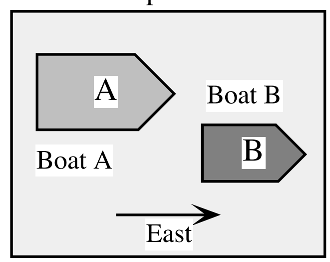
CONSERVATION OF MOMENTUM IN ONE DIMENSION

I. The concept of the momentum vector

The momentum vector, $\vec{p}$, of an object is defined as the product of its mass and its velocity ($\vec{p} = m\vec{v}$). The momentum vector of a system of multiple objects is defined as the sum of the momentum vectors of the individual objects ($\vec{p}_{\text{system}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$).

A. Two small boats, boat A and boat B, are in a lake. An observer on the shore sees the boats moving with the same velocity, 1 m/s, due east. The mass of boat A is 10 kg, and the mass of boat B is 5 kg.

Top view

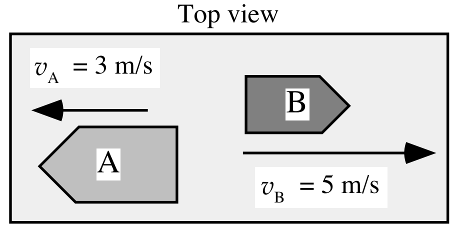


1. Find the magnitude and direction of the momentum vector of each boat.

2. Consider the system of boats A and B together. Call this system C. What is the magnitude and direction of the momentum vector of system C? Explain.

B. Much later, an observer on the shore sees the boats moving with different velocities. Boat A moves with velocity 3 m/s, due west, and boat B moves with velocity 5 m/s, due east.

Top view



1. Find the magnitude and direction of the momentum vector for each boat.

2. What is the magnitude and direction of the momentum vector for system C? Explain.

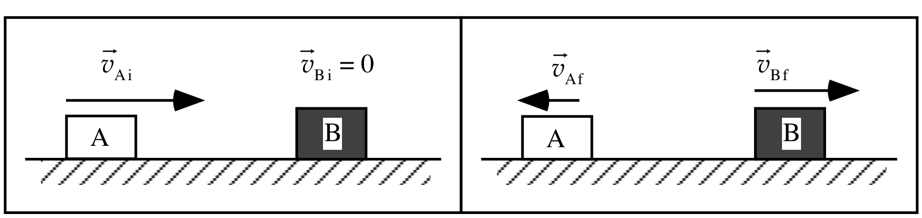
II. Momentum of objects in a collision

A. Experiment 1 is conducted with two gliders on a level, *frictionless* track. In Experiment 1, glider A moves toward glider B, which is initially at rest. After the collision, glider A has reversed direction.

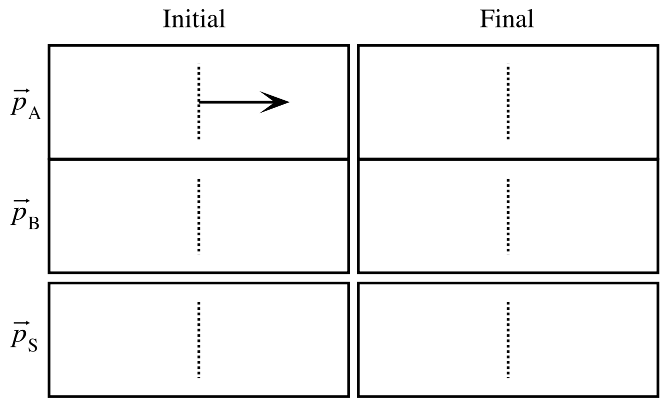
The mass of glider B is greater than the mass of glider A ($m_B > m_A$). Magnets are affixed to the gliders so that the gliders repel each other without touching.

Before collision After collision

Experiment 1



1. In the boxes at right, draw vectors to represent the momentum of glider A, glider B, and system S (the system of both gliders) both before and after the collision.



2. Is the magnitude of the final momentum of glider B ($|\vec{p}_{Bf}|$) *greater than, less than,* or *equal to* the magnitude of the final momentum of the system ($|\vec{p}_{Sf}|$)? Explain.

   Would your answer change if glider A had moved to the right after the collision? Explain.

3. Consider the following student comment about Experiment 1:

   "*Glider A is still moving after the collision. Glider A keeps some of the momentum that it had initially, giving B only a portion of it. There is a limited amount of momentum in the system, and the most any one object can have is all of the system's momentum. Therefore, p_{Bf} must be less than p_{Sf}.*"

   Do you agree or disagree with the student? Explain.

