<u>CH. 2: Kinematics: Describing motion.</u>

Our first goal is understanding the motion of objects. The first step is simple: merely DESCRIBING the motion of something. In Ch. 2, we'll keep life as easy as possible:

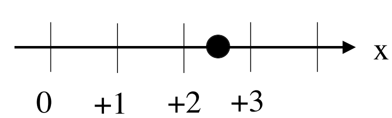
1) We'll only talk about "particles": pointlike objects, whose structure is irrelevant. It's an abstraction, a model for real-life objects. (Consider a spherical horse...)

2) We'll work in one dimension ("1-D"), e.g. a train moving back and forth on a straight track, or a marble tossed straight up and down. (We'll get to more realistic 3-D motion soon enough. The concepts really aren't very different, though)

To describe motion,we need a few basic and critical concepts, quantities, and definitions. We'll use English language words but define them rigorously, mathematically when possible.You'll see that words like "velocity, acceleration, force, energy, momentum..." (which are often sloppy, nebulous, even sometimes synonymous!), are, in physics, totally distinct and well defined.

<u>1) POSITION:</u> Where is the object? You need a *reference frame* to describe position conveniently. A reference frame means a choice of axis and coordinate system: where is the origin, what units will we use to measure length, which direction will we call positive? It's a convention, YOU choose the reference frame.

In this coordinate system (labeled arbitrarily by "x"), the object (the black dot) is at position x=+2.5.



Different people can make different choices! If your coordinate system is, say, shifted to the left from mine, the *number* used to *describe* the position will then be different, but the position itself will be the same for all observers. Your reference frame may even move (e.g., a physics lab set up in a train car).

In 1-D horizontal motion, I will usually pick an origin, and let the positive direction be to the right, like in a number line. If I don't state otherwise, you can assume this. But that's a convention. If there's a reason, I can make left be positive. For vertical motion, We sometimes let "up" be +, and sometimes "down". It's important to a draw coordinate system to define conventions in any problem!

Position has a SIGN in 1-D: x=+2.5 and x=-2.5 are totally different positions.(Position has a DIRECTION in more than 1-D. Position is a *vector*. More on this in Ch. 3)

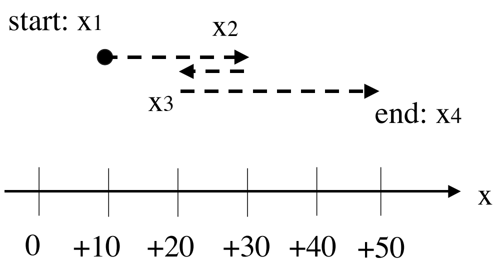
<u>2) TIME:</u> When does an event occur? You need a reference frame here too: when do you define "t=0" to occur? I label time by "t", which is an INSTANT or POINT in time.

E.g. 3:04:25 PM, or "t=2 sec" on a stopwatch.

<u>3a) DISPLACEMENT</u>: This is the net CHANGE in position. E.g. $\Delta x = +2\ m$: the object has *moved* 2 meters to the right (with my usual convention of "+" = "right") The Greek letter there is a "Delta", it always means "change".

$\Delta x = -2\ m$ means something different, the object has moved 2 meters to the *left*.

<u>3b) DISTANCE</u>. The total length of the path the object has traveled. It's different from displacement in several ways. It's a positive number, a scalar. If an object moves forward 2 meters and then back 2 meters, the DISTANCE traveled is 4 meters, but the displacement (the NET CHANGE) is zero!



<u>Example</u>: An object starts at +10, moves right to x=+30, moves back left to +20, then finally moves right to +50, as shown.

The overall displacement is $\Delta x = 50\ m - 10\,m = +40\,m$.

The overall *distance* is 20+10+30=60 m (do you see why?)

Mathematically: $\boxed{\Delta x = x_{final} - x_{initial}}$, or if you prefer, $\boxed{x_i \ + \ \Delta x \ = \ x_f}$

Position and displacement are useful, but when describing motion, you often care about more, e.g. how fast it's moving (which displacement alone doesn't tell you).

We DEFINE a useful measure of "how fast" to be

<u>4a) AVERAGE SPEED</u> = (distance traveled)/(time taken).

This is always +, it's a scalar. In the previous graph, if we started at t=0, and then point x2 was reached at 20 sec, point x3 at 30 sec, and the end was at 60 sec, then average speed = (60 m)/(60 sec) = 1 m/s.

4b) AVERAGE VELOCITY = (displacement)/(time taken)

This has a sign, or a direction: it's a vector. Given my conventions, v>0 will mean $\Delta x > 0$, i.e. the object is moving to the right. If v<0, the object is moving leftwards. The value of the position is TOTALLY irrelevant, only the CHANGE in position matters! (An object at x=-9,999 can be moving to the right or left!)

A car moving right at 10 m/s has a TOTALLY DIFFERENT velocity than a car moving left at 10 m/s! The SPEED is the same, but the left moving car has a velocity of -10 m/s.

