Fluids (= liquids or gases)

Two definitions:

$$\text{pressure} = \frac{\text{force}}{\text{area}}, \quad p \equiv \frac{F_\perp}{A}, \qquad \text{density} = \frac{\text{mass}}{\text{volume}}, \quad \rho \equiv \frac{m}{V},$$

$F_\perp$ = force perpendicular to area, $[p] = N/m^2$ = pascal (Pa)

(ρ = greek letter "rho") → CTFL-1

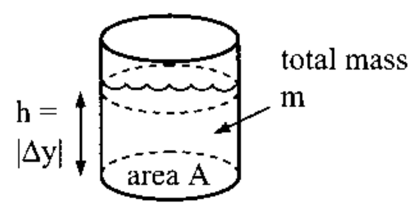
$[\rho] = kg/m^3$ or $gram/cm^3 = g/cm^3$

$$\rho_{water} = \frac{1\text{ gram}}{cm^3} \times \underbrace{\frac{1\text{ kg}}{10^3\text{ g}} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^3}_{\frac{10^6}{10^3} = 10^3} = 10^3\frac{kg}{m^3}, \qquad \rho_{iron} = 7.9\,g/cm^3 = 7900\,kg/m^3$$

"specific gravity" = $\dfrac{\rho}{\rho_{water}}$ =

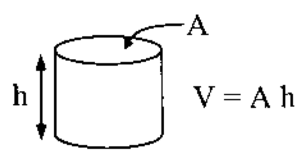
1	water
7.9	iron
11.9	Pb
2.7	Al

Consider a bucket of water. What is the pressure at the bottom of the bucket *due to the weight of the water*?



$$p = \frac{F_\perp}{A} = \frac{\text{weight}}{A} = \frac{mg}{A}$$

Will now show that $\boxed{p = \rho g h}$



$$\rho = \frac{m}{V} \Rightarrow m = \rho V,$$

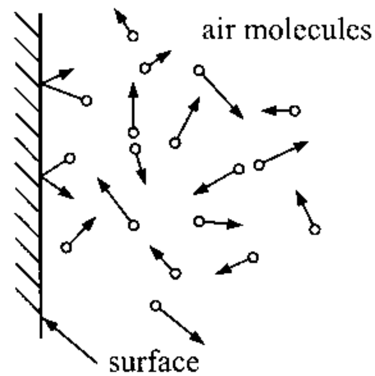
$$p = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho (A h) g}{A} = \rho g h$$

More generally, $\Delta p = -\rho g \Delta y$

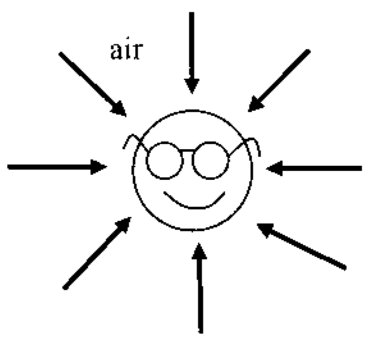
Minus sign because p increases as y decreases (go deeper). This derivation assumes that density ρ = constant, which it is for the case of water, because water is incompressible.

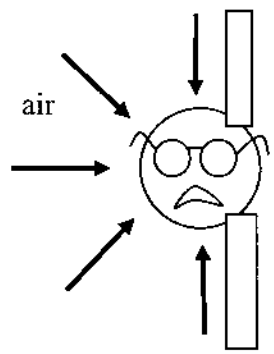
We are at the bottom of an ocean of air! $\rho_{air} = 1.29\,kg/m^3$ (at sea level) atmospheric pressure at sea level = $p_{atm} = 1\,\text{atm} = 1.01\times10^5\ \text{Pa} = 14.7\ \text{lb/in}^2$

Air consists mostly of oxygen and nitrogen molecules. At room temperature, the molecules have thermal energy and are moving around rapidly (speed ≈ 400 m/s), colliding with each other, and with every exposed surface. The pounding of the air molecules on a surface, like the pitter-pat of rain on the roof, adds up to a large force per area: $p_{atm} = 14.7$ psi.

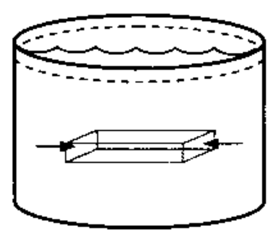


This is a big pressure! We are not ordinarily aware of this big pressure because the air pushes on us equally from all sides (even from our *insides* due to the air in our lungs). The big forces on us from all sides cancel out and there is not *net* force on us.





At a given depth, a fluid exerts the same pressure in every direction.



To see this, consider a block of water (outlined by an imaginary box) within a bucket of water. Since the water is in equilibrium, the forces (and therefore the pressures) on opposite sides of the block must cancel.

