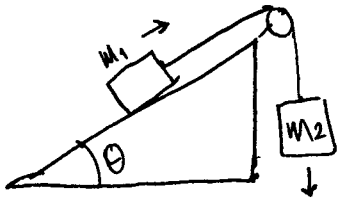
PHYSICS 2010 LECTURE 14

Solving problems involving $\vec{F}=m\vec{a}$.

1) Draw free-body diagrams for each object
2) Select axes
3) Form $\Sigma\vec{F}=m\vec{a}$ equations for each object
4) Grind through algebra to solve for appropriate variable

Problem:



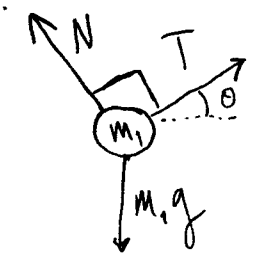
Know m_1, m_2, θ

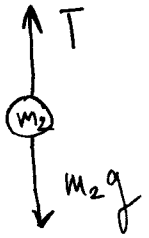
Seek T = tension in cord

a = acceleration of m_1

Frictionless surface, massless pulley.

Step 1: Free-body diagrams for each object:



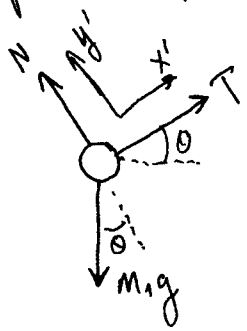


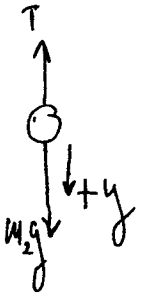
} Label the magnitude of forces.

T is same for both objects? It's one string, and if pulley is massless then it can't have any force on it: → NIII says T equal on m_1, m_2.

Why? →

Step 2: Axes: Often convenient to tilt them:





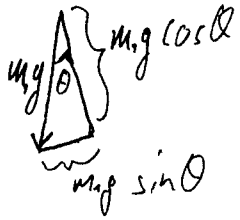
y' called "y-prime"

+y is downward.

Now each acceleration will be along a + axis.

Step 3: For each object, write force equation: $\sum \vec{F} = m\vec{a}$

Separate the components:



m_1, x' component: No normal force component

$$T - m_1 g \sin\theta = m_1 a$$

m_1, y' component: No acceleration

$$N - m_1 g \cos\theta = 0$$

} gravity is negative in x', y'.

m_2, y component:

$$+m_2 g - T = m_2 a$$

Note: $|a|$ is same for both objects. (string doesn't stretch)

Now, just a messy algebra problem.

(1) $T - m_1 g \sin\theta = m_1 a$

(2) $N - m_1 g \cos\theta = 0$

(3) $m_2 g - T = m_2 a$

N is easy: $N = m_1 g \cos\theta$

Remainder: 2 eqns (1 + 3), 2 unknowns (T, a)

Solve (1) for T, plug into (3):

(1): $T = m_1 g \sin\theta + m_1 a = m_1 (g \sin\theta + a)$

(3) $m_2 g - m_1 (g \sin\theta + a) = m_2 a$

solve for a:

$$m_2 g - m_1 g \sin\theta = m_1 a + m_2 a = a(m_1 + m_2)$$

$$a = \frac{m_2 g - m_1 g \sin\theta}{(m_1 + m_2)}$$

What is $a = 0$ condition?

$$\frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2} = 0 \implies m_2 = m_1 \sin\theta$$