Example: You go from x=0 to x=10 and back to x=0 in a total of 60 seconds. Your average velocity over the total time period is zero! (Total displacement is zero, x_final = x_initial =0)

Your average speed is not zero. (If you ran with steady speed, it's 20 m/60 sec = .33 m/s.)

Mathematically, $\boxed{\bar{v} \equiv \frac{\Delta x}{\Delta t}}$. The triple equals sign means "is defined to be".

The bar over the v is used to represent an *average*, this formula tells the average velocity over some time *interval*.

In our previous graphical example, when averaged over the first 20 seconds,

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{30\,m - 10\,m}{20\sec - 0\sec} = 1 \text{ m/s.}$$

However, if you average over the first 60 seconds instead,

$$\bar{v} = \frac{x_4 - x_1}{t_4 - t_1} = \frac{50\,m - 10\,m}{60\sec - 0\sec} = 0.67 \text{ m/s.}$$

The average velocity depends on the time interval considered! (Makes sense. If you turn around in a race and run back to pick up something, of course your average velocity will be lowered. Even if you run back very quickly!)

Units of velocity (indicated by square brackets) are [v] = [m/s] or [m*s^-1] or [mi/hr] or ...

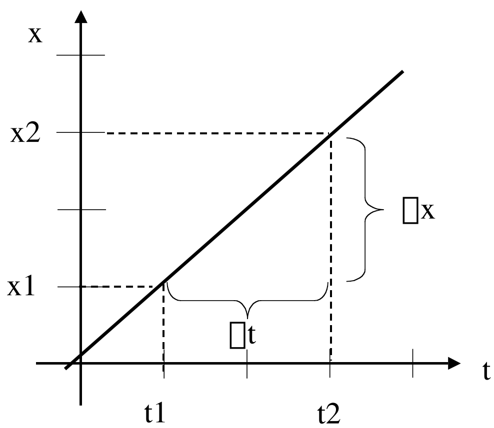
People say "miles per hour": 20 mi/hr = "20 miles per hour", you go 20 miles EACH hour. (You might not go for an hour, but that's what you WOULD travel in an hour...)

<u>Example</u>: Unit conversion: What's "1 m/s" in familiar driving units?

1 m/s * (1 km/1000 m) * (60 sec/min) * (60 min/hr) * (1 mi/1.6 km) = 2.3 mi/hr

Velocity lets you predict the future! if $\bar{v}$ =10 m/s, you'll go 10 meters to the right every second, and after some time interval, you've displaced : $\boxed{\Delta x = \bar{v}\,\Delta t}$.

Graphically, we most often draw "position of something as a function of time":



The slope of a line is rise over run, from the picture, you can see that the average velocity is the *slope* of the position vs. time curve:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

If this line is "tipped up" (positive slope), the average velocity is positive, the object is moving towards larger x. If the line is "tipped downwards" (negative slope), the object is moving to the left, towards smaller x. Then the average velocity is negative.

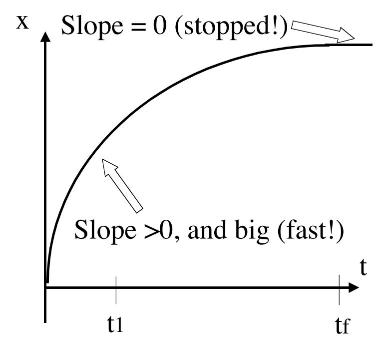
Average velocity is useful, but still not the whole story. It depends on the time *interval* considered. If I drive 50 km to Denver in 1 hr (assume a straight line path, with Boulder at x=0, and Denver at x=+50 km), then $\bar{v}$ =+50 km/hr. But watch the speedometer. (Which, by the way, is well named! It's *speed*, not velocity, that it tells you. Direction is irrelevant, speed is always +) It may vary with time, sometimes fast, sometimes slow.

You may care about your velocity (or speed) RIGHT NOW, which we call INSTANTANEOUS velocity (or speed). It's defined to be the velocity over a very short interval of time. In fact, an infinitesimally short interval of time (Your speedometer approximates your instantaneous speed by measuring the average speed over a *very* short time interval, namely the time for the tire to turn around once!)

Imagine an object moving right, rapidly at first, but slowing down to a halt.

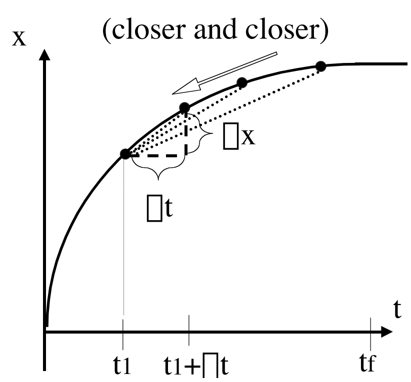
(E.g. my car, approaching a traffic jam!)

