of vector  
pointing direction = direction of vector

Vectors in physics: displacement (position), velocity, acceleration

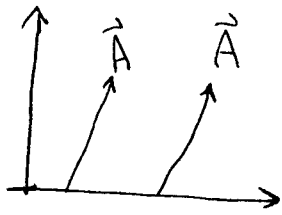
Notation: Handwritten vector:  $\vec{A}$

Printed vector: **A** boldface, not italic  
(Note - your text goes all-out and writes  $\vec{A}$ )

Magnitude:  $A$  (italic, not bold, no arrow) or  $|\vec{A}|$

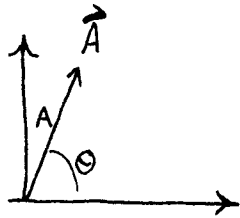
Note that vector magnitude is never negative.  
(can be zero)

Vector does not specify its location:



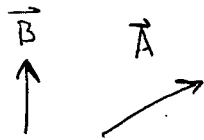
Same magnitude, direction = same vector.  
 (But "position vectors" are understood relative to a specific origin)

In 2D, need 2 numbers to specify a vector:

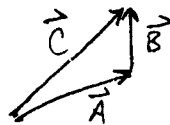


Magnitude  $A$ , direction angle  $\theta$  (always counterclockwise, usually from x axis)

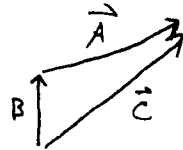
Vectors can be added:



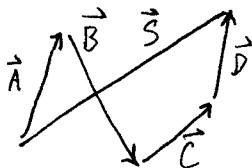
"tail-to-head" method



$$\vec{C} = \vec{A} + \vec{B}$$



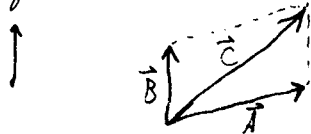
$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \quad \text{addition is commutative}$$



$$\vec{S} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

(Something like a treasure map: "20 steps north, 12 east,...")

Another way to add vectors: parallelogram method.



$$\vec{C} = \vec{A} + \vec{B} \quad \text{same result as tail-to-head.}$$

Can multiply a vector by a scalar (number):

- Magnitude is multiplied:  $|3\vec{A}| = 3A$

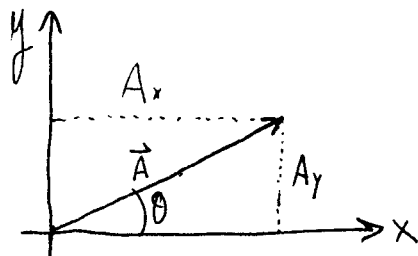
- Direction is unaffected if scalar positive  
reversed if scalar negative:

Negative of a vector: same magnitude, opposite direction.

Subtracting vectors = adding negative:  $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

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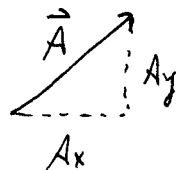
Components of a vector: "projections" of the vector on perpendicular axes: sort of like shadows.



$$A_x = A \cos \theta = \text{x-component of } \vec{A}$$

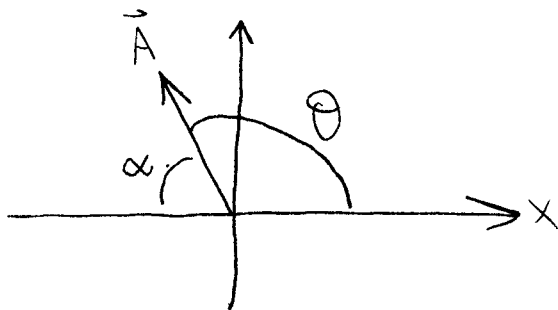
$$A_y = A \sin \theta = \text{y-component of } \vec{A}$$

These make a right triangle:



$$\text{so } |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

What if vector is pointing down or to left?



$$A_x = \cos \theta < 0$$

Can also write as  $A_x = -\cos \alpha$ ,  
which is often easier to calculate.

$A_x, A_y$  or  $A, \theta$  are equivalent ways to specify  $\vec{A}$ .

