Motion in one dimension (1D) [ Chapter 2 in Giancoli ]

In this chapter, we study speed, velocity, and acceleration for motion in one-dimension.

**speed and velocity**

\[
\text{speed} = \frac{\text{distance traveled}}{\text{time elapsed}}, \quad s = \frac{d}{t}, \quad \text{units are m/s or mph or km/hr or...}
\]

Speed \( s \) and distance \( d \) are both always positive quantities, by definition.

**velocity = speed + direction of motion**

Things that have both a magnitude and a direction are called vectors. More on vectors in Ch.3.

For 1D motion (motion along a straight line, like on an air track), we can represent the direction of motion with a +/- sign

\[
\begin{align*}
+ & = \text{going right} \rightarrow \quad \text{always!} \\
- & = \text{going left} \leftarrow
\end{align*}
\]

\[
\begin{align*}
v_A &= -10 \text{ m/s} \\
v_B &= +10 \text{ m/s}
\end{align*}
\]

Objects A and B have the same speed \( s = |v| = +10 \text{ m/s} \), but they have different velocities.

If the velocity of an object varies over time, then we must distinguish between the *average* velocity and the *instantaneous* velocity.

**Definition:** average velocity \( \bar{v} \) = \( \frac{\text{change in position}}{\text{change in time}} = \frac{\Delta x}{\Delta t} \)

\[
\begin{align*}
\bar{v} &= \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \\
\Delta x &= x_{\text{final}} - x_{\text{initial}} = \text{displacement (can be + or – )}
\end{align*}
\]
Notice that $\Delta$ (delta) always means "final minus initial".

$$\nabla = \frac{\Delta x}{\Delta t}$$ is the slope of a graph of $x$ vs. $t$

**Review:** Slope of a line

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Suppose we travel along the x-axis, in the positive direction, at constant velocity $v$:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{x_2 - x_1}{t_2 - t_1} = v$$

y-axis is $x$, x-axis is $t$. 
Now, let us travel in the negative direction, to the left, at constant velocity.

Note that \( v = \text{constant} \Rightarrow \text{slope of } x \text{ vs. } t = \text{constant} \Rightarrow \text{graph of } x \text{ vs. } t \text{ is a straight line}

But what if \( v \neq \text{constant} \)? If an object starts out going fast, but then slows down and stops...

The slope \textit{at a point} on the \( x \text{ vs. } t \) curve is the \textit{instantaneous velocity} at that point.

\textbf{Definition:} instantaneous velocity = velocity averaged over a very, very short (infinitesimal) time interval

\[
v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \text{slope of tangent line}
\]
Acceleration

If the velocity is changing, then there is acceleration.

**Definition:** acceleration = time rate of change of velocity

In 1D: **instantaneous acceleration** \( a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \)

**average acceleration** over a non-infinitesimal time interval \( \Delta t \): \( \bar{a} = \frac{\Delta v}{\Delta t} \)

units of \( a = [a] = \frac{m/s}{s} = \frac{m}{s^2} \)

Sometimes I will be a bit sloppy and just write \( a = \frac{\Delta v}{\Delta t} \), where it understood that \( \Delta t \) is either a very small time interval in the case of instantaneous \( a \) or \( \Delta t \) is a large time interval in the case of average \( a \).
\[ a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{v_2 - v_1}{t_2 - t_1} \]

- \( v = \text{constant} \Rightarrow \Delta v = 0 \Rightarrow a = 0 \)
- \( v \text{ increasing (becoming more positive)} \Rightarrow a > 0 \)
- \( v \text{ decreasing (becoming more negative)} \Rightarrow a < 0 \)

In 1D, acceleration \( a \) is the slope of the graph of \( v \) vs. \( t \) (just like \( v = \text{slope of } x \text{ vs. } t \))

**Examples of constant acceleration in 1D** on next page...
Examples of constant acceleration in 1D

**Situation I**
An object starts at rest, then moves to the right (+ direction) with constant acceleration, going faster and faster.

\[ v \quad \Delta v \quad \Delta t \quad t \]

\[ a > 0, \ a = \text{constant} \]
( since \( v \) vs. \( t \) is straight)

