PHYS 2010 LECTURE 29

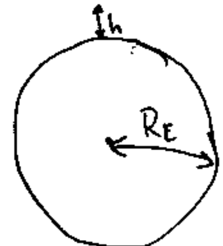
- Orbits: Recall $F_{grav} = \frac{GmM}{r^2}$ where m, M two masses, r = dist. btw. centers

$G = 6.7 \times 10^{-11}\ N m^2/kg^2$

Therefore, g depends on distance from center of earth.

g at altitude h:

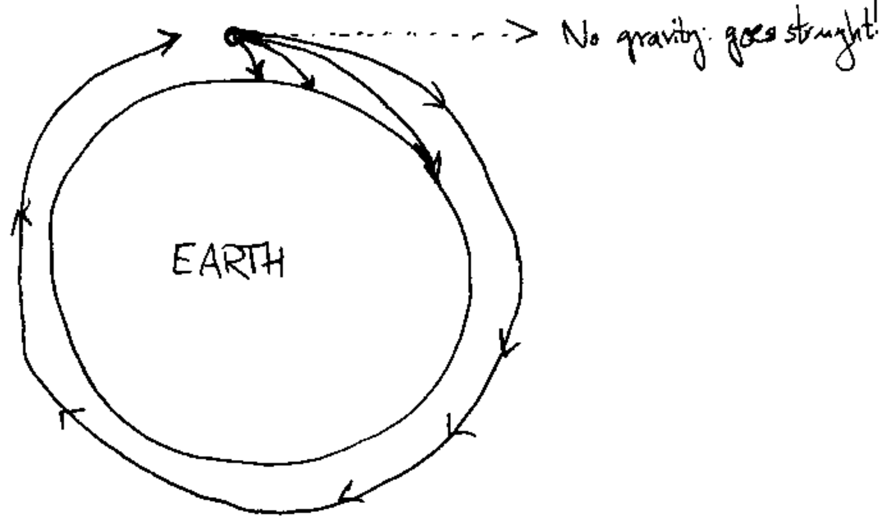
$$F_g = \frac{GmM_e}{(R_e+h)^2} \Rightarrow a_m = \frac{F}{m} = \frac{GM_e}{(R_e+h)^2}$$



So, g gets smaller (very slowly) with altitude.

Consider an object very slightly above earth's surface, (and atmosphere, and all obstructions): Throw it to right.

- (12)



Throw harder, and it lands farther away

Throw hard enough, and it orbits!

Circular orbit: constant speed, constant acceleration toward center of earth. Sounds like freefall!

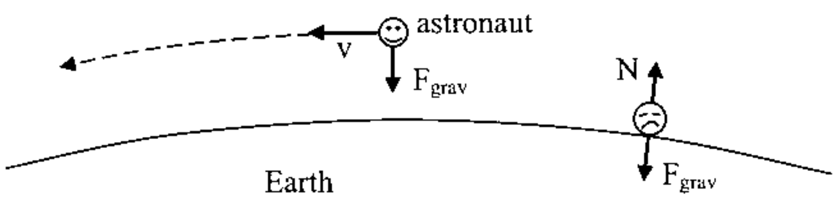
- $a = g = \frac{V^2}{R_o} \longrightarrow \frac{V^2}{R_o} = \frac{GM_e}{R_o^2}$ · For $R_o \approx R_e$, $\frac{V^2}{R_e} \approx g$: $V^2 = 10\frac{m}{s^2} \cdot 6400\,km$

(R_o ← orbit radius)

$\Rightarrow V = 8000\frac{m}{s} \approx 18{,}000$ mph.

"Weightless" does not mean "no gravity".
"Weightless" = "freefall" means the <u>only</u> force acting is gravity.

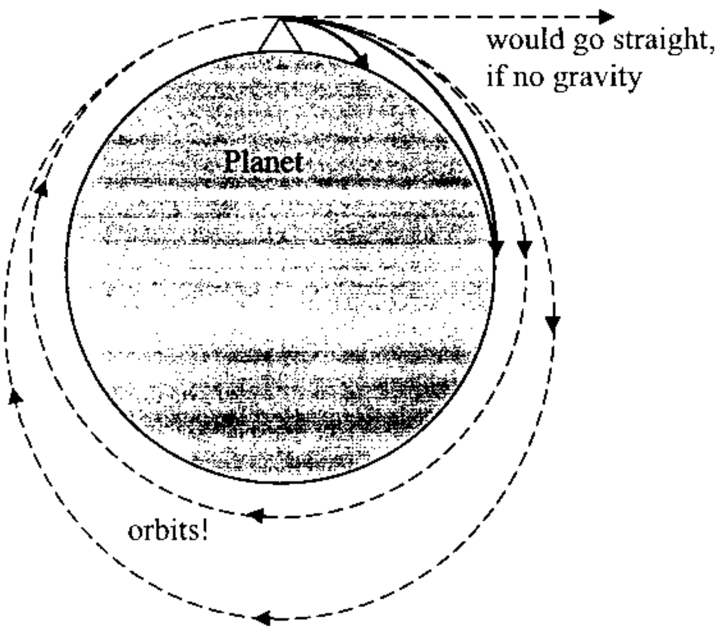
If you fall down an airless elevator shaft, you will feel exactly like the astronauts. You will be weightless, you will be in free-fall.



