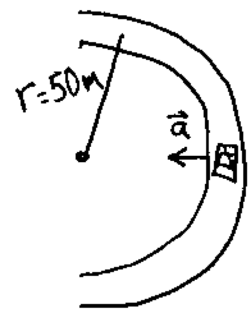
PHYS 2010 LECTURE 28

- Circular motion review:

Recall $a = \frac{v^2}{r}$, $F_c = \frac{mv^2}{r}$ for constant-speed circular motion.

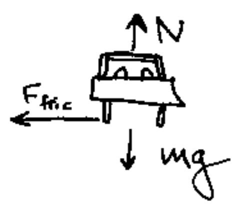
Example: How fast can a car turn along a highway ramp without skidding?



Typical radius: 50m ≈ 170 feet.

Friction must provide centripetal force.

Free-body diag.:



No vertical accelleration: $\Rightarrow N - mg = 0$.

$F_{net} = F_{fric} = ma$

$F_{fric} \leq \mu_s N$ as usual. Car will skid if reaches static fric. limit, where $F_{fric} = \mu_s N$.

$$a_{max} = \frac{V_{max}^2}{r} = \frac{F_{fric}}{m} = \frac{\cancel{m} g \mu_s}{\cancel{m}}$$

$$\text{so } \frac{V_{max}^2}{r} = \mu_s g \Rightarrow V_{max} = \sqrt{\mu_s g r}$$

Look at what happens with numbers:

Dry pavement: $\mu_s \approx 0.95$.

$$V_{max} = \sqrt{0.95 \cdot 10\tfrac{m}{s^2} \cdot 50m} = 22\,\tfrac{m}{s} \approx 49 \text{ mph}$$

Wet pavement: $\mu_s \approx 0.7$:

$$V_{max} = \sqrt{0.7 \cdot 10\tfrac{m}{s^2} \cdot 50m} = 19\,\tfrac{m}{s} \approx 42 \text{ mph}$$

Icy pavement: $\mu_s \approx 0.1$

$$V_{max} = \sqrt{0.1 \cdot 10\tfrac{m}{s^2} \cdot 50m} = 7\,\tfrac{m}{s} \approx \underline{\underline{\cancel{18}^{\,16} \text{ mph}!}}$$

$1\,m = 0.622 \times 10^{-3}\,mi$

$1\,hour = 3600\,sec$

$\Rightarrow 1\,\tfrac{m}{s} = 2.24\,mph$

Gravity!

Newton's Universal Law of Gravitation (first stated by Newton): any two masses m_1 and m_2 exert an attractive gravitational force on each other according to

$$\boxed{F = G \frac{m_1 m_2}{r^2}}$$

G = universal constant of gravitation = 6.67×10^{-11} N m^2 / kg^2 (very difficult to measure G!)

Don't confuse G with g.

Newton showed that the force of gravity must act according to this rule in order to produce the observed motions of the planets around the sun, of the moon around the earth, and of projectiles near the earth. He then had the great insight to realize that this same force acts between *all* masses. [That gravity acts between all masses, even small ones, was experimentally verified in 1798 by Cavendish.]

Newton couldn't say *why* gravity acted this way, only *how*. Einstein (1915) General Theory of Relativity, explained why gravity acted like this.

Example: Force of attraction between two humans. 2 people with masses $m_1 \cong m_2 \cong 70$ kg a distance r = 1 m apart.

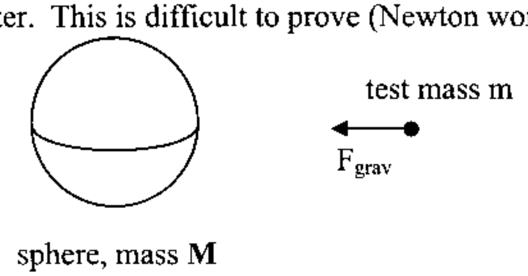
$$F = G\frac{m_1\, m_2}{r^2} = \frac{(6.67\times10^{-11})(70)^2}{1^2} = \cancel{3.2\times10^{-11}}\ \text{N} \quad 3.27\times10^{-7}\ \text{N}$$

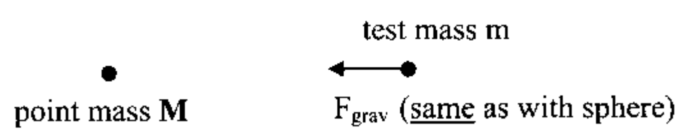
~~3.2×10^{-9} g~~ 33×10^{-6} g

This is a very tiny force! It is the weight of a ~~34×10^{-6} gram~~ mass. A hair weighs 2×10^{-3} grams – the force of gravity between two people talking is about ~~1/60~~ the weight of a single hair. 1/160

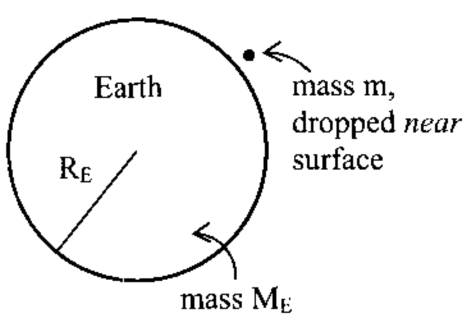
Computation of g

Important fact about the gravitational force from spherical masses: a spherical body exerts a gravitational force on surrounding bodies that is the same as if all the sphere's mass were concentrated at its center. This is difficult to prove (Newton worried about this for 20 years.)





We can now *compute* the acceleration of gravity g ! (Before, g was experimentally determined, and it was a mystery why g was the same for all masses.)



$$F_{grav} = m\,a = m\,g$$

$$G\frac{M_E m}{{R_E}^2} = m\,g$$

(since $r = R_E$ is distance from m to center of Earth)

m's cancel ! $\Rightarrow$ $\boxed{g = \dfrac{G\,M_E}{{R_E}^2}}$

$M_E = 5.98\times10^{24}$ kg
$R_E = 6.38\times10^{6}$ m $\Longrightarrow$ $g = 9.80\ \text{m/s}^2$