Physics 2010 – Spring 2006
Laboratory 5: Rotational Dynamics

NAME ______________________________
Section Day (circle): M Tu W Th F
Section Time: 8a 10a 12p 2p 4p

This lab will cover the concepts of moment of inertia, rotational kinetic energy and angular momentum, which play the same roles in rotational motion as mass, kinetic energy and momentum do for straight-line motion.

Instructions

- **Your first priority in any lab is safety.** This experiment uses fairly heavy objects that roll. There is a risk of toe injury as well as damage to equipment.
- **DO NOT BRING FOOD OR DRINK TO THE LAB.**
- Work the prelab problems *before* the lab. Failure to turn in the prelab before the lab will result in a 2-point penalty (out of 10) for the lab. You are encouraged to make a copy of your prelab responses, so you can refer to them in the lab.
- In the lab, use the space provided for short answers, and attach your own paper for extended analysis and commentary.
- A measurement is *wrong* if it has units and they are not specified. As always, keep track of the precision of your measurements and make sure to report an appropriate number of significant figures.

1 Background Information

1.1 Moment of Inertia

The moment of inertia I of a body is a measure of how hard it is to get it rotating about some axis. The moment I is analogous to mass m for motion of the center of mass. The larger the I, the more work is required to get the object spinning to a certain angular speed, just as the larger the mass m, the more work required to get it moving at a given speed in a straight line.

The moment of inertia is always defined with reference to a particular axis of rotation – often a symmetry axis, but it can be any axis, even one that is outside the body. The moment of inertia of a body about a particular axis is defined as:

$$I = \sum_i m_i r_i^2 \tag{1}$$

where the sum is over all parts of the body (labeled with the index i), m_i is the mass of part i, and r_i is the distance from part i to the axis of rotation. Performing this sum is straightforward if the body consists of discrete point masses. But if the body is a continuous object of some arbitrary shape, then performing the sum requires techniques of calculus. In this course, we simply tell you the answer for various shapes. For a disc with an axis through the center of symmetry, the moment of inertia is

$$I_{\text{disc}} = \frac{1}{2} M R^2 \tag{2}$$

where M is the mass of the entire disc and R is its radius (see Fig. 1). Notice that the thickness of the disc doesn't enter into the expression for I_{disc}, which depends only on the mass and radius.

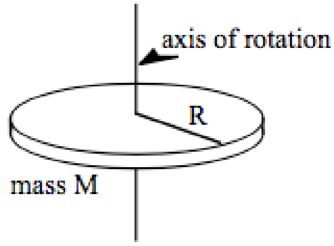

Figure 1: Solid disc with axis of symmetry and uniform mass density.

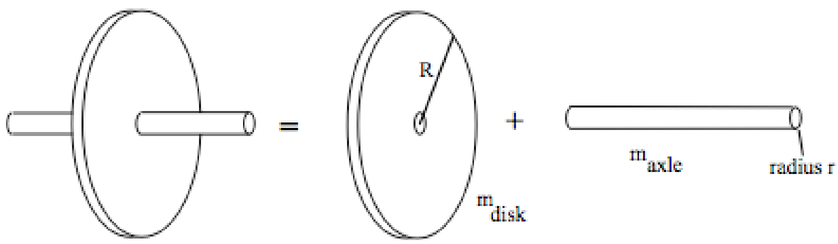

Figure 2: Axle and disc combine to form a single system, a wheel.

Therefore, this expression works as well for a solid cylinder — which is just a very thick disc. (To see this, imagine making a disc thicker and thicker until the thickness is bigger than the radius and becomes the length of a cylinder.)

Note that the expression in equation 2 is only true for a *uniform* disc – if the mass is a hollow circle or a thin-walled cylinder instead, then the formula doesn't apply! (In fact, in that case the moment of inertia is $I = mR^2$ because all the mass is at the same radius from the axis.)

In this experiment you will measure I for a disc (radius R, mass $m_{\text{disc}} \equiv M$) mounted on an axle (radius r, mass $m_{\text{axle}} \equiv m$), as shown in Fig. 2. The axle is a uniform cylinder (remember, this is just a thick disc), so its moment of inertia is $I_{\text{axle}} = m\,r^2/2$. The disc's moment of inertia is $I_{\text{tot}} = MR^2/2$. The moment of inertia for the combined system is simply the sum of the moments of the two components:

$$I_{\text{tot}} = I_{\text{disc}} + I_{\text{axle}} = \frac{1}{2}(MR^2 + m\,r^2) \tag{3}$$

Be careful here: there are *two different radii* in the problem, R and r! If you mix them up, your calculations will be very wrong.

1.2 Rotational Energy

Your experiment will consist of allowing the system to roll down an inclined track, and using conservation of energy to find the moment of inertia experimentally. The test will be the comparison of the moment measured this way, and the moment calculated from equation 3.

