PHYS 2010 LECTURE 37

HARMONIC MOTION:

- Looks like the x- or y- component of uniform circular motion: $x = A\cos(\omega t)$
  $\quad A \leftarrow$ some constant = <u>Amplitude</u>

(recall from Monday that $x = r\cos\theta = r\cos\omega t$ for circular motion.)

- Occurs when <u>Restoring force</u> proportional to displacement from equilibrium: $F = -kx$

- $\omega = \frac{2\pi}{T}$ is independent of amplitude
  $T \leftarrow$ Period

- Velocity is also sinusoidal: go back to circular motion to see:

$\theta = \omega t$

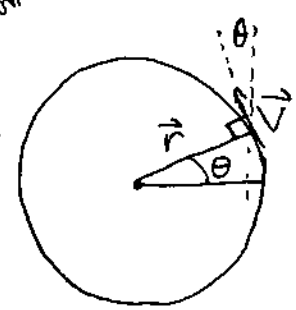
r is amplitude of oscillation in x.

$x = r\cos\theta = r\cos\omega t$

$|v| = \omega r$

$V_x = -\omega r \sin\omega t$

$\quad = -\omega A \sin\omega t$ for oscillator with amplitude A.



$\vec{V}$ is perp. to $\vec{r}$

Know magnitude $v = \omega r$

Use analogy with circular motion to find relationship between "force constant" k and period ω:

Centripetal force $|F_c| = \frac{mv^2}{r}$, in $-\vec{r}$ direction (toward center).

x-component: So $F_x = \frac{mv^2}{r}(-\cos\theta)$ But if $x = r\cos\theta$, then

$F_x = -\frac{mv^2}{r^2}(x)$. Recall that $\omega = \frac{v}{r}$: $F_x = -m\omega^2 x$.

So, match this up with spring constant equation:

Circular motion: $F_x = -m\omega^2 X$

spring (Hooke's Law): $F = -kX$

So now can say $k = m\omega^2$

$$\omega^2 = \frac{k}{m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$ for spring oscillation!

big k, small m → faster oscillations.

In this case, then:

- Amplitude $\longleftrightarrow |X_{max}| = |X_{min}|$ = max. displacement from equilibrium

We can set A to whatever we want by stretching the spring before releasing it.

Position $X = A\cos\omega t$

Velocity $V = -\omega A \sin\omega t$

Acceleration $a = \frac{F}{m} = \frac{-\cancel{m}\omega^2 x}{\cancel{m}} = -\omega^2 A\cos\omega t$

(Consistency check: Needs to preserve $F = -kx \Rightarrow ma = -kx$

$ma = -m\omega^2 A\cos\omega t$ but $A\cos\omega t = x$

so $ma = -m\omega^2 x$ ...and we found $k = m\omega^2$

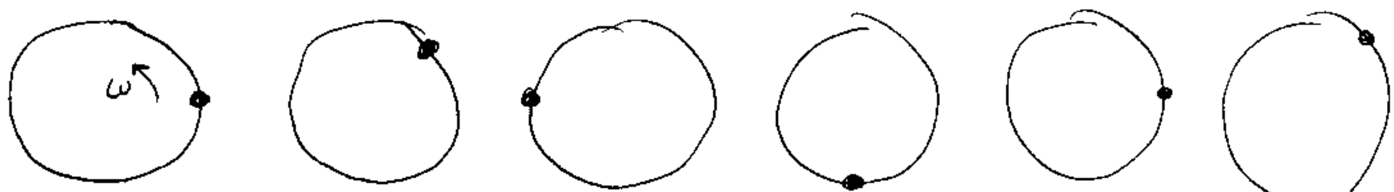
$= -kx$ ✓ Still consistent!)

Caveat: Not all oscillations follow these equations: only harmonic oscillations. Ex.: ball bouncing on floor doesn't have characteristics of harmonic osc.!

Caveat 2: We're assuming max. extension $X = A$ at $t=0$.

Note on the "phase angle" $\theta = \omega t$. In what sense is it really an angle?

Circular motion: it's the angular position of the object:



$\theta = 0$	$\theta = \frac{\pi}{4}$	$\theta = \pi$	$\theta = \frac{3\pi}{2}$	$\theta = 2\pi$	$\theta = 2\pi + \frac{\pi}{4}$
$t = 0$	$t = \frac{\pi}{4\omega}$	$t = \frac{\pi}{\omega}$	$t = \frac{3\pi}{2\omega}$	$t = \frac{2\pi}{\omega}$	$= \frac{9\pi}{4}$
	$= \frac{T}{8}$	$= \frac{T}{2}$	$= \frac{3T}{4}$	$= T$	$= \frac{9}{8}T$

(remember $T = \frac{2\pi}{\omega}$)

- So θ doesn't reset after one period!

$$\text{But } \sin(\theta + 2\pi) = \sin\theta$$
$$\cos(\theta + 2\pi) = \cos\theta$$

so x (and y) do "reset" after a period.

Now in 1-D oscillations (i.e. springs) we can define a more abstract "angle" $\theta = \omega t$ to keep the math the same, so $x = A\cos\theta$, $v = -\omega A \sin\theta$, $a = -\omega^2 A\cos\theta$. This is a mathematical angle, but not necessarily a spatial or physical angle.

- The "phase angle" here is an abstract concept! Can think of it as "what the angle would be if the spring we're looking at were really the x-component of a circular motion."