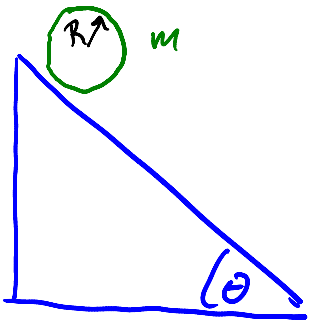
Rolling down an incline

Wednesday, December 06, 2006
5:50 PM

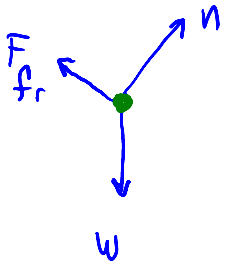
The multiple-choice question regarding torque was probably the hardest of those on the exam. It is a good example of a multiple-choice question which should be answered by treating it as a written question.

An extended object rolls down an inclined plane without slipping.



What is the torque on this object?

FBD:



Three forces act on the object: weight, normal, and friction

$$F_{net_{\parallel}} = mg\sin\theta - F_{fr} = ma$$

Net torque $\tau_{net} = RF_{fr} = I\alpha$

Rolling without friction $\alpha = a/R$

so $RF_{fr} = I\frac{a}{R}$

or $F_{fr} = I\frac{a}{R^2}$

So $ma = mg\sin\theta - I\frac{a}{R^2}$

$$\Rightarrow a = \frac{g\sin\theta}{1 + I/mR^2}$$

Thus

$$\tau = I\frac{a}{R} = \frac{g}{R}\sin\theta \frac{I}{1 + I/mR^2}$$

$$= gmR\sin\theta \frac{I/mR^2}{1 + I/mR^2}$$

Let $c = \frac{I/mR^2}{1 + I/mR^2}$

c is <u>maximum</u> when $I = mR^2$, so torque is also maximum for $I = mR^2$, which is true for the hoop