**Situation II**
An object starts at rest, then moves to the left (– direction) with constant acceleration, going faster and faster.

\[ v \quad \Delta v \quad \Delta t \quad t \]

\[ a < 0, \ a = \text{constant} \]
( since \( v \) vs. \( t \) has constant, negative slope)

**Situation III**

\[ v \quad \Delta v \quad \Delta t \quad t \]

\[ a < 0, \ a = \text{constant}!! \]
( since \( v \) vs. \( t \) has constant, negative slope)
The direction of the acceleration

For 1D motion, the acceleration, like the velocity, has a sign ( + or – ). Just as with velocity, we say that positive acceleration is acceleration to the right, and negative acceleration is acceleration to the left. But what is it, exactly, that is pointing right or left when we talk about the direction of the acceleration?

Acceleration and velocity are both examples of \textbf{vector} quantities. They are mathematical objects that have both a magnitude (size) and a \textbf{direction}. We often represent vector quantities by putting a little arrow over the symbol, like \( \vec{v} \) or \( \vec{a} \).

direction of \( \vec{a} \) \( \neq \) direction of \( \vec{v} \)

direction of \( \vec{a} \) = the direction which the velocity is \textit{tending towards}

Reconsider Situation I (previous page)

\[ \begin{array}{c}
1 \quad 2 \\
\end{array} \]

\( \downarrow \) \( \downarrow \quad 1 \) is an earlier time, 2 is a later time

\( \vec{v}_1 = \) velocity at time 1 = \( \vec{v}_{\text{init}} \)
\( \vec{v}_2 = \) velocity at time 2 = \( \vec{v}_{\text{final}} \)

\( \Delta \vec{v} = \) "change vector" = how \( \vec{v}_1 \) must be "stretched" to change it into \( \vec{v}_2 \)

\( \begin{array}{c}
\vec{v}_1 \\
\vec{v}_2 \\
\end{array} \)

\( \Delta \vec{v} \)

\( \begin{array}{c}
direction of \vec{a} \quad = \quad direction of \Delta \vec{v} \\
\end{array} \)

\( \begin{array}{c}
direction of \vec{a} \quad = \quad \frac{\Delta \vec{v}}{\Delta t} \\
\end{array} \)

( This has been a preview of Chapter 3, \( \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \) )

Our mantra:
"Acceleration is \textbf{not} velocity, velocity is \textbf{not} acceleration."
Constant acceleration formulas (1D) (relates)

(a) \( v = v_0 + at \) \( (v, t) \)

(b) \( x = x_0 + v_0 t + \frac{1}{2}at^2 \) \( (x, t) \)

(c) \( v^2 = v_0^2 + 2a(x-x_0) \) \( (v, x) \)

(d) \( \bar{v} = \frac{v_0 + v}{2} \)

\( x_0, v_o = \) initial position, initial velocity \( x, v = \) position, velocity at time \( t \)

Reminder: all of these formulas are only valid if \( a = \) constant, so these are special case formulas. They are not laws. (Laws are always true.)

**proof** of formula (a) \( v = v_0 + at \)

Start with \( a = \frac{\Delta v}{\Delta t} = \frac{v_2-v_1}{t_2-t_1} \) (definition of acceleration \( a \))

Notice that since \( a = \) constant, there is no difference between average acceleration \( \bar{a} \) and instantaneous acceleration at any time.

\[
\begin{align*}
  &\left\{ \begin{array}{c}
  v_1 = v_0, \quad v_2 = v \\
  t_1 = 0, \quad t_2 = t
  \end{array} \right. \\
  \Rightarrow \quad a = \frac{v-v_0}{t} \\
  \Rightarrow \quad v = v_0 + at
\end{align*}
\]

(See the appendix or your text for proofs of the remaining formulas.)

**Example: Braking car.** A car is moving to the right with initial velocity \( v_0 = +21 \text{ m/s} \). The brakes are applied and the car slows to a stop in \( t = 3 \text{ s} \) with constant acceleration. What is the acceleration of the car during braking?

\[
a = \frac{\Delta v}{\Delta t} = \frac{v-v_0}{t} = \frac{0-21 \text{ m/s}}{3 \text{ s}} = -7 \text{ m/s}^2
\]

(Do you understand why we have set \( v = 0 \) in this problem?)