Consider the disc and axle together as a wheel, as in figure 2. Let the wheel roll down an inclined set of rails after starting from rest at the top, as in Fig. 3. The total energy at any given point on the ramp is the sum of: translational kinetic energy (motion of the center of mass); rotational kinetic energy; and gravitational potential energy:

$$E_{\text{tot}} = KE_{\text{trans}} + KE_{\text{rot}} + PE\,. \tag{4}$$

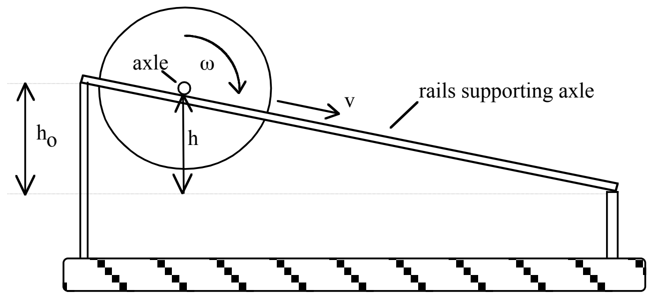

Figure 3: Axle and disc rolling down an idealized track.

The kinetic energy of translation is just

$$KE_{\text{trans}} = \frac{1}{2}(M+m)v_{\text{CM}}^2\,, \tag{5}$$

where v_{CM} is the velocity of the center of mass, and we have used the *total mass* $M+m$. The gravitational potential energy is also familar:

$$PE = (M+m)gh\,, \tag{6}$$

where h is the height of the center of mass, and again it is the total mass that counts. Finally the kinetic energy of rotation is:

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2\,. \tag{7}$$

This is similar to KE_{trans} but with the moment of inertia I replacing the total mass, and the velocity of the center of mass replaced by ω, the *angular velocity* of the rolling motion: $\omega = \Delta\phi/\Delta t$ with $\Delta\phi$ the angle rotated (in radians per second) in time Δt.

Putting it all together, the total energy is

$$E_{\text{tot}} = \frac{1}{2}(M+m)v_{\text{CM}}^2 + \frac{1}{2}I\omega^2 + (M+m)gh\,. \tag{8}$$

We are going to derive an expression for I by noting that energy is conserved (the effects of friction are small and can be neglected) and hence is the same at the beginning and end of the motion, $E_i = E_f$ where i stands for "initial" and f stands for "final".

Initially, the wheel is at rest at initial height h_0, so its initial kinetic energy (both translational and rotational) is zero and its energy is all potential: $E_i = PE_i = (M+m)gh_0$.

Then, when the wheel reaches the bottom of the rail, $h = h_f = 0$, and the energy is all kinetic:

$$E_f = KE_{\text{trans}} + KE_{\text{rot}} = \frac{1}{2}(M+m)v_f^2 + \frac{1}{2}I\omega_f^2. \tag{9}$$

Here v_f and ω_f are the velocity of the center of mass and rotational velocity, respectively, at the end of the motion.

The rest of the preparation will be worked out in the prelab.

Prelab Questions

1. In order to go further, we need to know how to relate translational velocity to angular velocity. Since the circumference of a circle of radius r is $2\pi r$, every time the center of mass moves by

$2\pi r$ the object also rotates by a full circle (2π radians). So, say this happens in a time t_{rot}. We know that:

$$\omega = \frac{\Delta\phi}{\Delta t} = \frac{2\pi}{t_{\mathrm{rot}}} \tag{10}$$

and

$$v_{\mathrm{CM}} = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{t_{\mathrm{rot}}}. \tag{11}$$

Combine these equations to find an expression for ω in terms of v_{CM}.

2. When applying the previous expression to our system, which radius (r or R) should you use? Why?

3. Use conservation of energy $E_i = E_f$, the expressions for E_i and E_f from the background discussion, and the results of the previous prelab questions to solve for I in terms of $(M + m), r, g, v_f$, and h_0.

4. Justify the general statement for any object moving under constant acceleration that if $v_{\mathrm{initial}} = 0$, then $v_{\mathrm{average}} = v_{\mathrm{final}}/2$.

5. The expression you derived for I involves g, the acceleration of gravity. So, you might imagine the result would be different on another planet where g is different. But the definition of I (Eq. 1) doesn't involve gravity at all: it's the same on Earth and the moon. Can you resolve this apparent paradox? Think about this now, and discuss and answer it with your lab partners when you have your lab section.

2 Pre-setup (5 min)

Check off and confirm with your TA the answers to pre-lab questions 1-4. Make sure these are right before continuing.