Here's a possible graph of position vs. time:

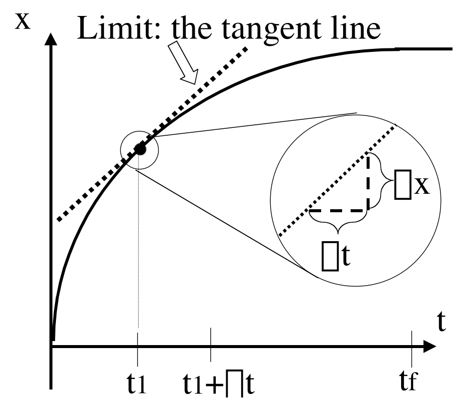


You could ask what the average velocity was, over the whole time interval t=0 to t=tf. Or, over the shorter interval t=0 to t=t1. (this would be *larger*, because we were going faster in those earlier times)

But you might want to know the instantaneous velocity right at the moment t1. That would be given, graphically, by the average velocity for a TINY little delta t, right around time t1. On the graph, you want the slope as you "zoom in" on the curve at that time. If you try to draw little "chords" (whose slope gives average velocity) and make the two end-points very close together, you will get a better and better approximation to the slope at the point.



The instantaneous velocity is, as you can see, the slope of the TANGENT LINE to the curve at the time t1. Mathematically, this slope is the



5a) INSTANTANEOUS VELOCITY at time t1: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$.

We say v is the "derivative of position with respect to time". If I use the word "velocity" alone, I generally mean instantaneous velocity. (If I intend "average", I'll explicitly say so) We will also refer to

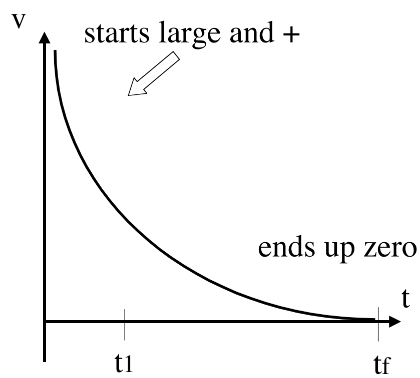
5b) INSTANTANEOUS SPEED (or just speed), defined to be the size of v (the magnitude of the velocity). Your speedometer is measuring the size of your instantaneous velocity.

Speed and velocity are closely related, but technically different. (Velocity has a direction, or in 1-D, a SIGN, which is meaningful!) I will *try* hard not to mix them up, but because they are synonyms in "common usage", I will surely mess up from time to time.

In that last curve, the slope of the tangent line varies from place to place. First, it's quite steep (high velocity), but as time goes by, the tangent gets less and less steep. By the end, (time tf) the curve is horizontal, the tangent line has 0 rise over run, slope=0. Velocity has gone to zero. It makes sense, x isn't increasing with time there, the car is stopped!

It can be very useful to graph velocity vs. time. It's a graph of the SLOPE of the previous plot, i.e. the derivative of the position vs. time plot.

In that last example, as we just said, it starts off large, and decrease towards zero, and might look something like this:



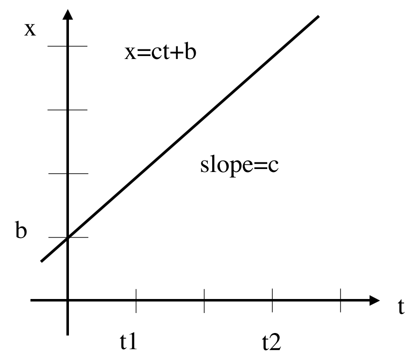
If you have a graph (or table) of x versus time, you just measure the slope of the tangent. If you have a *formula* for x versus time, i.e. the function x(t), then calculus tells us immediately what the velocity is, v = dx/dt. The Appendix A-2 of your text has a lot more details. (See also the appendix at the end of these CH. 2 notes.) The most common formula we'll use is for the derivative of a simple polynomial:

If $x = c\,t^n$, (where c and n are some constants), then v = dx/dt = $(c \cdot n)\ t^{n-1}$.

Also, the derivative of a sum is the sum of derivatives.

<u>Example</u>: If x = c*t+b (this is a straight line, it means x is increasing linearly with time), then v= dx/dt = $(c \cdot 1)\ t^{1-1} + 0 = c$.

(This makes sense, a straight line has constant slope, given here by c, of course!)



If v is constant, (as in this previous example), life is predictable, simple, and natural. Position is steadily increasing, it's like a cart on a frictionless air track sliding along with constant speed forever. It's also very DULL. Nothing is really happening! When you sit in an airplane going 700 mi/hr, you might think that should be kind of exciting, but inside the plane, you're totally unaware of this speed, because it is *constant*. Inside the plane, life is kind of boring. It only gets interesting when the velocity CHANGES, when you hit turbulence or go into a dive or something.

So, it's Δv that's interesting. But not JUST Δv. The amount of time for that change matters too. If you're in the back of the car, reading, and the car goes from 60 mph to 0 in a minute, you barely notice. (Must be time for a pit stop..) But if the car goes from 60 mph to 0 in half a second, that's a serious crash!

What's interesting is the ratio (change in velocity)/(amount of time). We give this a name:

6a) AVERAGE ACCELERATION, $\boxed{\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}}$. And, as before, we define

6b) INSTANTANEOUS ACCELERATION, $\boxed{a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}}$.