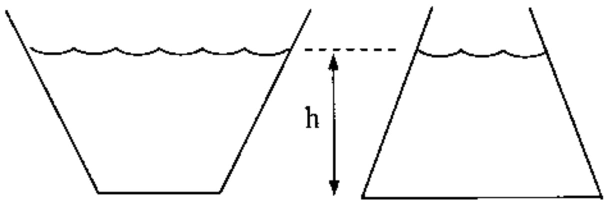
Example: At the surface of a swimming pool the pressure on a swimmer (due to the air) is p_{atm} = 1 atm. At what depth below the surface of the water is the total pressure on the swimmer = 2 atm? Answer: when the pressure <u>due to the weight of the water alone</u> is 1 atm, then the total pressure will be 2 atm. $|\Delta p| = \rho g |\Delta y| \;\Rightarrow\; |\Delta y| = \dfrac{|\Delta p|}{\rho g} = \dfrac{1.01\times10^5\,\text{Pa}}{(10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)} = 10.3\,\text{m}$

When computing the <u>total pressure</u> at a depth h below the surface of a liquid, we must include the pressure due to the atmosphere above the liquid:

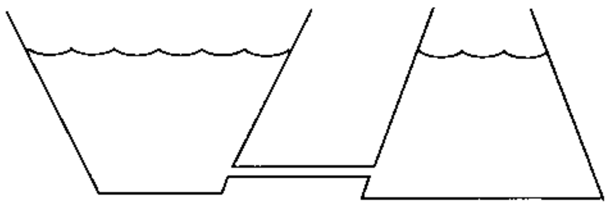
$$p_{tot} = \rho g h + p_{atm}$$

total p = p due to fluid above + p due to atmosphere above fluid

The pressure at a given depth is the same regardless of the shape of the container.



Same h ⇒ same p at bottom

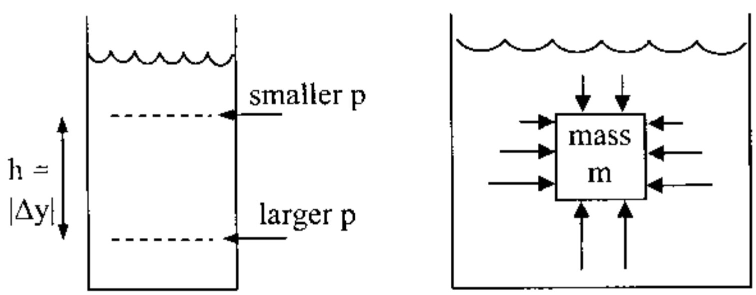


Must be same level (due to same pressure at bottom) or else could have a perpetual water fall ⇒ free energy ⇒ impossible

Archimedes' Principle

A solid body, either partially or totally submerged in a fluid experiences an upward buoyant force = weight of the displaced fluid

Before computing the buoyant force, let's first ask: *Why* is there an upward buoyant force?



$|\Delta p| = \rho g |\Delta y|$

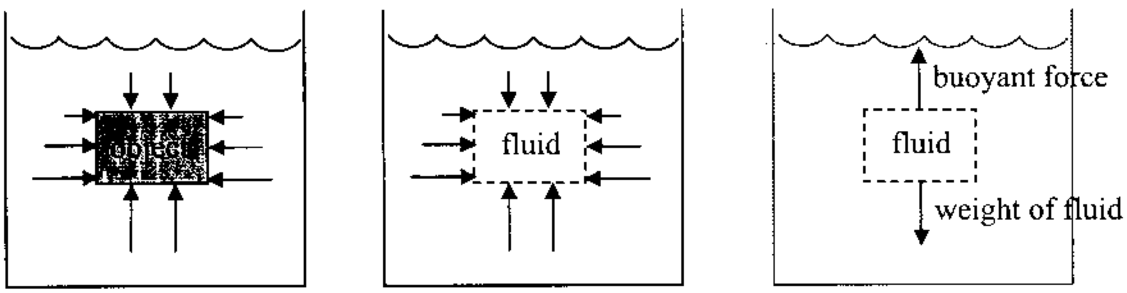
net upward force on submerged object = buoyant force

If buoyant force > mg, then object floats.
If buoyant force < mg, then object sinks.

