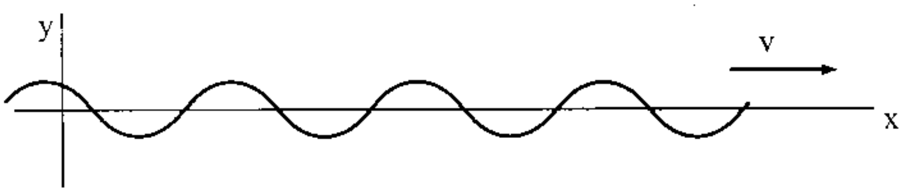
Mathematical description of traveling sinusoidal waves

Sinusoidal waves have a wavelength λ and a frequency $f = 1/T$. (Impulse waves have neither.)

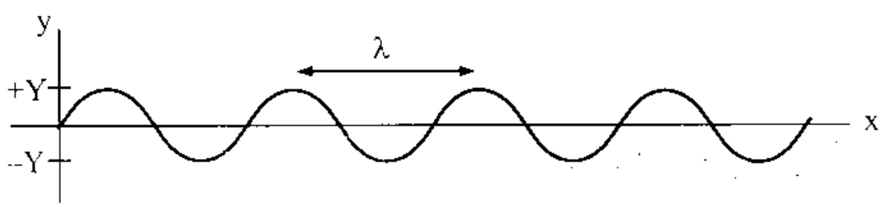


$y = y(x, t)$

y = displacement

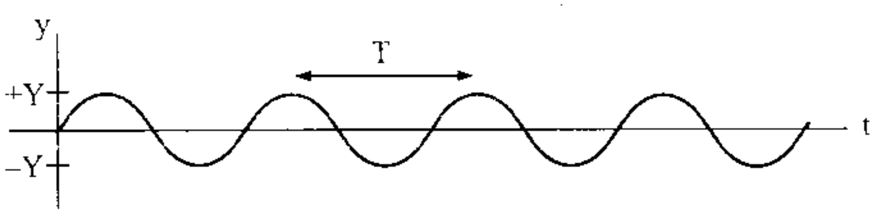
y = displacement from the equilibrium (no wave) position

Snapshot in time: freeze time at, say, $t = 0$



$$y(x, t=0) = Y \sin\left(2\pi \frac{x}{\lambda}\right)$$

Now freeze position, watch wave go by at position $x = 0$:

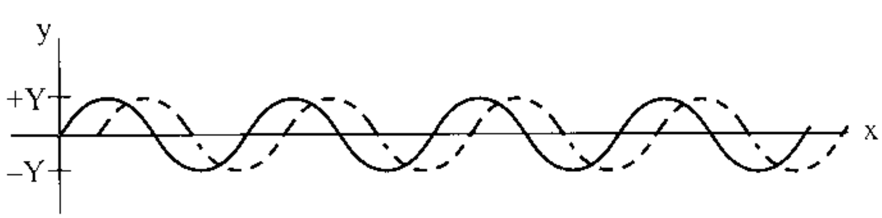


$$y(x=0, t) = Y \sin\left(2\pi \frac{t}{T}\right)$$

Wave traveling to the right:

$$\boxed{y(x,t) = Y \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]}$$

Notice: When position x changes by distance λ or time t changes by period T, the sine function goes through one complete cycle. When x increases by one λ AND t increases by one T, then the sine function stays at the same *phase*; we are then riding along with the wave.



The *argument* of the sin function $\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right]$ is called the *phase*. A point on the traveling wave (traveling along with the wave) corresponds to a particular value of the phase $\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right]$. As t increases, x must increase in order to keep $\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right]$ a constant value, hence a point of constant phase corresponds to a point moving to the right (increasing x).

Could also have a wave traveling to the left: $y(x,t) = Y\sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)\right]$

We could have used cosine instead of sine for the form of the wave. The only difference between sin and cos here is where we put the zero of time.

Wave speed is $v = \dfrac{\text{distance}}{\text{time}} = \dfrac{1\text{ wavelength}}{\text{time for }1\,\lambda\text{ to go by}} = \dfrac{\lambda}{T}$

$$\boxed{v = \frac{\lambda}{T} = \lambda f}$$ (since frequency $f = 1/T$)

Another way to see that our formula for the wave $y = y(x,t)$ corresponds to a wave moving right with speed $v = \lambda/T$ is to rewrite the formula like so:

$$y(x,t) = Y\sin\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right] = Y\sin\left[\frac{2\pi}{\lambda}\left(x-\frac{\lambda}{T}t\right)\right] = Y\sin\left[\frac{2\pi}{\lambda}(x-vt)\right]$$

A point traveling with the wave is a point with $(x - v\,t) = \text{constant}$ or $x = vt + \text{const}$. This is the equation for a point moving right with speed v (graph of x vs t has slope $\Delta x/\Delta t = v$)

Generally, the wave speed v is a constant, *independent* of λ and T.
The wave speed v depends on the properties of the medium, *not* on the properties of the wave.
Examples:
medium = string, properties = tension, mass per length
medium = air, properties = temperature, mass per molecule, etc

$v = \lambda f = \text{constant} \Rightarrow \lambda$ increases as f decreases, λ decreases as f increases