If I talk about "acceleration", I mean instantaneous acceleration, at some instant in time.

Why is it fun to drive a Porsche, even if you never break the law and exceed 65 mi/hr? Because it can have a large acceleration. You can change velocity in a short amount of time. Acceleration is NOT velocity. It is the CHANGE in velocity, over some time. (Just like velocity is NOT position. It's the change in position/time.) You can have a very large acceleration even if velocity is ZERO, it just means the velocity is rapidly CHANGING.

Units of acceleration are [a]=[(m/s)/s] = [m/s^2] or [(km/hr)/sec], or [(mi/hr)/sec], etc....

<u>Example</u>: My Honda Civic can go from 0 to 60 mi/hr in 12 seconds.

It has an average acceleration of $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{60 \text{ mi / hr}}{12 \sec} = 5 \text{ (mi / hr) / sec}$

You can use acceleration to "predict the future"! If I start from rest, and maintain a constant acceleration, I use $\boxed{\Delta v = v_f - v_i = \bar{a}\,\Delta t}$ to predict speed at later times:

In the last example, with a=5 (mi/hr)/s, (starting at rest, v_i=0):

At time t=1 sec, I’m going v= (5 mi/hr)/sec*1 sec = 5 mi/hr

At time t=2 sec, I'm going v= (5 mi/hr)/sec * 2 sec = 10 mi/hr

At 3 sec, I'm going 15 mi/hr.

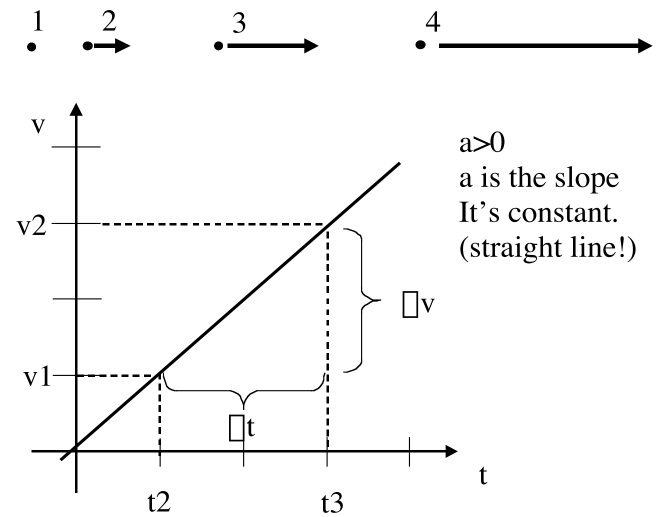
(Notice that the CHANGE in the last second was 5 mi/hr. But, I had been going 10 mi/hr, so now I'm going 15 mi/hr)

At 4 sec, v=20 mi/hr. etc.

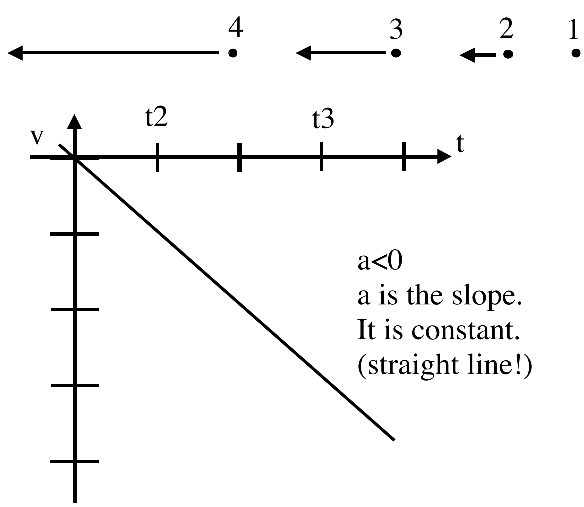
At 12 seconds, as stated, I'm going v=(5 mi/hr)/sec*12 sec = 60 mi/hr.

Acceleration has a sign, which is meaningful. (It's a vector) “a” is the SLOPE of the graph of velocity versus time. The sign is a bit tricky to interpret, but it is IMPORTANT, you need to think about it. If a>0, that means that the CHANGE in v must be positive, i.e. v is increasing. If v starts out zero, and a>0, it means v will be getting bigger and bigger with time. You will move faster and faster to the right. But (important): a>0 does NOT mean that v has to be positive! You might have started off with a big negative v, i.e. moving rapidly to the left, but *less rapidly with time*: That’s still a>0, it means v is GROWING MORE POSITIVE. (or less negative, same thing!)

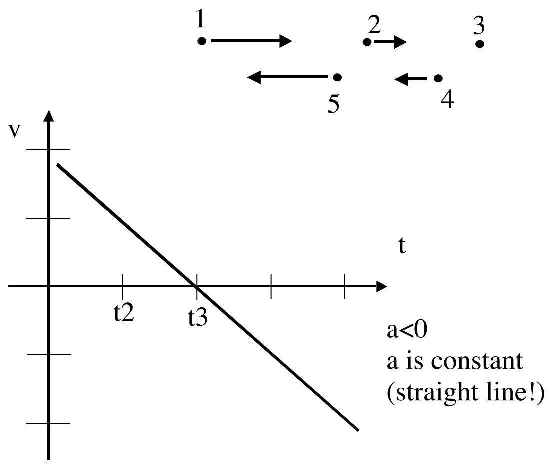
Example I: (simplest) Object starts at rest, moves right (+ direction) faster and faster.



