PHYS 2010 LECTURE 15:

More ~~&~~ forces, acceleration:

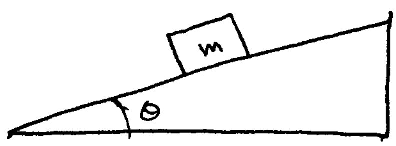
Start with a block in free-fall. ~~What steps?~~
What is its acceleration?

① Free-body diagram: 

only one force $F_{net} = mg$

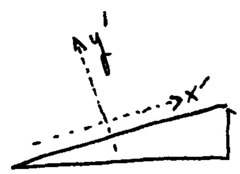
(2,3,4) $F_y = -mg = ma_y \rightarrow a_y = -g.$

Now, move on to a block on a frictionless inclined plane:



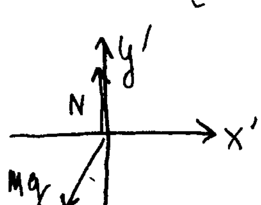
(think of an air track.

① Free-body diagram: 

② Select axes: 

x', y' are rotated CCW by θ from x, y.

③ Make F=ma equations in new axes:



	x'	y'
Gravity:	$-mg\sin\theta$	$-mg\cos\theta$
Normal:	0	$+mg\cos\theta$

How did we know $N = mg\cos\theta$? N must oppose all other forces normal to the surface, to keep $(F_{net})_{y'} = 0$.

So our acceleration equations are:

x' $\quad F_{net} = -mg\sin\theta = ma_{x'} \Rightarrow a_{x'} = -g\sin\theta$

y' $\quad F_{net} = -mg\cos\theta + mg\cos\theta = 0 \Rightarrow a_{y'} = 0$

Does this make sense? Look at $|a| = |g\sin\theta|$

mg is a force

g is a number with acceleration units.

Digression!

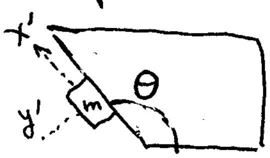
Take limit $\theta \to 0$: $|a| \to g\sin 0 = 0$

makes sense → no net force on level plane.

Take limit $\theta \to 90°$: $|a| \to g\sin 90° = g$

makes sense → if plane is vertical, block is in free-fall.

Take $\theta > 90°$:



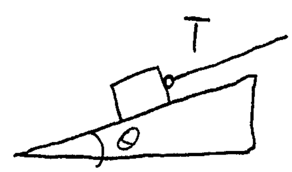
$a_{x'} = g\sin\theta < 0$

$a_{y'} = 0$

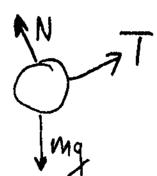
Makes no sense. Block won't stick to surface!

Why? Normal force can only push, not pull. So we set up the equations in a way that doesn't apply for $\theta > 90°$. Be careful of situations like this!

Now, add a string with constant tension:



(1) Free-body diagram:



(2) Axes: same as before; T is in $+x'$ direction.

(3) Forces:

	x'	y'
Gravity	$-mg\sin\theta$	$-mg\cos\theta$
Normal	0	$+mg\cos\theta$
Tension	T	0

④ Solve:

$$\sum F_{x'} = ma_{x'} : \quad T - mg\sin\theta = ma_x$$

$$\frac{T}{m} - g\sin\theta = a_x$$