Physics 3210

Week 2 clicker questions
What is the Lagrangian of a pendulum (mass $m$, length $l$)? Assume the potential energy is zero when $\theta$ is zero.

A. $\mathcal{L}(\theta, \dot{\theta}, t) = \frac{1}{2} m l^2 \dot{\theta}^2 - mg\ell (1 - \cos \theta)$

B. $\mathcal{L}(\theta, \dot{\theta}, t) = \frac{1}{2} m l^2 \dot{\theta}^2 + mg\ell (1 - \cos \theta)$

C. $\mathcal{L}(\theta, \dot{\theta}, t) = \frac{1}{2} m \dot{\theta}^2 + mg \ell (1 - \cos \theta)$

D. $\mathcal{L}(\theta, \dot{\theta}, t) = \frac{1}{2} m \dot{\theta}^2 - mg \ell (1 - \cos \theta)$
For which of these systems could you use Lagrange’s equations of motion?

1. A double pendulum: a pendulum (mass m, length l) has a second pendulum (mass m, length l) connected to its bob.
2. A projectile moves in two dimensions with gravity and air resistance.
3. A bead slides without friction on a circular, rotating wire.

A. 1 only
B. 2 only
C. 3 only
D. 1 and 2
E. 1 and 3
What would be a good choice of generalized coordinates for the double pendulum? (Assume the pendula are constrained to move in the x-y plane.)

A. the Cartesian coordinates of the bob positions: $x_1, y_1$ (first bob) and $x_2, y_2$ (second bob)
B. the Cartesian coordinates of the first bob: $x_1, y_1$ and the Cartesian coordinates of the second bob, treating the first bob as the origin: $x'_2, y'_2$
C. the angles made between each pendulum rod and the vertical: $\theta_1$, (first bob) $\theta_2$ (second bob)
D. the angles made between a line drawn from each pendulum bob to the pivot and the vertical: $\alpha_1$, (first bob) $\alpha_2$ (second bob)
What is the kinetic energy of a particle sliding on the parabola $y=x^2$?

A. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2$

B. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2(1+x)$

C. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2(1+4x)$

D. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2(1+2x^2)$

E. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2(1+4x^2)$
What is the generalized force of a particle sliding on the parabola $y = x^2$?

A. $-mgx$

B. $-2mgx$

C. $4mx\dddot{x} - 2mgx$

D. $2mx\dddot{x}^2 - mgx$

E. $4mx^2\dddot{x} - 2mgx$
A bead of mass \( m \) slides on a circular wire of radius \( R \). The wire rotates about a vertical axis with angular velocity \( \Omega \). What is the kinetic energy of the bead?

A. \( T(\theta, \dot{\theta}, t) = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\Omega^2 \)

B. \( T(\theta, \dot{\theta}, t) = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\cos^2\theta\Omega^2 \)

C. \( T(\theta, \dot{\theta}, t) = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\sin^2\theta\Omega^2 \)

D. \( T(\theta, \dot{\theta}, t) = \frac{1}{2}mR^2\dot{\theta}^2 - \frac{1}{2}mR^2\Omega^2 \)

E. \( T(\theta, \dot{\theta}, t) = \frac{1}{2}mR^2(\dot{\theta} + \Omega\cos\theta)^2 \)
A bead of mass $m$ slides on a circular wire of radius $R$. The wire rotates about a vertical axis with angular velocity $\Omega$. When the bead is at angle $\theta$, how high is the bead above the lowest point of the wire?

A. $h = R \cos \theta$

B. $h = R \sin \theta$

C. $h = R (\sin \theta - \cos \theta)$

D. $h = R (1 - \sin \theta)$

E. $h = R (1 - \cos \theta)$
A bead of mass \( m \) slides on a circular wire of radius \( R \). The wire rotates about a vertical axis with angular velocity \( \Omega \). The equation of motion of the bead is

\[
\ddot{\theta} + \frac{g}{R} \sin \theta - \Omega^2 \sin \theta \cos \theta = 0
\]

What are the equilibrium value(s) of \( \theta \)?

A. \( \theta = 0 \)

B. \( \theta = \cos^{-1} \left( \frac{g}{R\Omega^2} \right) \)

C. \( \theta = \sin^{-1} \left( \frac{g}{R\Omega^2} \right) \)

D. \( \theta = 0 \) and \( \theta = \cos^{-1} \left( \frac{g}{R\Omega^2} \right) \)

E. \( \theta = 0 \) and \( \theta = \sin^{-1} \left( \frac{g}{R\Omega^2} \right) \)
A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity $\Omega$. The equation of motion for small motions about the equilibrium $\theta_0 = \cos^{-1}\left(\frac{g}{R\Omega^2}\right)$ is

$$\ddot{\theta} + \Omega^2 \sin^2 \theta_0 \, \delta \theta = 0$$

What is the oscillation frequency of the bead?

A. $\omega = \Omega$

B. $\omega = \Omega \sin \theta$

C. $\omega = \Omega \sin \theta_0$

D. $\omega = \Omega^2$

E. $\omega = \Omega^2 \sin^2 \theta_0$
Which of these constraints is holonomic?

1. A particle is constrained to slide on the inside of a sphere.
2. A disk rolls without slipping down an inclined plane (in one dimension).
3. A disk rolls without slipping on a table (in two dimensions).
4. A moving car is constrained to obey the speed limit.

A. None
B. Only one
C. Exactly two
D. Exactly three
E. All four
A particle of mass $m$ slides on the outside of a cylinder of radius $a$. A good choice of generalized coordinates is $(r, \theta)$. What is the constraint equation?

A. $f(r, \theta) = r$
B. $f(r, \theta) = a$
C. $f(r, \theta) = r + a$
D. $f(r, \theta) = r - a$
A particle of mass \( m \) slides on the outside of a cylinder of radius \( a \). What is the force of constraint (in the radial direction) when \( \theta=0 \)?

A. \( Q_r(\theta=0) = mg \)
B. \( Q_r(\theta=0) = mg/2 \)
C. \( Q_r(\theta=0) = 0 \)
D. \( Q_r(\theta=0) = -mg/2 \)
E. \( Q_r(\theta=0) = -mg \)
A particle of mass m slides on the outside of a cylinder of radius a. What happens to the (magnitude of the) force of constraint as \( \theta \) increases?

A. The constraint force increases.
B. The constraint force decreases.
C. The constraint force is constant.
A particle of mass \( m \) slides on the outside of a cylinder of radius \( a \). What are the kinetic and potential energy of the particle?

A. \( T = \frac{1}{2} mr^2 \dot{\theta}^2 \)
   \( U = mgr \cos \theta \)

B. \( T = \frac{1}{2} mr^2 \dot{\theta}^2 \)
   \( U = mgr \sin \theta \)

C. \( T = \frac{1}{2} m (r^2 + \dot{\theta}^2) \)
   \( U = mgr \sin \theta \)

D. \( T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) \)
   \( U = mgr \cos \theta \)

E. \( T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) \)
   \( U = mgr \cos \theta \)
A particle of mass \( m \) slides on the outside of a cylinder of radius \( a \). What condition must be satisfied by the force of constraint at the point (angle \( \theta_0 \)) where the particle leaves the cylinder?

A. \( Q_r = mg \)
B. \( Q_r = mg \sin \theta_0 \)
C. \( Q_r = 0 \)
D. \( Q_r = -mg \)
E. \( Q_r = -mg \sin \theta_0 \)