Now we will discuss circuits with batteries, resistors and capacitors. → “RC Circuits”

Simplest case is just a Capacitor C, charged to a Voltage \( V_0 = \frac{Q_0}{C} \), attached to a Resistor R and a switch.

Initially one has \( +Q_0 \) and \( -Q_0 \) on the Capacitor plates. Thus, the initial Voltage on the Capacitor \( V_0 = \frac{Q_0}{C} \).

What do you think happens when the switch is closed?

Close the switch at \( t=0 \), then current \( i \) starts to flow.

At \( t=0 \), \( i_0 = \frac{V_0}{R} \)

Later \( i(t) = -\frac{dQ}{dt} \)

* Negative sign since \( Q \) is decreasing.

Voltage across \( C \) = Voltage across \( R \)

\[
\frac{Q}{C} = iR = -\frac{dQ}{dt}R
\]

We now need to solve this differential equation.

Solving the differential equation:

\[
\frac{dQ}{dt} = -\frac{1}{RC}Q \\
Q(t) = Q_0e^{-t/RC}
\]

Check solution by taking the derivative...

\[
\frac{dQ}{dt} = Q_0\left(-\frac{1}{RC}\right)e^{-t/RC} = -\frac{1}{RC}Q
\]

Also \( Q(t) = Q_0 \) at \( t=0 \).

\[
Q(t) = Q_0e^{0} = Q_0
\]

Exponential Decay

\[
Q(t) = Q_0e^{-t/RC}
\]

After a time \( t = RC \), Q has dropped by \( e^{-1} = 1/e \).

After a time \( t = 2RC \), Q has dropped by \( e^{-2} = 1/e^2 \)

Thus \( t = RC \) is often called the time constant and has units [seconds].
### Clicker Question

A capacitor with capacitance 0.1 F in an RC circuit is initially charged up to an initial voltage of $V_o = 10$ V and is then discharged through an R=10Ω resistor as shown. The switch is closed at time $t=0$. Immediately after the switch is closed, the initial current is $i_o = \frac{V_o}{R} = \frac{10}{10} = 1$ A. What is the current $i$ through the resistor at time $t=2.0$ s?

A) 1 A  
B) 0.5 A  
C) $\frac{1}{e}A = 0.37$ A  
D) $\frac{1}{e^2}A = 0.14$ A  
E) None of these.

Answer: $\frac{1}{e^2}A = 0.14$ A. The time constant for this circuit is $RC=(10\Omega)(0.10F) = 1.0$ sec. So at time $t=2.0$ sec, two time constants have passed. After one time constant, the voltage, charge, and current have all decreased by a factor of $e$. After two time constants, everything has fallen by $e^2$. The initial current is 1 A. So after two time constants, the current is $\frac{1}{e^2}A = 0.135$ A.

### More Complex RC Circuit: Charging C with a Battery

Before switch closed $i=0$, and charge on capacitor $Q=0$.

Close switch at $t=0$.

Try Voltage loop rule.

$$V_b + V_R + V_C = 0$$

$$V_b - iR - \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} = \frac{V_b}{R} - \frac{Q}{RC}$$

$$Q(t) = CV_b\left(1 - e^{-t/RC}\right)$$

Although no charge actually passes between the capacitor plates, it acts just like a current is flowing through it.

Uncharged capacitors act like a "short": $V_C=Q/C=0$

Fully charged capacitors act like an "open circuit". Must have $i_C = 0$ eventually, otherwise $Q \to \infty$. 

$$\frac{dQ}{dt} = \frac{V_b}{R} - \frac{Q}{RC}$$

$$Q(t) = CV_b\left(1 - e^{-t/RC}\right)$$
An RC circuit is shown below. Initially the switch is open and the capacitor has no charge. At time $t=0$, the switch is closed. What is the voltage across the capacitor immediately after the switch is closed ($t = 0$)?

A) Zero  B) 10 V  C) 5V  D) None of these.

**Clicker Question**

**House Wiring**

Wall socket = 3 prong plug

- "Hot" (~120 Volts)
- "Cold" or "Neutral" (0 – 3 Volts)
- "Ground" (0 Volts)

"Hot" Voltage is AC (Alternating Current) which is sinusoidal and not DC (Direct Current) which is flat with time.

$|V(\text{peak})| = 170 \text{ V}$

$\sqrt{\langle V^2 \rangle} = 120\text{ V}$

Frequency = $1/T = 60 \text{ Hz}$

AC Voltage is more economical to distribute from power plants to homes than DC Voltage (gigantic batteries). In fact this decision resulted from a famous debate between Edison and Tesla. More about this later.

**Standard Electrical Wiring Colors**

- Black = Hot (burnt black)
- White = Cold (like snow)
- Green = Ground (like grass)

!!! Never assume house wiring is correct. Always test first !!!
Fuse Breakers

If $|i(fuse)| > 15$ Amps, breaker opens, preventing the wires from overheating.

![Diagram of fuse breakers showing hot and cold sides with a toaster symbol and current path through a ground safety feature.]

Clicker Question

Does turning the Toaster switch on, increase the chances of the fuse blowing?  A) yes, B) no, C) no change.

![Diagram of a clicker question with a similar setup as the fuse breakers but focusing on the toaster switch and current paths.]