An astronaut falls toward the earth, as she moves forward, just as a bullet fired horizontally from a gun falls toward earth.

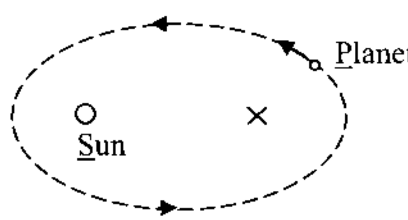
Orbits

Consider a planet like Earth, but with no air. Fire projectiles horizontally from a mountain top, with faster and faster initial speeds.



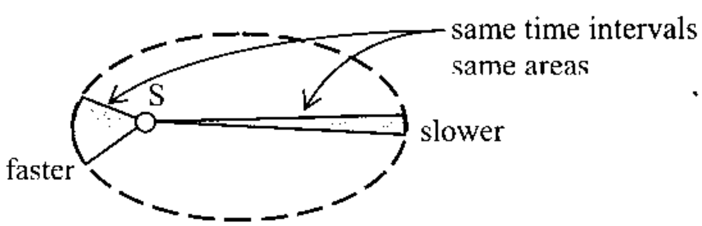
The orbit of a satellite around the earth, or of a planet around the sun obeys *Kepler's 3 Laws*.

Kepler, German (1571-1630). <u>Before</u> Newton. Using observational data from Danish astronomer Tycho Brahe ("Bra-hay"), Kepler discovered that the orbits of the planets obey 3 rules.



KI : A planet's orbit is an ellipse with the Sun at one focus.

KII : A line drawn from planet P to sun S sweeps out equal areas in equal times.



KIII: For planets around the sun, the period T and the mean distance r from the sun are related by $\frac{T^2}{r^3}$ = constant. That is for any two planets A and B, $\frac{T_A^{\ 2}}{r_A^{\ 3}} = \frac{T_B^{\ 2}}{r_B^{\ 3}}$. This means that planets further from the sun (larger r) have longer orbital periods (longer T).

Kepler's Laws were empirical rules, based on observations of the motions of the planets in the sky. Kepler had no theory to explain these rules.

Newton (1642-1727) started with Kepler's Laws and NII ($\mathbf{F}_{net} = m\mathbf{a}$) and deduced that

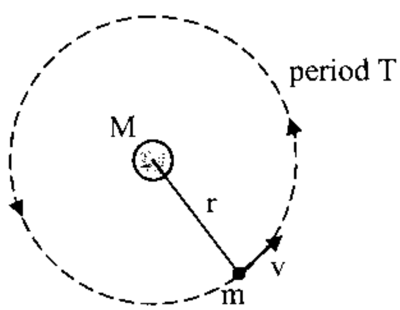
$$\underset{(\text{Sun-planet})}{F_{grav}} = G\frac{M_S\, m_P}{r_{SP}^{\ 2}}.$$

Newton applied similar reasoning to the motion of the Earth-Moon system (and to an Earth-apple system) and deduced that $\underset{(\text{Earth-mass m})}{F_{grav}} = G\frac{M_E\, m}{r_{Em}^{\ 2}}$.

Newton then made a mental leap, and realized that this law applied to any 2 masses, not just to the Sun-planet, the Earth-moon, and Earth-projectile systems.

Starting with $\mathbf{F}_{net} = m\mathbf{a}$ and $F_{grav} = G\,Mm / r^2$, Newton was able to derive Kepler's Laws (and much more!). Newton could explain the motion of everything!

Derivation of KIII (for special case of circular orbits). Consider a small mass m in circular orbit about a large mass M, with orbital radius r and period T. We aim to show that T^2 / r^3 = const.



Start with NII: $F_{net} = m\,a$

The only force acting is gravity, and for circular motion $a = v^2 / r \;\Rightarrow$

$$G\frac{M\not{m}}{r^2} = \not{m}\frac{v^2}{\not{r}} \;\Rightarrow\; G\frac{M}{r} = v^2 = \left(\frac{2\pi r}{T}\right)^2$$

[recall the v = dist / time = $2\pi r$ / T]

$$\Rightarrow\quad G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2} \quad\Rightarrow\quad \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = \text{constant, independent of m}$$

(Deriving this result for elliptical orbits is much harder, but Newton did it.)

An extra result of this calculation is a formula for the speed v of a satellite in circular orbit: $v = \sqrt{\frac{GM}{r}}$. For low-earth orbit (few hundred miles up), this orbital speed is about 7.8 km/s $\cong$ 5 miles/second.