Example II: Object starts at rest, moves left (- direction) faster and faster.



<u>Example</u> III: Object starts moving to the right, but it's slowing down. It comes to a halt, and then moves to the left, now going faster and faster.



Think about this one! Even though it was SLOWING down at first, and SPEEDING up later, this is a case of constant, negative acceleration. a is the same the whole time. (Slope of a straight line is constant) Even at the moment when the object came to rest (v=0), the slope of the v vs. t curve is still the same constant negative value!

The direction of acceleration is the direction of the CHANGE of v, not the same as the direction of v itself. Here's a different way of thinking, pictorially, about the sign (or direction) of a.

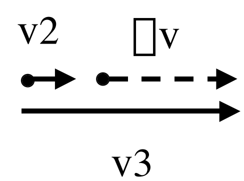
Use the fact that $v_i + \Delta v = v_f$. (Acceleration and Δv are in the same direction, i.e. the same sign, since $\Delta v = \bar{a}\,\Delta t$, and change in time is always a positive number)

If you want to know which way "a" is, just draw v_init, and v_final, and look which way the CHANGE is. Go back to those same last three examples:

Example I (let v_init be at time t2, and v_final be at t3)

Delta v is POSITIVE, or to the right.

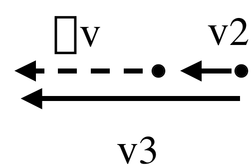
It's a positive acceleration. The *change* in v is to the right



Example II (let v_init be at time t2, and v_final be at t3)

Here, Delta v is NEGATIVE, or to the left.

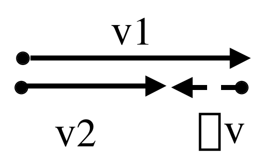
It's a negative acceleration. The *change* in v is to the left.



Example III (let v_init be at time t1, and v_final be at t2)

Here, Delta v is NEGATIVE again, leftwards.

It's a negative acceleration. The *change* in v is to the left, even though v itself (both initial and final) is still rightwards.



(In this last example, suppose v1=+10 m/s, and v2=+5 m/s, then

$\Delta v = v_2 - v_1 = 5\,\frac{\text{m}}{\text{s}} - 10\,\frac{\text{m}}{\text{s}} = -5\,\frac{\text{m}}{\text{s}}$ is negative, just like the picture shows.)

Remember, acceleration is NOT velocity, the direction of a is not the same as the direction of v! ("Slowing down right" motion has the same sign "a" as "speeding up left" motion!)

$a>0$ means velocity is getting "more rightward" or "less leftward",

$a< 0$ means velocity is getting "more leftward" or "less rightward".

(Replace with "up" and "down" appropriately, if motion is vertical)

I said before that it's change in velocity that you care about. You can FEEL acceleration. You cannot feel velocity. (Think of being in that plane at a steady 700 mph. If you don't look out the window, you'll never know) If you accelerate forwards in a car, you *feel* it - you feel a pressure in your backside. There are (at least) 2 ways to accelerate forwards in a car: 1) start at rest and floor it. You feel pressure in your back. 2) Move in reverse, fast, and then slam on the brakes! Again, you feel the SAME direction of pressure in your back! In both cases, it's the forwards acceleration. Your velocity might be forwards, or zero, or backwards, and you can still feel the *same* acceleration.

How about *backwards* acceleration? If you are moving forward and slam on the brakes, that's a negative acceleration. You feel the seat belt tugging you back! Or, equivalently, you might start at rest, put the car in reverse, and floor it. Again, the seat belt tugs... These are equivalent, but each is very different than the forward acceleration situations above. The key point: the CHANGE in something is not the same as the something. The change in velocity is NOT the same as velocity!

Going back to those examples with constant "a", the graph of "v vs. time" was a straight line. However, if velocity is CHANGING, that means the graph of "x vs. time" must NOT be a straight line, the velocity (slope) is changing. Let's work this all out analytically.

Constant Acceleration formulas in 1-D

$$i)\ v_f = v_i + a\,\Delta t$$

$$ii)\ x_f = x_i + v_i\,\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$iii)\ v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$iv)\ \bar{v} = \frac{v_i + v_f}{2}$$

These formulas are ONLY true if "a" is constant!

When acceleration is *zero*, formula *ii* reduces to $x_f = x_i + v_i\,\Delta t$ (often useful too)

Note: I use a slightly different notation than the book ("i" for init and "f" for final) just for clarity. You can use any subscripts you want! I will frequently leave off the "f", and e.g. just call final position "x". (We usually set t_init=0, by convention, so that $\Delta t = t_f - t_i = t - 0 = t$)

These formulas are all very useful, we'll use them all semester long. They're worth memorizing, and understanding! The proofs all come straight from the definitions of "v", "$\bar{v}$", and "a".