Negative acceleration means that the acceleration is to the left.
Let’s stare at the formula \( x = x_o + v_o t + (1/2) a t^2 \) until it start to make sense. You should always stare at new formulas, turning them over in your mind, until they start to make a little sense.

\[
\frac{x - x_0}{\text{how far you travel}} = \frac{v_0 t}{\text{how far you would travel if } v = \text{constant}, \ a = 0} + \frac{(1/2) a t^2}{\text{how much more } (a > 0) \text{ or less } (a < 0) \text{ you travel compared to how far you would have gotten if } a = 0}
\]

**Gravitational acceleration**

**Experimental fact:** In free-fall, near the surface of the earth, all objects have a constant downward acceleration with magnitude \( g = +9.8 \text{ m/s}^2 \). (\( g > 0 \) by definition)

The term *free-fall* means that the **only** force acting on the object is gravity — no other forces are acting, no air resistance, just gravity. Only if air resistance is small enough to ignore, is a falling object in free-fall.

(Later, when we study gravity, we will find out why \( g = \text{constant} = 9.8 \text{ m/s}^2 \) for all objects, regardless of mass.)

Things to notice:
- The acceleration during free-fall is always straight **down**, even though the velocity might be upward. Repeat after me: "Acceleration is not velocity, velocity is not acceleration."
- All objects, regardless of mass, have the same-size acceleration during free-fall. Heavy objects and light objects all fall with the same acceleration (so long as air resistance is negligible).

**Example: Object is dropped from rest.** What is the position, velocity, and acceleration at 1 s intervals as the object falls?

Choose downward as the (+) direction, so that \( a = +g \). If we instead chose upward as the positive direction, then the acceleration would be in the negative direction, and we would have \( a = -g \). Remember, the symbol \( g \) is defined as the **magnitude** of the acceleration of gravity. \( g > 0 \) always, by definition.

\[
x_o = 0, \ v_o = 0 \Rightarrow x = (1/2) a t^2, \quad v = a t \quad \text{(from constant acceleration formulas)} \\
g \approx 10 \text{ m/s}^2 \Rightarrow x = 5 t^2, \quad v = 10 t
\]
<table>
<thead>
<tr>
<th>t (s)</th>
<th>x (m)</th>
<th>v (m/s)</th>
<th>a (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

Notice that you can compute acceleration \( a \) by taking any pair of \((t, v)\) values and computing
\[
a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}.
\]
You always get \( a = 10 \text{ m/s}^2 \).

**Example Projectile Motion:** A projectile is fired straight up from the ground with an initial speed of \( |v_0| = 10 \text{ m/s} \).

♦ Describe the velocity vs. time. (Assume negligible air resistance.)

Choose upward as the (+) direction and set the ground at \( y = 0 \).

\[
y = 0, \quad v = +10 \text{ m/s}
\]
\[
a = -g = -9.8 \text{ m/s}^2
\]
\[
v = v_0 + a t = v_0 - g t
\]

Graph of \( v \) vs. \( t \):

♦ What is time to reach maximum height, \( y_{\text{max}} \)?

At the maximum height, \( v = 0 \Rightarrow 0 = v_0 - g t \), \( t = \frac{v_0}{g} = \frac{10 \text{ m/s}}{10 \text{ m/s}^2} = 1.0 \text{s} \)

♦ What is \( y_{\text{max}} \)?

Method I: \( y(t) = \frac{y_0}{0} + v_0 t + \frac{1}{2} a t^2 = v_0 t - \frac{1}{2} g t^2 \)

At \( t = 1 \text{s} \), \( y = y_{\text{max}} = v_0 t - \frac{1}{2} g t^2 = 10(1) - (0.5)(10)(1)^2 = 10 - 5 = 5 \text{ m} \)
Method II: Use $v^2 = v_o^2 + 2a(y - y_o)$.