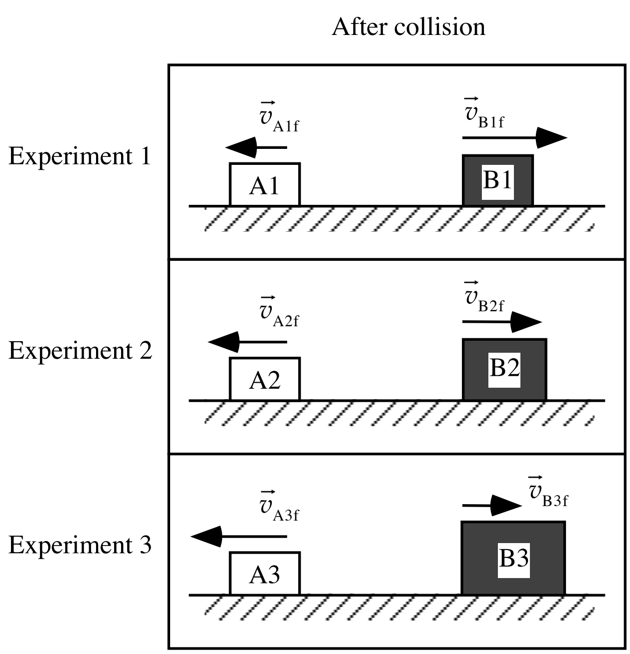
4. Check that your answer to question 2 is consistent with your answer to question 3 and with the law of conservation of momentum.

B. Two new experiments, 2 and 3, are performed. They are identical to Experiment 1, except that mass has been added to the target gliders B2 and B3 so that $m_{B3} > m_{B2} > m_{B1}$. Gliders A1, A2, and A3 are identical and have the same initial velocity.

Repelling magnets are used in all three experiments.

It is *observed* that, after the collisions:

- gliders A1, A2, and A3 move to the left with $v_{A3f} > v_{A2f} > v_{A1f}$.
- gliders B1, B2, and B3 move to the right with $v_{B1f} > v_{B2f} > v_{B3f}$.



Rank $|\vec{p}_{B1f}|$, $|\vec{p}_{B2f}|$, and $|\vec{p}_{B3f}|$ from greatest to least, where $|\vec{p}_{B1f}|$ is the magnitude of the final momentum of glider B1, *etc*. Draw vector diagrams to support your ranking.

C. Experiments 4, 5, 6,…100, *etc*., are set up in the same pattern as the first three experiments. The mass of glider B100 is *much larger* than the mass of glider A100.

It is *observed* in Experiment 100, that:

- the final speed of glider B100 is very nearly zero.
- glider A100 moves to the left with a final speed very nearly equal to its initial speed.

1. Is the magnitude of the final momentum of glider B100 *greater than, less than*, or *equal to* the magnitude of the initial momentum of glider A100? Explain.

2. Three students discuss question 1, about the final momentum of glider B100:

Student 1: *"The final speed of A100 is almost the same as its initial speed. This means that A100 kept almost all of the momentum, giving B100 almost no momentum."*

Student 2: *"Right. You can also see that the momentum of B100 has to be nearly zero because momentum is mass times velocity. If the velocity is very small, then the momentum also must be very small."*

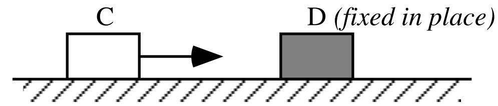
Student 3: *"I don't think we can say anything about the momentum of B100 without actual numbers. As we go through the experiments, the mass of B increases, while the final speed of B decreases. The product of m_B and v_{Bf} might be the same for different experiments, or it might be different."*

With which student, if any, do you agree? For any statements that are incorrect, discuss the errors in reasoning in those statements.

D. Suppose that glider A100 has initial momentum 3 kg•m/s, to the right. After the collision, glider A100 has final momentum very nearly equal to 3 kg•m/s, to the left. What is the magnitude and direction of the final momentum of glider B100? Show your work.

III. Changes in momentum of a system of multiple objects

A. A new experiment is performed with two gliders, C and D, of *equal mass*. Glider D is fixed in place. Glider C is launched toward glider D and rebounds with the same speed that it had initially.



1. In the spaces provided, draw separate free-body diagrams for each glider and for the system of the two gliders at an instant during the collision in this new experiment.

Free-body diagram for glider C	Free-body diagram for glider D	Free-body diagram for system S

Explain how the fact that glider D is fixed in place is reflected in your free-body diagrams.

2. Does the momentum of each of the following change during the collision? Explain.

- glider C
- glider D
- system S

B. Consider the experiments described in parts II and III. When the momentum of an object or system of objects did not change:

- were *external* forces exerted on the object or system?
- was there a *net* force on the object or system?

C. When the momentum of an object or system of objects does not change with time, the momentum of the object or system is said to be *conserved*.

On the basis of your results above, describe how you can tell if the momentum of an object or system is conserved by inspecting the free-body diagram for that object or system.

D. Consider the following student discussion about the above experiment.

Student 1: *"This experiment is just like Experiment 100. The momentum of glider C is the same before and after the collision - only the direction of motion is different."*

Student 2: *"Right. Glider D has no momentum afterwards, just like glider B100. So the momentum of the system is conserved."*

Describe the *error* in each statement.