<u>Example</u>: Proof of *i*) just uses the definition of acceleration: $\boxed{\bar{a} \equiv (v_f - v_i)/\Delta t}$.

Multiply both sides by Δt, add v_init to both sides, and that's it.

(Notice that if "a" is constant, there's no difference between "a" and "$\bar{a}$".)

The other proofs are similar, and done in the book. Try to work them out yourself.

Example: A car starts at rest, and accelerates uniformly. After 5 secs, it passes the 25 m mark on the track. What was the acceleration?

Soln: Let me choose t_init=x_init=0. It started at rest, so v_i=0 is given.

After 5 sec, ∏*t*=5 s, and x_final=25 m is given. I want "a".

Eq. *i* might look ok, but I didn't tell you v_f! Instead, eq. *ii* is the best, it contains only known quantities and the one unknown, "a". Plugging into *ii* gives

25 m = 0 + 0*t + (1/2)*a*(5 sec)^2. Solve this for a, getting

a = 2*(25 m)/(25 sec^2) = 2 m/s^2

(Notice that in the t^2 term, the 5 got squared, it's very easy to forget that!

Same with the units, the secs get squared.)

Example continued: What was the velocity at 5 sec?

Soln: Now we can use equation *i*, because we found "a". Just plug in,

v_final = 0 + (2 m/s^2)*(5 sec) = 10 m/s. Notice how the units work out.

It's VERY useful to include units in all quantities. It's a great way to catch mistakes! Units MUST work out right, or you've used a wrong equation, or made some algebra mistake.

Puzzle regarding this example: We just found v=10 m/s. So, why did we not travel distance = v*t = 10 m/s * 5 sec = 50 m? (Remember, we only went 25 m)

<u>Resolution of that puzzle:</u> We weren't going 10 m/s the whole time! (We started at rest) You'd only go 50 m if you had traveled 10 m/s the *whole time*. The average speed is (1/2)*(0+10 m/s) = 5 m/s, and indeed, 5 m/s for 5 seconds takes us 25 m.

<u>An alternate soln to previous "example continued":</u> Use *iii*:

v^2 = 0^2 + 2*(2 m/s^2)*(25 m) = 100 m^2/s^2.

Take the square root of both sides: v = ± 10 m/s, (you have to think about the ambiguity in the sign, so the previous solution is probably better)

<u>Next example:</u> At t=5 sec, the driver above releases the gas, and slams on the brakes. She stops after traveling an additional 15 m. What was "a" during the braking period?

<u>Soln:</u> It's basically a new problem - we have a new "init", which is at t=5 sec. So I will use v_init=10 m/s, instead of 0! I didn't tell you how long she braked, so I look for the equation that does NOT involve time, namely *iii*. Final" is now when the car stops, so v_final=0. Plugging into *iii* gives

(0 m/s)^2 = (10 m/s)^2 + 2 a (x_final-x_init) = (100 m^2/s^2) + 2 a (15 m)

(I don't really need to know x_final or x_init, just the difference, and that's precisely what I gave in the problem, that's what the "additional 15 m" means.)

Solving for "a": a = -100 m^2/s^2 / (2 * 15 m) = -3.33 m/s^2.

The sign is CORRECT, don't ignore it, or try to "fudge it away"!

The car is slowing down, acceleration really is negative, i.e. to the left

<u>GRAVITY</u> Objects falling without friction (or objects sliding down a frictionless ramp) have constant acceleration. This is an experimental fact. It *may* seem counterintuitive (?) When you slide down a ramp, it seems like you have constant velocity (after awhile) i.e. a=0. But that's because of friction. Let's make a definition: FREELY FALLING OBJECTS are objects which feel no force *except* gravity. So, no friction. Their "history" is basically irrelevant. They may have been dropped, launched, thrown up, or down, but once they're "on their own", they are called freely falling (even if they're still going up!)

Galileo discovered, by careful experiments, that all freely falling objects have the SAME acceleration, namely g = 9.8 m/s^2 downwards. (I *define* the symbol "g" to be the positive quantity +9.8 m/s^2) g is "the acceleration of gravity" (not "gravity") and I will frequently approximate it as 10 m/s^2. For homework, use 9.8 m/s^2. (If you're designing bridges or cars or rockets, find out the next decimal place or two..) It's about 32 ft/s^2.

This is a remarkable fact of nature: all objects, heavy or light, accelerate at the *same* rate, if they freely fall. Many smart people did not believe or understand this for many 1000's of years of human history! To be fair, air resistance confuses the story - if there is friction, the object is not "freely falling". But for *many* real life situations, air friction is surprisingly negligible. Aristotle argued that heavy rocks must "obviously" fall faster than light ones. It's interesting that neither he, nor anyone for almost 2000 years bothered to pick up some rocks and just try it!