Here $v = 0$ (at apex), $a = -g$, and $(y - y_o) = (y_{\text{max}} - 0) = y_{\text{max}}$, so we have

$$0 = v_o^2 - 2gy_{\text{max}}, \quad y_{\text{max}} = \frac{v_o^2}{2g} = \frac{10^2}{2(10)} = 5\text{ m}$$

**Comments about projectile motion:**
- The acceleration is constant (straight down, magnitude $g$), only if we can ignore air resistance. Real projectiles, like cannonballs moving through air, are strongly affected by air resistance. Play with a simulation of projectile motion with and without air resistance. Go to [http://phet.colorado.edu](http://phet.colorado.edu) and find the simulation called Projectile Motion.
- The formula $y = y_0 + v_0 t - \frac{1}{2}gt^2$ is a quadratic equation, so there are two solutions, that is, two values of $t$ for a given value of $y$. These two times correspond to on the way up and on the way down.
- Recall that the direction of $\vec{a}$ = direction of $\Delta\vec{v}$ (not the direction of $\vec{v}$). How does this square with vertical projectile motion?

![Diagram of projectile motion](image)

Notice that "Delta-v" is always downward, regardless of the direction of the velocity.

**Qualitative comments about acceleration**
- You can feel acceleration; you cannot feel constant velocity. If you are in an airplane traveling at constant velocity (heading NW at 600 mph, say) and the ride is smooth, then you can eat dinner, juggle, fall asleep, … exactly as if you were at rest. In fact, if the ride is perfectly smooth, there is no way to tell that you are moving, except by looking out the window.

But you can tell right away if you are accelerating. If you are accelerating forward, you feel yourself being pushed back into the seat. (Actually, the seat is pushing you forward; it just feels like you are being pushed back — more on that later).
If you are in a car, there are two different ways that you can be accelerating forward:
1) Start at rest, and then floor it!
2) Move in reverse at high constant velocity, and then apply brakes!

In both these cases, you are accelerating in the forward direction. In both these cases, you feel exactly the same thing: you feel yourself being pressed back into the seat. Puzzle for later: when you are in a chair that is accelerating forward, why does it feel like there is a force pushing you backwards.

Acceleration and velocity are completely different things. Acceleration is the time rate of change of velocity. Be aware that…

\[
\text{the rate of change of something } \neq \text{ the something}
\]

\[
\begin{align*}
\dot{\vec{a}} &= \text{ rate of change of } \vec{v} = \vec{v} \\
\end{align*}
\]

Example: The radio weatherman says, "The temperature is 48° and the temperature is falling at 10° per hour."

\[
T = 78°, \quad \frac{\Delta T}{\Delta t} = -10°/\text{hr} = -10° \text{ hr}^{-1}
\]

If all you know is \( \frac{\Delta T}{\Delta t} \), then you know nothing at all about \( T \). And if all you know is \( T \), you know nothing at all about \( \frac{\Delta T}{\Delta t} \). They are completely different; knowledge of one tells you nothing about the other.
Appendix:

Proving the constant acceleration formulas:

We can derive the constant acceleration formulas on page 8, without using calculus. Let's derive 
\[ x = x_0 + v_0 t + \frac{1}{2} a t^2. \]

When \( a = \) constant, a plot of \( v \) vs. \( t \) is a straight line (since \( a \) is the slope of \( v \) vs. \( t \)). In this case, the average velocity is half-way between the velocities at the start and finish: 
\[ \bar{v} = \frac{v + v_0}{2} \]

We can also write the average velocity using the definition 
\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0}. \]

Some rearranging and substituting gives 
\[ x = x_0 + \bar{v} t = x_0 + \frac{1}{2} (v_0 + v) t. \]

Now substitute \( v = v_0 + a t \) (proven on p.8): 
\[ x = x_0 + \frac{1}{2} (v_0 + v_0 + a t) t = x_0 + v_0 t + \frac{1}{2} at^2. \]

Done!

To prove \( v^2 = v_0^2 + 2a(x-x_0) \), combine the formulas \( v = v_0 + a t \) and 
\[ x = x_0 + v_0 t + \frac{1}{2} a t^2, \] eliminating time \( t \). (I'll let you do that.)