Claim: An object of mass m and volume V, submerged in a fluid with density ρ_{fluid}, will experience an upward buoyant force with magnitude = weight of displaced fluid =

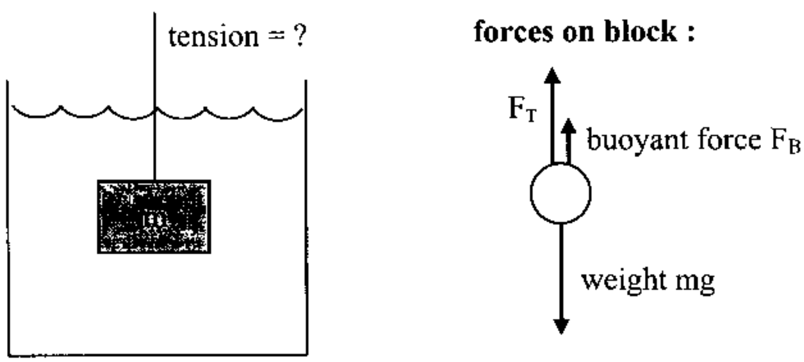
$$\boxed{F_{buoy} = m_{fluid} \cdot g = \rho_{fluid} \cdot V \cdot g}$$

Proof: This is a simple but slightly subtle argument. We note the forces on a submerged object from the surrounding fluid (the buoyant force) are exactly the same as the forces on a block of fluid with the same size and depth as the submerged object.



In equilibrium, the buoyant force on the block of fluid from the surrounding fluid must be equal in magnitude to the weight of the fluid, otherwise the block would not be in equilibrium. Therefore, the magnitude of the buoyant force on the submerged object is the same as the weight of the displaced fluid.

Example: A block of copper (Cu) with mass m = 400 g and density $\rho_{Cu} = 8.9\ \text{g/cm}^3$ is suspended by a string while under water. How does the tension in the sting compare to the weight of the copper block?



Since the block is not moving, the net force on it is zero and we can write:

$F_T + F_B = mg$ (since | upward forces | = | downward forces |)

So we have $F_T = mg - F_B$ We must now compute the magnitude of the buoyant force F_B.

Archimedes says that F_B is the weight of the *displaced water* $= m_{water}\,g = \rho_{water} V g$ where V is the volume of the displaced water = volume of the copper block. We get V from

$$\rho_{Cu} = \frac{m}{V} \Rightarrow V = \frac{m}{\rho_{Cu}} \quad \text{so} \quad F_B = \rho_{water} V g = \rho_{water} \frac{m}{\rho_{Cu}} g = \frac{\rho_{water}}{\rho_{Cu}} m\,g$$

$$F_T = mg - F_B = mg - \left(\frac{\rho_{water}}{\rho_{Cu}} m\,g \right) = mg\left(1 - \frac{\rho_{water}}{\rho_{Cu}} \right) = mg\left(1 - \frac{1}{8.9} \right) = 0.89\ mg$$

So the tension in the string is only 89% of the weight of the copper block. The buoyant force is helping support the weight of the block, so the tension is less than the full weight of the block.

This calculation can be turned around and used to compute the density of a block, given its mass and the tension in the string. Legend has it that Archimedes used this technique to determine the density of the king's crown (the king of Syracuse, a Greek colony in Sicily). The king was worried that the crown was not pure gold, and Archimedes was able to show that the density of his crown was considerably less than the density of gold ($\rho_{Au} = 19.3\ g/cm^3$), confirming the king's suspicion.

Archimedes (Greek, 287-212 B.C.) was the greatest mathematician and scientist of antiquity. He was also a brilliant engineer and inventor. His accomplishments in math, especially in geometry, were unmatched until the work of Isaac Newton, nearly 2000 years later. Archimedes should not be confused with a lesser scientist of ancient times: Aristotle (Greek. 384-322 B.C.). In stark contrast to Archimedes, Aristotle was a lousy mathematician and an inept experimentalist. Aristotle was the first scholar to make a serious attempt to produce a rational model of the natural world — a world of physical laws discovered through experiment to replace the world of superstition and magic. Although Aristotle was on the right track, he was not good with quantitative arguments. Unlike Aristotle, Archimedes actually *made* things and performed careful measurements. Archimedes knew that nature could not be fooled; if his computations were incorrect, then his devices simply would not work.

Archimedes (Greek, 287-212 B.C.)

P2010 LECTURE 43

Bernoulli's equation:

in a moving fluid:

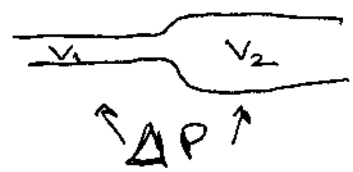
$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

($\rho g y_1$ and $\rho g y_2$) → often similar.

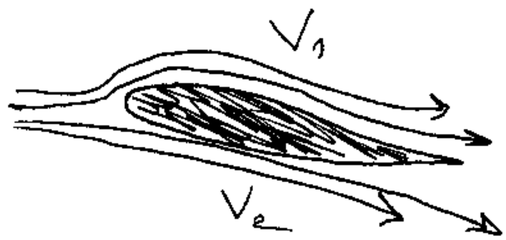
At constant altitude: $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

P goes up $\Longleftrightarrow$ V goes down.

→ constricted pipe:



→ Airplane wing:



In airplane's reference frame

$V_1 > V_2$

so $P_1 < P_2$

$\Delta P \cdot$ area = upward force on wing = lift.

(Also upward force from N3 — air forced downward.)