Acceleration of gravity is constant and down, even if "v" is up! Because “a” is constant, you can use those kinematics formulas to solve for the position and velocity of any freely falling object. The only trick is this: you must choose a coordinate system. If you decide to call "up" positive, then you MUST plug in "a=-9.8 m/s^2", or "a=-g", into those formulas, because acceleration is DOWN! You have to be consistent - in this case, an object moving upwards has positive v, a downwards moving object has negative v. You can set "x=0" to be anywhere you want, but with these conventions, higher objects will have more positive values of x.

If instead you decide to call "down" positive, you should plug in "a=9.8 m/s^2", or "a=g" into the formulas (which is most people's instinct.) But to be consistent, an object moving up now has NEGATIVE velocity!! Think about this, if you get these signs wrong, it's a disaster. (With this convention, the higher an object is, the smaller, or more negative its position, x.)

<u>Example</u>: Drop a ball from rest. How fast is it going after 1 sec? 2 sec? 3 sec?

<u>Soln</u>: Let's set "up" to be positive, and use the formula v_f = v_i + a*t.

Here, v_i =0 and a=-g=-9.8 m/s^2 (roughly -10 m/s^2) giving $v_f = -g\,t$

At 1 sec: v_f = -10 m/s^2*1 sec = -10 m/s. (Speed = |v| = 10 m/s)

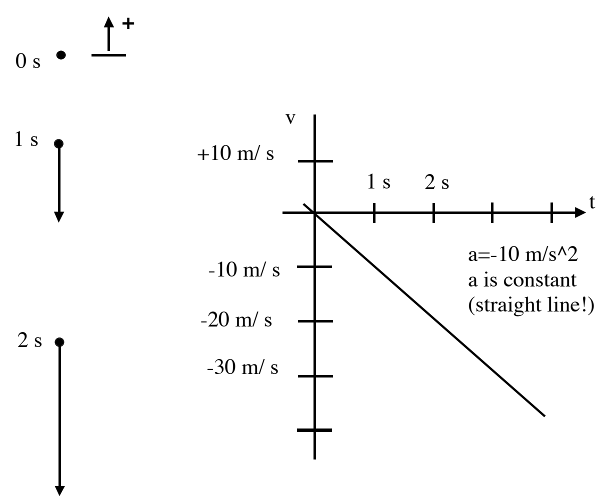
At 2 sec: v_f = -20 m/s. (Speed = |v| = 20 m/s)

At 3 sec v_f = -30 m/s.

The velocity is negative, it's going down, makes sense. It's going faster and faster.

10 m/s *faster*, each second. That's what an acceleration of 10 (m/s)/s means!

Graphically, that last example looks like this:



<u>Next example</u>: Throw a ball up, with initial speed 10 m/s. Find velocity AND position as a fn of time as it rises (and falls).

<u>Soln</u>: Once again, I set "up" to be positive. For velocity use the formula $v_f = v_i + a*t$. Now, $v_i = +10$ m/s. (It's thrown up, so that's positive). We have

$v(t) = v_i - g*t$ (notice the sign in front of g!)

$= +10 \text{ m/s} - g*t = +10 \text{m/s} - 10 \text{ m/s}^2 * t$.

At t=0, v(0) = 10 m/s - 0 = 10 m/s (of course, that was the given initial condition)

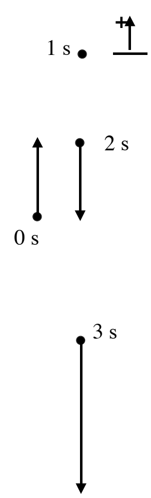
At t=1 s: v(1) = 10 m/s - 10 m/s^2* (1 sec) = 10 m/s-10 m/s = 0 m/s.

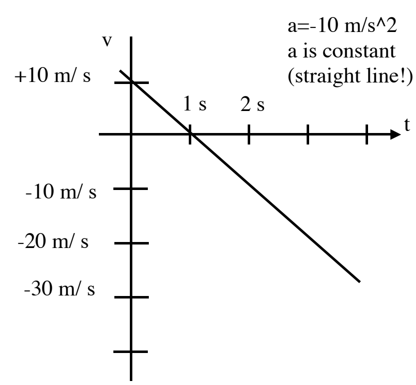
(It's at rest, it reached the top!)

At t=2 s: v(2) = 10 m/s - 10 m/s^2 * (2 sec) = 10 m/s - 20 m/s = -10 m/s.

(On it's way back down....)

Graphically, that last example looks like this:





The graph is similar to the previous one, though it *seems* like a different problem. But, really only v_init is different!

At 1 sec, the graph passes through v= 0. At that time, a = -10 m/s^2 as always. There's nothing SPECIAL about t=1 sec in this graph! Many people think that when it reaches the top of its trajectory (when v=0, at 1 sec), that maybe a=0, but it is NOT, the object was on its way up, it's about to go down, its velocity is CHANGING, same as always, by 10 m/s every second.