3 Weigh and measure components to predict I (15 min.)

- Gently slide the axle out of the disc and weigh both separately to find their masses. Measure their diameters to find their radii, r for the axle and R for the disc. Measure the diameter of the axle very carefully three or four times with the calipers. Use the average of your measurements and estimate the uncertainty in r. If you don't know how to use the calipers, ask your instructor. Check that you understand how to use the calipers by using them to measure something that you already know the size of, for example, something you can measure precisely with a ruler. Write your measurements and uncertainties in a table below:

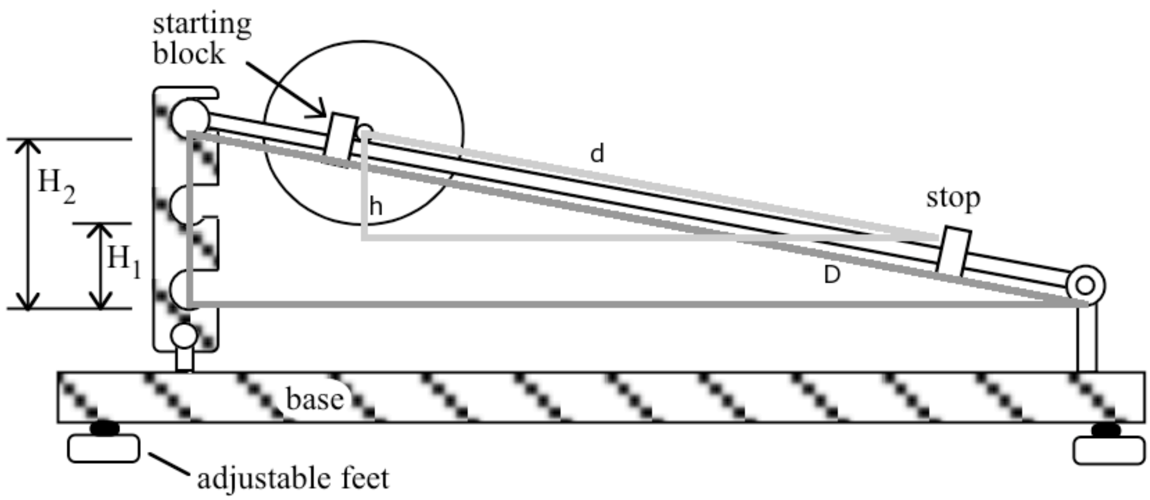

Figure 4: The rolling track you will use in the lab, including similar triangles you can use to find the height drop of the axle.

- Using Eq. 2, find I_{disc} and I_{axle} separately and then add them to get I_{total}: Note that using MKS units, your moment of inertia will be a small number. Be sure to keep track of an appropriate number of significant figures; this is best done in scientific notation.

- Does the axle contribute significantly to the total mass? To the total I?

4 Measurement of I using energy conservation (60 min.)

One end of the rails can be raised and lowered to one of three positions (see Fig. 4). Place the rails in the lowest position, at which they are approximately level, and then using the adjustable screws in the base, make the rails exactly level. Use the bubble level to get a rough level and then place the wheel on the rails to get a precise level. (If the rails are exactly level, a stationary wheel will not start rolling.)

Raise the movable end of the rails to one of the two upper positions and then fix the two starting blocks, one on each rail, at some convenient position near the top of the track. Make sure that the starting blocks are level with each other, so that the axle can be started resting against both blocks and will roll straight down the track when released. Using the meter stick attached to one rail, record the positions of the sharp tip of the axle in the starting and stopping positions and compute the distance d along the track through which the wheel rolls. Leave the starting blocks fixed from now on, so that the value of d is the same for all timings.

To determine the heights h_1 and h_2 through which the wheel descends, begin by measuring the height changes H_1 and H_2 of the end of the rail when it is raised from the level position to the two upper positions. H_1 and H_2 can be measured quite precisely by measuring the separations of the notches that hold up the end of the rail.

Unfortunately, H_1 and H_2 are not the actual heights through which the wheel descends, since it does not roll the whole length of the rail. Instead, the situation is as shown in Fig. 4, where d is the distance traveled by the wheel, while D is the total length of the track (D is measured from the center of the pivot at the bottom to the center of the support at the top). The two triangles (pale gray and dark gray) shown are similar triangles. Therefore,

$$\frac{h}{H} = \frac{d}{D}\,, \tag{12}$$

where H can be either H_1 or H_2 (it's H_2 in the figure). Use this relation to calculate h_1 and h_2 from measured values of d, D, H_1, and H_2:

Now use the stopwatch to measure the time t_1 for the wheel to roll down the rail when it is in notch 1 (notch height H_1). This is best done with the same person operating the stopwatch and releasing the wheel. Make a few trial runs to determine the best procedure. Have each member of your team measure t_1 a few times, and record all values here. From your measurements, determine an average value of t_1 and estimate its uncertainty.

Repeat this whole procedure for t_2, measured when the rail is in notch 2 (height H_2). From your measurements, determine an average value of t_2 and estimate its uncertainty.

Now calculate the wheel's final speed v_f at the bottom of its travel for each of the two positions. Be careful! The quantity d/t is the *average* speed of the wheel. Use the result from Prelab question 4 to find the final velocities v_{1f} and v_{2f}:

Finally, with all your measurements, using the result from Prelab question 3, compute I for each of the two positions of the rail. Be careful to display the correct number of significant figures in your final answers.

5 Final Analysis (15 min)

Display your three final values for I: your value from Sec. 3 and your two values from Sec. 4 with the axle rolling from different heights. If there is a large discrepancy among the values, comment on possible sources of experimental error.