How about position? The formula says $x_f = x_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2$. This means

x(t) = v_i *t - (1/2) * g * t^2

= +10 m/s*t - (5 m/s^2) * t^2

At t=0, x(0) = 0 (of course! We defined x_i=0. It's nice to check that your formulas give you back your initial conditions, though)

at t=1 s: x(1) = 10 m/s*1 s - 5 m/s^2 * (1 s)^2 = +5 m

at t=2 s: x(2) = 10 m/s*2s - 5 m/s^2 * (2s)^2 = 0 m

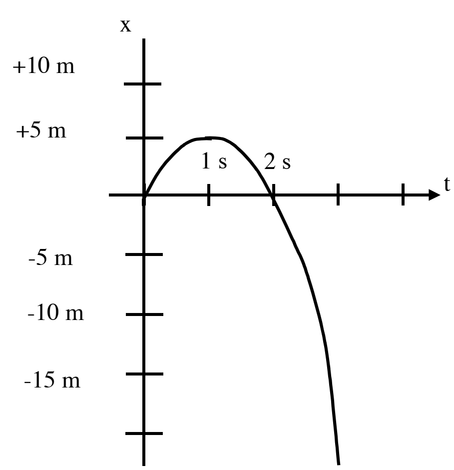
at t=3 s: x(3) = 10 m/s*3 s - 5m/s^2 * (3s) ^2 = -15 m

At 1 second, x is positive, the object is above where it started. Makes sense. That's the top!

At 2 sec, it's back down to the origin.

At 3 sec, x is negative. As time goes by it falls farther and farther. (I'm assuming there's no floor to stop it! If there were, acceleration would not be constant...)

The graph is a parabola (it's supposed to be, my drawing isn't perfect) Once it's on its way down, it goes farther and farther ever second, because its speed is getting larger and larger. It's accelerating!



<u>Next example:</u> In the previous situation, suppose the ball hits the ground at -40 m. (i.e., suppose my "launch point", which I'm calling x=0, is really 40 meters above ground level) How *long* will it take, total, to hit the ground?

<u>Soln:</u> We already found the formula x(t) = $\boxed{v_i t - \frac{1}{2} g t^2}$

= +10 m/s*t - (5 m/s^2) * t^2.

Setting x(t)=-40 m (and then adding 40 m to both sides) we then need to solve

-5 m/s^2*t^2 + 10 m/s*t +40 m = 0, which is a quadratic equation.

There are two solns to ax^2+bx+c=0, namely $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

In this case, the two solutions are

$$\frac{-10\ \mathrm{m/s} \pm \sqrt{100\ \mathrm{m^2/s^2} - 4(-5\ \mathrm{m/s^2})(40\ m)}}{2(-5\ \mathrm{m/s^2})}$$ = (-10 m/s +/- 30 m/s)/(-10 m/s^2)

(look at *all* the - signs in there, make sure you understand each one!)

There are two answers then: +4 s, or else -2 s.

Negative time doesn't make sense - it hits AFTER we throw it - so use the positive answer, 4 s. (although if you think about it, if it *had* been launched earlier, from somewhere way below, then at t=-2 seconds it would've been at x=-40 m and *climbing,* still on its way up to x=0 (and v=+10) at t=0, which was *our* starting point. So there is some physics to that negative answer after all.)

It's a good exercise to go through these kinds of problems using the opposite sign convention (down is positive) and see that you get the SAME physical answers to all the questions.

Little Calculus Appendix:

Let's look at the function f(x) = c x^2, and prove that df/dx = 2*c*x.

(This is a special case of our formula dx/dt = $(c \cdot n)\ t^{n-1}$, if you have $x = c\,t^n$,

where c and n are some constants).

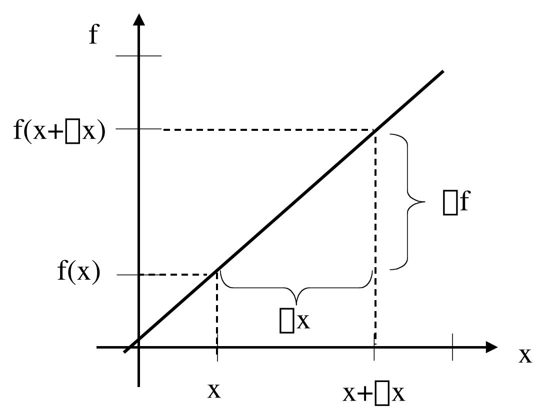
The basic definition of the derivative is df/dx =

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

Let's forget about the "lim", and just imagine Delta x is very small.

So, zoom in on a graph of f(x) vs x.

No matter how curvy a graph is, when you really zoom in, it always looks straight. Here's what f(x) (that parabola, c x^2) looks like, at some random x.



Mathematically, if f(x)=cx^2, then

$$f(x+\Delta x) = c(x+\Delta x)^2 = cx^2 + 2c\,x\,\Delta x + c\,\Delta x^2.$$

Plugging in,

$$\frac{df}{dx} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(cx^2 + 2c\,x\,\Delta x + c\,\Delta x^2) - (cx^2)}{\Delta x}$$

$$= 2cx + c\,\Delta x.$$

In the limit that second term (with c*Delta x) gets tiny, you just have 2cx, as advertised.